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A STUDY ON RELATIONS and THEIR PROPERTIES

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Abstract

This paper deals with the study on properties of relations. Special attention given to difference between symmetric and antisymmetric relations.

Keywords: Antisymmetric, Reflexive, Relation, Symmetric, Transitive.

Introduction

The most direct way to express a relationship between elements of two sets is to use ordered pairs made up of two related elements. For this reason, sets of ordered pairs are called binary relations. In this section we introduce the basic terminology used to describe binary relations.

Relation (binary relation)

Let A and B be sets. A binary relation from A to B is a subset of A×B.

In other words, a binary relation from A to B is asset of R of ordered pairs where the first element of each ordered pair comes from A and the second element comes from B. We use the notation a R b to denote that (a, b)∈ R. Moreover, when (a, b) belongs to R, a is said to be related to b by R.

Example: Let A be the set of all cities, and let B be the set of the states in India. Define the relation R by specifying that (a, b) belongs to R if city ais in state b. For instance, (Trivandrum, Kerala), (Kanyakumari, Tamil Nadu), (Bangalore, Karnataka), (Jaipur, Rajasthan), (Mumbai, Maharashtra), and (Srinagar, Jammu &Kashmir) are in R.

Relations on a set

A relation on the set A is a relation from A to A.

In other words, a relation on a set of A×A.

Example: Let A be the set {2,4,5,6}. Which ordered pair are in the relation R = {(a, b): a divides b}?

Solution: A×A = {(2, 2), (2, 4), (2, 5), (2, 6), (4, 2), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 5), (5, 6), (6, 2), (6, 4), (6, 5), (6, 6)}

R = {(a, b): a divides b}

= {(2, 2), (2, 4), (2, 6), (4, 4), (5, 5), (6, 6)}.

Properties of Relations

1, Reflexive relation

A relation R on a set A is called reflexive if (a, a)∈ R for every element a∈ A.

An example of reflexive relation is the relation "is equal to" on the set of real numbers, since every real number is equal to itself.

2, Irreflexive relation

A relation R on the set A is irreflexive if for every a∈ A, (a, a)∉ R. That is, R is irreflexive if no element in A is related to itself.

An example of irreflexive relation is the relation is the" greater than" relation on the real numbers.

The binary relation the product of x and y is even is reflexive on the set of even numbers, irreflexive on the set of odd numbers, and neither reflexive nor irreflexive on the set of natural numbers.

Result

A relation is irreflexive if and only if its compliment is reflexive.

3, Symmetric relation

A relation R on a set A is called symmetric if (b, a)∈ R whenever (a, b)∈R, for all a, b∈ A.

An example of symmetric relation is the relation "is equal to" because if a = b is true then b = a is also true.

If R^T represent the converse of R, then R is symmetric if and only if R = R^T.

4, Asymmetric relation

A relation R is called asymmetric if (a, b) ∈ R implies that (b, a)∉ R.

An example of an asymmetric relation is the "less than" relation between real number. If x< y then necessarily y is note less than x.

Result

Asymmetry is not the same thing as "not symmetric"; the less than-or-equal relation is an example of a relation that neither symmetric nor asymmetric.

5, Antisymmetric relation

A relation R on a set A such that for all a, b ∈ A, if (a, b) ∈ R *and* (b, a) ∈ R, then a = b is called antisymmetric.

An example of antisymmetric is: for a relation "is divisible by" which is the relation for ordered pairs in the set of integers. For relation, R, an ordered pair (x, y) can be found where x and y are whole numbers and x is divisible by y.

Result

If a relation is not symmetric that does not mean it is antisymmetric.

6, Transitive relation

A relation R on a set A is called transitive if whenever (a, b) ∈ R and (b, c) ∈ R then (a, c) ∈ R, for all a, b, c ∈ A.

"Is greater than" and "is equal to" are transitive relation on various sets, for instance, the set of real numbers or the set of natural numbers: whenever x > y and y > z, then also x > z, and whenever x = y and y = z, then also x = z.

Result

The complement of a transitive relation need not be transitive. For instance, while "equal to" is transitive, "not equal to" is only transitive on sets with at most one element.

A transitive relation is asymmetric if and only if it is irreflexive.

A transitive relation need not be reflexive.

Important Points

- Symmetric and antisymmetric relation are not opposite because a relation R can contain both the properties or may not
- A relation is asymmetric if and only if it is both antisymmetric and irreflexive.
- Number of Reflexive relations on a set with "n" elements is $2^{n(n-1)}$.
- Number of Symmetric relations on a set with "n" elements is $2^{n(n+1)/2.}$
- Number of Antisymmetric relations on a set with "n" elements is $2^{n}.3^{n(n-1)/2}$.
- Number of Asymmetric relations on a set with "n" elements is $3^{n(n-1)/2}$.
- Number of Irreflexive relations on a set with "n" elements is $2^{n(n-1)}$.

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