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Impact of modulation on the beginning of convection in a couple stress fluid saturated permeable layer with g-jitter

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Abstract:

The stability of a liquid soaked permeable layer warmed from below is inspected for the situation of a time dependent gravity by making a linear stability analysis. The perturbation method is used to work out the critical Rayleigh and wave numbers for small amplitude gravity modulation. The correction Rayleigh number is estimated as a function of frequency of the modulation, viscosity ratio, Prandtl number, porous parameter and couple stress parameter. We found that it is possible to progress or postpone the onset of convection in systems with couple stress fluid by proper regulating of the frequency of the gravity modulation. The impact of various parameters on the stability of the system is brought out.

Key words: Convection, Couple stress fluid, Porous medium, , Gravity, Linear stability analysis, Modulation.

1. Introduction

The natural convection issue in a horizontal liquid soaked permeable layer driven by buoyancy and warmed from below and cooled from above has been concentrated widely throughout the long term, both hypothetically and tentatively as a result of its specialized and geophysical applications. A great survey of these investigations has been accounted for in a new book by Nield and Bejan (1999).

There have been numerous examinations concerning how a time-dependent boundary temperature affects the onset of Rayleigh-Benard convection. The majority of the discoveries pertinent to these issues have been surveyed by Davis (1976). In case of small amplitude of the surface temperature, a linear stability analysis has been carried out by Venezian (1969).

The convection of a liquid layer in the presence of complex body forces is a significant kind of issue in classical hydrodynamics. A physically important class of problems involves convection in a fluid layer in the presence of complex body forces, such forces can arise in a number of ways. For example, when a system with density gradient is subjected to vibrations, the resulting buoyancy forces, which are produced by the interaction of the density gradient with the gravitational field, have complex spatio-temporal structures. The time-dependent gravitational field is of interest in space laboratory experiments, in areas of crystal growth and different applications. It is additionally of significance in the huge scope convection of climate.

The effect of gravity modulation on a convectively stable configuration can significantly influence the stability of a system by increasing or decreasing its susceptibility to convection. In general, a distribution of stratifying agency that is convectively stable under constant gravity field is introduced. Certain combinations of thermal gradients, physical properties and modulation parameters may led to parametric resonance and, hence, to the instability of the system.

Gershuni et al., (1970) and Grasho and Sani (1970) have studied the effect of gravity modulation on the stability of a heated fluid layer. Clever et al., (1993) studied the problem of two-dimensional oscillatory convection in a gravitationally modulated fluid layer. Farooq and Homsy (1986) studied linear and non-linear convection in a vertical slot under gravity modulation. Malashetty and Padmavathi (1997a, b) studied the effect of small amplitude gravity modulation on the onset of convection in a fluid and fluid saturated porous layer.

It is presently basic information that liquids with suspended particles are a working medium in numerous a application. The fluids most often do not subscribe to a Newtonian description and the advent of micromomentum field theories in such a situation threw open new fields of application. Rayleigh-Benard convection in fluids where stress is non-linearly proportional to velocity gradient is studied by few researchers (Siddheshwar and Sri Krishana 2002, 2003). The motivation behind the current part accordingly, is to analyze the effect of small amplitudes of gravity modulation on the onset of Rayleigh-Benard convection in a couple stress fluid saturated porous medium considering the Brinkman flow model with the effective viscosity bigger than the fluid viscosity.

2. Mathematical formulation

Consider a Boussinesq couple stress fluid saturated porous medium restricted between two infinite horizontal walls situated at $z = 0$ and $z = d$, which is heated from below and cooled from above, under the influence of a periodically varying vertical gravity field.

$$\mathbf{g} = g_0(1+\varepsilon\cos\Omega t)\hat{\mathbf{k}} \;, \tag{1}$$

where g_0 is the mean gravity, ε is the small amplitude of gravity modulation, Ω is the frequency, t is the time and $\hat{\mathbf{k}}$ is the unit vector in the vertical direction.

With the Boussinesq approximations, the governing equations for a couple stress fluid flow through a permeable medium are (Stokes, 1966)

$$\nabla\cdot\mathbf{q} = 0, \tag{2}$$

$$\frac{1}{\delta}\frac{\partial\mathbf{q}}{\partial t} + \frac{1}{\delta^2}(\mathbf{q}\cdot\nabla)\mathbf{q} = -\frac{1}{\rho_R}\nabla p + \frac{\rho}{\rho_R}g_0\,(1+\varepsilon\cos\Omega\mathrm{t})\hat{\mathbf{k}} - \frac{\mu_f}{\rho_R k}\mathbf{q} + \frac{\mu_e}{\rho_R}\nabla^2\mathbf{q} - \frac{\eta}{\rho_R}\nabla^4\mathbf{q}\,, \tag{3}$$

$$\gamma\frac{\partial T}{\partial t} + (\mathbf{q}\cdot\nabla)T = \kappa_T\nabla^2 T\;, \tag{4}$$

where $\mathbf{q} = (\mathrm{u}, \mathrm{v}, \mathrm{w})$ is the velocity, T is the temperature, p is the pressure, $\mathbf{g}$ is the acceleration due to gravity, μ_e is the effective viscosity, μ_f is the viscosity of the fluid, η is the material constant, δ is the porosity, γ is the ratio of specific heats and κ_T is the thermal diffusivity, ρ_R is the reference density, β is the coefficient of thermal expansion.

2.1 Basic state

In the undisturbed state, the fluid is quiescent and is depicted by

$$\mathbf{q} = \mathbf{q_b} = (0,0,0),\ T = T_b(z),\ p = p_b(z),\ \rho = \rho_b(z). \tag{5}$$

This clearly differs from the one in the time dependent wall temperature case. The basic state temperature T_b satisfies the equation $\nabla^2 T_b = 0$ and the pressure p_b balances the buoyancy force.

2.2 Linear stability analysis

The analysis of Venezian (1969) has been followed; hence the non-dimensional linearized perturbation equations are

$$\left(\frac{1}{\Pr}\frac{\partial}{\partial t} - M\nabla^2 + F + C\nabla^4\right)\nabla^2 w = R_T(1+\varepsilon\cos\omega t)\nabla_1^2 T\,, \tag{6}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T = w\,, \tag{7}$$

In the above equations the following dimensionless parameters are appeared

$\Pr = \dfrac{\nu}{\kappa_T}$ (Prandtl number), $R = \dfrac{\beta_1 g \Delta T d^3}{\nu\kappa_T}$ (Rayleigh number), $F = \dfrac{d^2}{k}$ (porous parameter), $M = \dfrac{\mu_e}{\mu_f}$ (viscosity ratio), $C = \dfrac{\eta}{\mu_f d^2}$ (couple stress parameter).

Equations (6) and (7) are combined to obtain a single equation in terms of the perturbed temperature T as

$$\left(\frac{1}{\Pr}\frac{\partial}{\partial t} - M\nabla^2 + F + C\nabla^4\right)\left(\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2 T = R_T\,(1+\varepsilon\cos\omega t)\nabla_1^2 T\,. \tag{8}$$

The boundary conditions can also be expressed in terms of T by making use of equation (7), which requires $(\partial^2 T/\partial z^2) = 0$ at the boundaries. Thus equation (8) has to be solved subjected to the homogeneous boundary conditions

$$T = \frac{\partial^2 T}{\partial z^2} = \frac{\partial^4 T}{\partial z^4} = \ldots = 0 \ \text{ at } \ z = 0,\ 1. \tag{9}$$

Following the approach of Venezian (1969), we obtain R_{T0} and R_{T2} in the form

$$R_{T0}^{(n)} = \frac{(\alpha^2 + n^2\pi^2)^2}{\alpha^2}\left[F + M(\alpha^2 + n^2\pi^2) + C(\alpha^2 + n^2\pi^2)^2\right]. \tag{10}$$

For a fixed wave number α, the least eigenvalue occurs for $n = 1$. R_{T0} assumes the minimum value R_{T0c}, when $\alpha = \alpha_c$, where α_c is given by

$$3C(\alpha_c^2)^3 + (2M + 5C\pi^2)(\alpha_c^2)^2 + (F + M\pi^2 + C\pi^4)\alpha_c^2 - (F + M\pi^2 + C\pi^4)\pi^2 = 0. \tag{11}$$

$$R_{T2} = -\frac{\alpha^2 R_0^2}{2}\sum_{n=1}^{\infty}\left(D_n / G_n\right). \tag{12}$$

where $L(\omega, n) = B_1 + B_2$, with

$$B_1 = (\alpha^2 + n^2\pi^2)\left((\omega^2/\mathrm{Pr}) - F(\alpha^2 + n^2\pi^2)\right) - (\alpha^2 + n^2\pi^2)^3(M + C(\alpha^2 + n^2\pi^2) + R_{T0}\alpha^2,$$

$$B_2 = \omega\left((\alpha^2 + n^2\pi^2)^2\left(M + (1\backslash \mathrm{Pr}) + C(\alpha^2 + n^2\pi^2)\right) + F(\alpha^2 + n^2\pi^2)\right).$$

where $D_n = \dfrac{\left[L(\omega,n) + L^*(\omega,n)\right]}{2} = B_1$ and $G_n = |L(\omega,n)|^2 = B_1^2 + B_2^2$.

3. Minimum Rayleigh number for convection

The value of the thermal Rayleigh number R_T obtained by this procedure is the eigen value corresponding to the eigen function w which, though oscillating remains bounded in time. Since R_T is a function of the horizontal wave number α and amplitude of perturbation ε, we may take

$$R_T(a,\varepsilon) = R_{T0}(\alpha) + \varepsilon^2 R_{T2}(\alpha) + \ldots, \tag{13}$$

$$\alpha = \alpha_0 + \varepsilon^2\alpha_2 + \ldots. \tag{14}$$

The critical value of the Rayleigh number R_T is computed upto $\mathrm{O}(\varepsilon^2)$ by evaluating R_{T0} and R_{T2} at $\alpha = \alpha_0$ given by equation (11). It is only when one wishes to evaluate R_{T4} that α_2 must be taken into account.

4. Results and discussion

The impact of time dependent body force on the beginning of convection in a couple stress fluid-saturated porous medium is investigated. A perturbation technique is used to find the critical thermal Rayleigh number as a function of frequency of the modulation, couple stress parameter, viscosity ratio, porous parameter and Prandtl number.

The examination introduced in this paper based on the assumption that the amplitude of the modulating gravity is small. We presently talk about the outcomes showed up at in the paper.

The impact of Prandtl number Pr on the stability of the system in the presence of gravity modulation is shown in figure 1. It is found that the effect of increasing Prandtl number is to minimize the impact of gravity regulation.

Figure 2 is the plot of R_{T2c} with ω for different values of porous parameter F. We find that an increase in the value of F also increases the value of R_{T2c} negatively, indicating that the effect porous parameter F advances the beginning of convection.

The impact of viscosity ratio M on the stability of the couple stress fluid saturated system in the presence of gravity modulation is shown in figure 3. We notice that, an increase in the value of M increases the value of R_{T2c} negatively, indicating that the effect is destabilizing one.

Figure 4 represents the plot of R_{T2c} with ω for different values of the couple stress parameter C in case of gravity modulation. We observe that R_{T2c} is negative over the whole range of frequencies in presence of gravity modulation, indicating that the gravity modulation is destabilizing in couple stress fluid saturated systems. We also find that an increase in the value of C, increases the value of R_{T2c} negatively, indicating that the effect of increasing couple stress parameter C is to progressing the onset of convection.

It is found that the impact of gravity modulation vanishes for large frequencies. The results of the present study are expected to be useful in controlling convection by time dependent body force in systems with couple stress fluid saturated porous medium. Finally we conclude that the low-frequency gravity modulation has a significant effect on the stability of the system. The consequences of the investigation illuminate outside methods for controlling convection, either progressing or postponing convection by regulation.

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Figures:

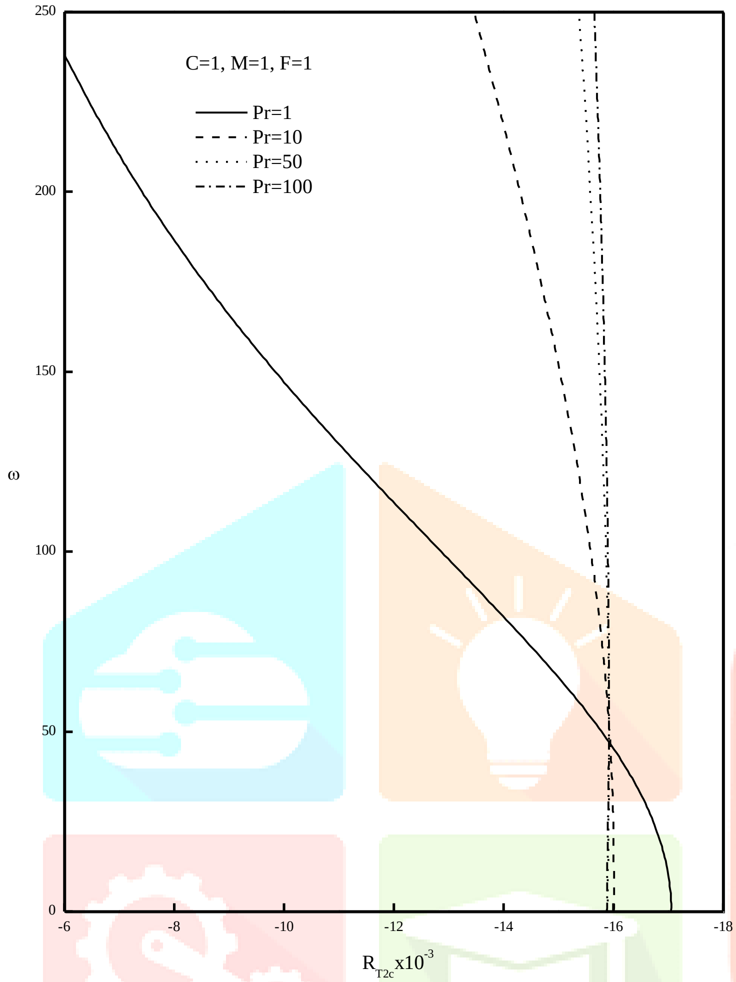

Fig.1. Impact of Pr on R_{T2c} with ω.

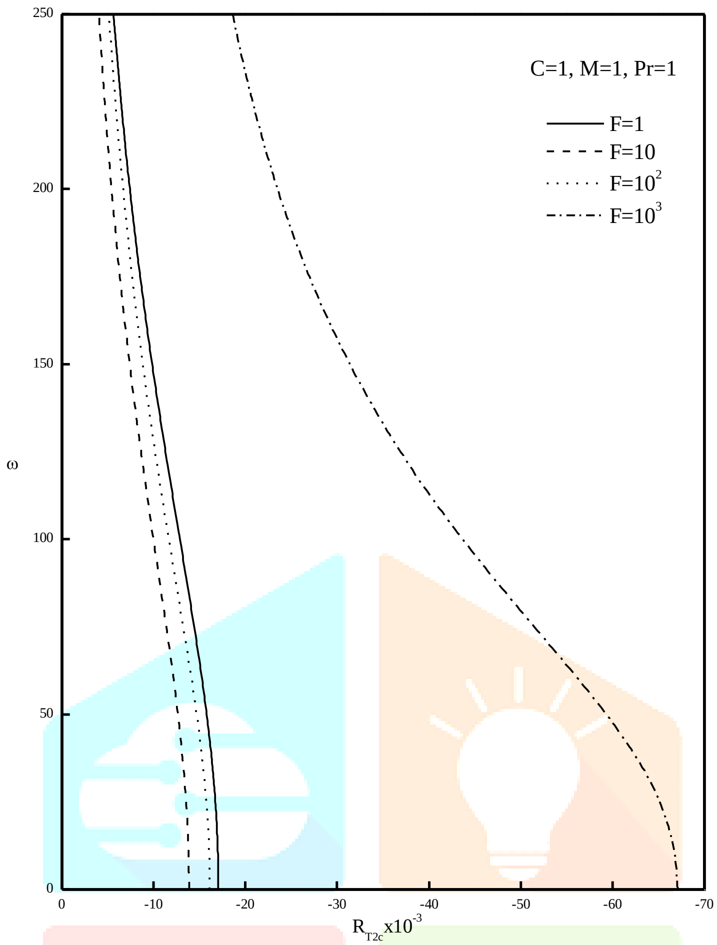

Fig.2. Impat of F on R_{T2c} with ω.

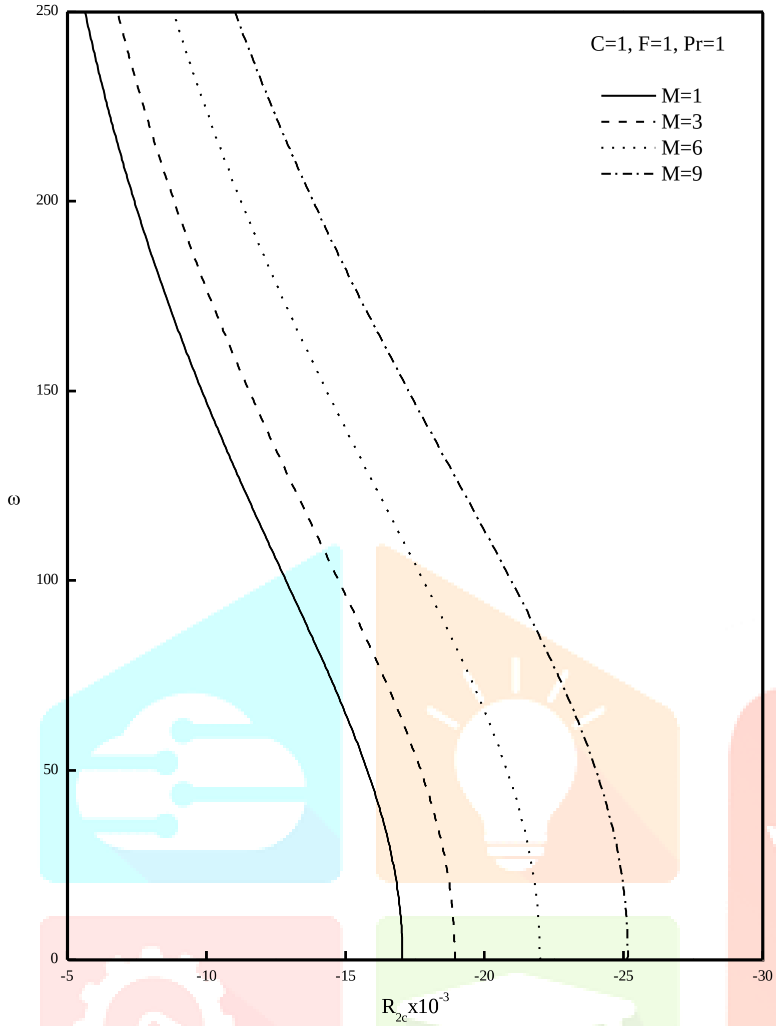

Fig.3. Impact of M on R_{T2c} with ω.

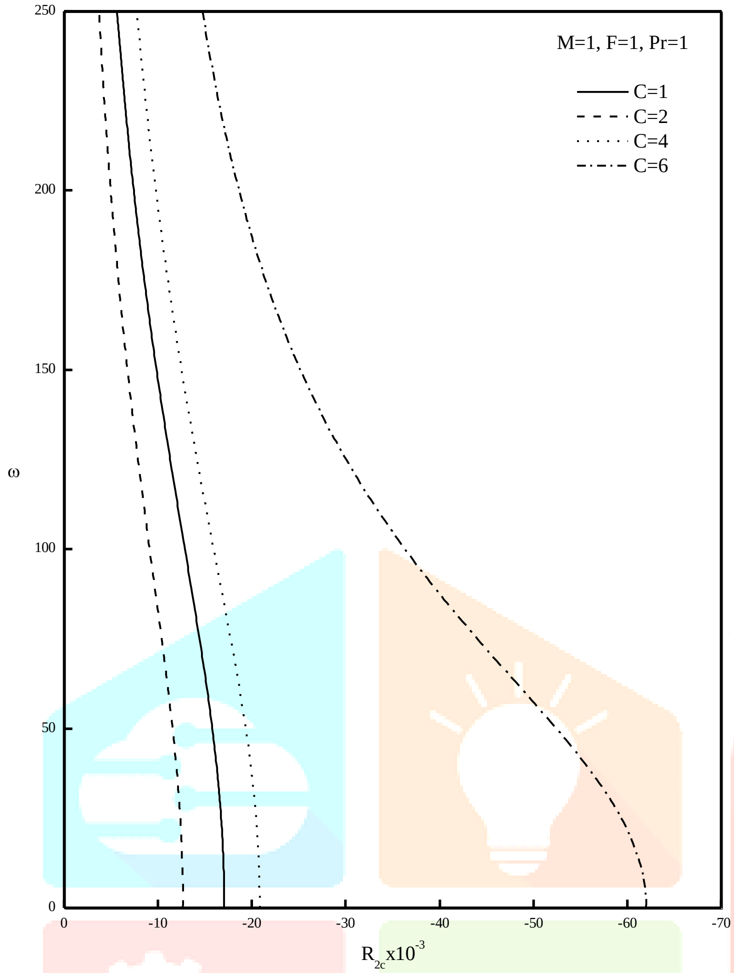

Fig.4. Impact of c on R_{T2c} with ω.