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Neighborhood Prime Labeling Of Some Graphs

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Abstract:Let G be the graph with n vertices, a bijective function $f: V(G) \rightarrow \{1,2, \dots, n\}$ is said to be a neighborhood prime labeling if for every vertex $v \in V(G)$, $\deg(v) > 1$. $\gcd\{f(p) | p \in N(w)\} = 1$. A graph which admits neighborhood prime labeling is called neighborhood prime graph.

Here we discuss about the coconut tree graph, the Jewel graph, $K_{1,3} * K_{1,n}$ graph, The Jelly fish graph which admits an NPL.

Keywords: Neighborhood prime labeling , Neighborhood prime graph, $K_{1,3} * K_{1,n}$ graph , Adjacent vertices

I. Introduction: Roger Entringer introduced the concept of prime labeling and was introduced in 1980' s by Tout et al [6] which paved way for many researches in this zone. Motivated by the study of prime labeling, S.K.Patel and N.Shrimali in [3] introduced the concept of neighbourhood prime labeling in 2015, in which they have recognized the enough condition for a graph to admit neighbourhood prime labeling and proved that paths, cycles, helm, closed helm and flower have neighbourhood prime labeling.

II. BASIC TERMINOLOGY OF NEIGHBOOURHOOD PRIME LABELING OF SOME GRAPHS

Definition1.1: Let G be the graph with n vertices, a bijective function $f: V(G) \rightarrow \{1,2, \dots, n\}$ is said to be a neighborhood prime labeling if for every vertex $v \in V(G)$, $\deg(v) > 1$.$\gcd\{f(p) | p \in N(w)\} = 1$. A graph which admits neighborhood prime labeling is called neighborhood prime graph.

Definition1.2: The double star graph $(K_{1,n,n})$ is a tree getting from the star graph $K_{1,n}$ by attaching new pendent edges of the exiting n pendent vertices which consisting total $2n + 1$ vertices and $2n$ edges.

Definition1.3[3]: Coconut tree graph is obtained by identifying the middle vertex of $K_{1,m}$ with a pendent vertex of the pathP_n.

Definition1.4[6]: The Jewel graph J_n is the graph with vertex set $V(J_n) = \{u, v, x, y, u_j | 1 \leq j \leq n\}$ and the edge set $E(J_n) = \{ux, uy, vx, vy, xy, uu_j, vv_j | 1 \leq j \leq n\}$

Definition1.5[2]: Let $G = K_{1,3} * K_{1,n}$ be the graph obtained from $K_{1,3}$ by joining root of a star $K_{1,n}$ at each pendent vertex of $K_{1,3.}$

Definition1.5[5]: The Jelly fish graph $J(n, m)$ is obtained from a 4-cycle $(v_1, \dots, v_4)$collected with an edge $v_1 v_3$ and affixing$n$ pendent edges to v_2 and m pendent edges to v_4.

III. NEIGHBOOURHOOD PRIME LABELING OF SOME GRAPHS

Theorem 3.1: Every Double star graph $(K_{1,n,n})$ is a neighboourhood prime labeling.

Proof: Let $G = (V(G), E(G))$ be a graph of double star graph $(K_{1,n,n})$ with

vertex set $\{v, v_i, u_i; i \in 1,2, \dots n\}$ obtained from the suppose $\{v, v_1, v_2, \dots, v_n\}$ and $\{u_1, u_2, \dots, u_n\}$ be the vertices and $\{e_1, e_2, \dots, e_{n-1}\}$ be the edges which are denoted in figure.

Note that the path consist n vertices and $n - 1$ edges.

If $G = (K_{1,n,n})$ then total number of vertices $2n + 1$ and total number of edges $2n$.

Now we define a vertex labeling $f: V(G) \rightarrow \{1,2, \dots, p\}$ as follows:

$f(v) = 1$

$f(v_j) = j + 1; 1 \leq j \leq n$

$f(u_j) = n + j + 1; 1 \leq j \leq n$

Here the vertices which are greater than one are

$f(v) = 1$,$f(v_1) = 2, f(v_2) = 3f(v_3) = 4, f(v_4) = 5, f(v_5) = 6$

$f(u_1) = 7, f(u_2) = 8, f(u_3) = 9, f(u_4) = 10, f(u_5) = 11$

Case 1:If$w = v$ then $\gcd\{f(p)|p \in N(w)\} = 1$, since $f(v) = 1$ such that which is relatively prime to remaining numbers.

Case2: If $w = v_j$; $1 \leq j \leq n$ then $\gcd\{f(p)|p \in N(w)\} = 1$

Since $f(v) = 1$ and $f(u_j) = n + j + 1: 1 \leq j \leq n$ such that which is relatively prime to remaining numbers.

Then the double star graph admits NPL. Hence it is Neighborhood prime graph.

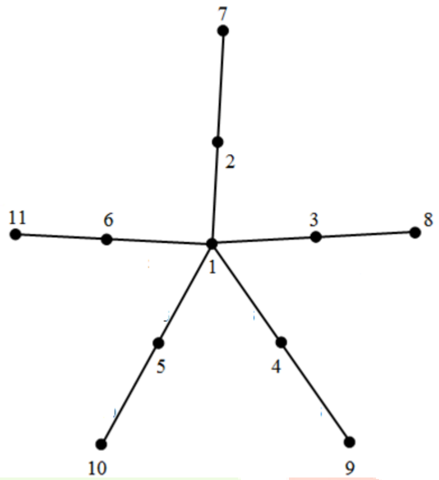

Fig 1:NPL of double star graph $(K_{1,n,n})$

Theorem 3.2: The coconut tree graph $CT_{m,n}$ admits a neighboourhood prime labeling.

Proof: Let $V = \{v_i|1 \leq i \leq m\} \cup \{u_j|1 \leq j \leq n\}$ be the vertex set of coconut tree where v_i are the vertices of the path P_m and andu_j are the n new pendent vertices at an end vertex of the path P_m.

Let $E = \{e_i = v_i v_{i+1}|1 \leq i \leq m\} \cup \{e_{ij} = v_i u_j|i = m, 1 \leq j \leq n\}$ be the edge set of coconut tree.Here the coconut tree has $|V(G)| = m + n$ vertices and $|E(G)| = m + n - 1$ edges.

An Injective function$f: V(G) \to \{1,3, \ldots ,2(m + n) - 1\}$ such that

Case(i): $m \equiv 1(mod2)$

$$f(v_i) = \begin{cases} i, ifiisodd \\ m + i, ifiiseven \end{cases}$$

$f(u_j) = 2m + 2j - 1, 1 \leq j \leq n$

The vertices with degree greater than one are $N(u_i) \supset \{u_{i-1}, u_{i+1}\}$where $2 \leq i \leq m - 2$.

Where $f(u_{i-1}) = i - 1$ &$f(u_{i+1}) = i + 1$

Then the $\gcd\{f(p)|p \in N(u_i)\} = 1$

Now $N(u_m) \supset \{u_{m-1}, v_i\}$ here $1 \leq i \leq m$

Here $f(u_{m-1}) = 5$ and $f(u_j) = 2m + 2j - 1; 1 \leq j \leq n$

Then the $\gcd\{f(p)|p \in N(u_m)\} = 1$

Then the coconut tree graph $CT_{m,n}$ admits NPL. Hence it is Neighborhood prime graph.

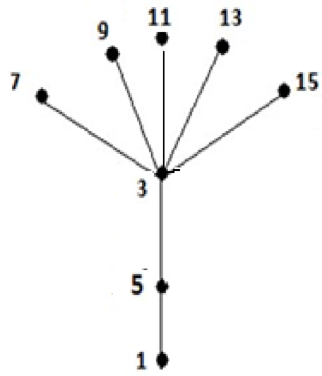

Fig 2:NPL of $CT_{3,5}$

Theorem 3.3: Let G be the Jewel graph admits an NPL.

Proof: Suppose G^* be the graph defined by

$G^* = G - \{uw_j | 1 \le j \le m-2\}$. Here $V(G^*) = \{u, v, x, y, w_j | 1 < j \le m\}$ and

$E(G^*) = \{ux, vx, uy, vy, uw_{m-1}, uw_m, vw_j | 1 \le j \le m\}$. Then $|V(G^*)| = m + 4$ and

$|E(G^*)| = m + 6$. Let $f: V(G^*) \to \{1,2, \ldots ,2m + 9,2(m + 6)\}$ is defined as follows:

$f(u) = 1$ $\quad f(v) = 3$

$f(w_j) = 2m + 9 - 2j; 1 \le j \le m - 2$

$f(w_{m-1}) = 9$ $\quad f(w_m) = 5$

$f(x) = 2(n + 6)\, f(y) = 2n + 9$

The vertices with degree grater than one are

$N(x) \supset \{f(u), f(v)\}$

Here $f(u) = 1$ and $f(v) = 3$.

Then the the gcd$\{f(p) | p \in N(x)\} = 1$

Now consider $N(y) = \{f(u), f(v)\}$

Here $f(u) = 1$ and $f(v) = 3$.

Then the gcd$\{f(p) | p \in N(y)\} = 1$

Now, $N(u) \supset \{f(x), f(y), f(w_{n-1}), f(w_n)\}$

Here $f(x) = 2m + 12$ $\quad f(y) = 2m + 9$

$f(w_{m-1}) = 9$ $\quad f(w_m) = 5$

Then the gcd$\{f(p) | p \in N(u)\} = 1$

Now, $N(v) \supset \{f(x), f(y), f(w_{m-1}), f(w_m), f(w_j)\}; 1 \le j \le m - 2$

Here $f(w_j) = 2m + 9 - 2j; 1 \le j \le m - 2$

$f(x) = 2(m + 6)$ $\quad f(y) = 2m + 9$

$f(w_{m-1}) = 9$ $\quad f(w_m) = 5$

Then the gcd$\{f(p) | p \in N(w_j)\} = 1$

Now $N(w_{m-1}) \supset \{f(u), f(v)\}$

Here $f(u) = 1$ and $f(v) = 3$

Then the $\gcd\{f(p)|p \in N(w_{m-1})\} = 1$

Now $N(w_m) \supset \{f(u), f(v)\}$

Here $f(u) = 1$ and $f(v) = 3$

Then the $\gcd\{f(p)|p \in N(w_m)\} = 1$

Here note that either n is odd or n is even.

The Jewel graph which admits NPL. Hence it is neighborhood prime graph.

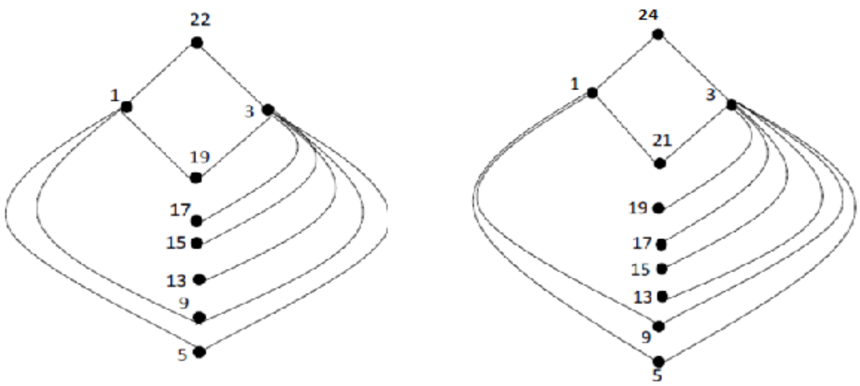

Fig 3: NPL of Jewel graph for odd and even numbers.

Theorem 3.4: The graph $k_{1,3} * K_{1,n}$ admits an NPL for all $n \geq 2$.

Proof: Let $G = k_{1,3} * K_{1,n}$ with $V(G) = \{x, u, v, w, v_j, w_j | 1 \leq j \leq n\}$ and

$E(G) = \{xu, xv, xw, uu_j vv_j, ww_j | 1 \leq j \leq n\}$.Hence $|V(G)| = 3n + 4$ and $|E(G)| = 3(n + 1)$

Define $f: V(G) \to \{1,2, \ldots, 6n + 7\}$ by

$f(u) = 1$ $f(v) = 3$ $f(w) = 2n + 5$

$f(x) = 4n + 7$ $f(u_j) = 6n + 9 - 2j; 1 \leq j \leq n$

$f(v_j) = 4n - 2j + 7; 1 \leq j \leq n$

$f(w_j) = 2n - 2j + 5; 1 \leq j \leq n$

The vertices with degree greater than one are

$N(u) \supset \{f(x), f(u_j)\}; 1 < j \leq n$

Here $f(x) = 4n + 7$ and $f(u_j) = 6n - 2j + 9$

Then the $\gcd\{f(p)|p \in N(u)\} = 1$

$N(v) \supset \{f(x), f(v_j)\}\,; 1 \leq j \leq n$

Here $(x) = 4n + 7$, $f(v_j) = 4n - 2j + 7$

Then the $\gcd\{f(p)|p \in N(v)\} = 1$

Now $N(w) \supset \{f(x), f(w_j)\}; 1 \leq j \leq n$

Here $(x) = 4n + 7$, $f(w_j) = 2n - 2j + 5$

Then the $\gcd\{f(p)|p \in N(w)\} = 1$

Now $N(x) \supset \{f(u), f(v), f(w)\}$

Here $f(u) = 1$ which is relatively prime to remaining numbers.

So, The graph $K_{1,3} * K_{1,n}$ admits NPL. Hence it is Neighborhood prime graph.

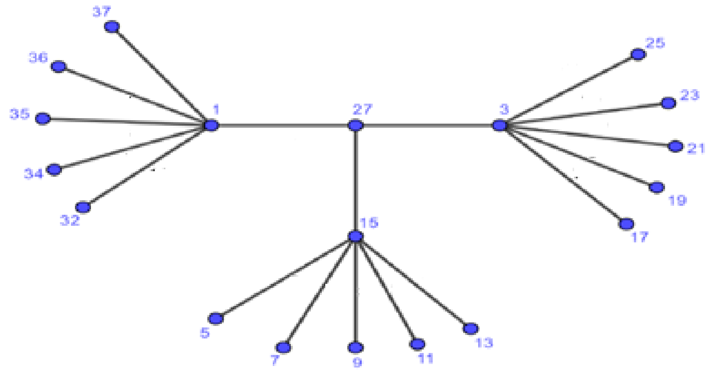

Fig 4:NPL of The graph $K_{1,3} * K_{1,n}$

Theorem 3.5: The Jelly fish $J(n, m)$ admits an NPL.

Proof: Let G be the Jelly fish$J(n, m)$ graph. Let $V(G) = \{u, v, x, y, u_i, v_j | 1 \leq i \leq n, 1 \leq j \leq m\}$

And $E(G) = \{xu, xv, yu, yv, xy\} \cup \{uu_i | 1 \leq i \leq n\} \cup \{vv_j | 1 \leq j \leq m\}$

Then $|V(G)| = n + m + 4$ and $|E(G)| = n + m + 5$

Define $f: V(G) \to \{1, 2, \ldots, 2(n + m + 4), 2n + 2m + 11\}$ as follows:

$f(u) = 2n + 2m + 11$ $\quad$ $f(v) = 2n + 2m + 7$

$f(x) = 2n + 1$ $\quad$ $f(y) = 2n + 3$

$f(u_i) = 2i - 1; 1 \leq i \leq n$

$f(v_j) = 2n + 2j + 3; 1 \leq j \leq m$

The vertices with degree greaterthan one as follows:

Now $N(u) \supset \{f(u_i), f(x), f(y)\}; 1 \leq i \leq n$

Here $f(u_i) = 2i - 1$ $\quad$ $f(x) = 2n + 1$ $\quad$ $f(y) = 2n + 3$

Then the $\gcd\{f(p) | p \in N(u)\} = 1$

Now $N(v) \supset \{f(v_j), f(x), f(y)\}; 1 \leq j \leq m$

$f(v_j) = 2n + 2j + 3$ $\quad$ $f(x) = 2n + 1$ $\quad$ $f(y) = 2n + 3$

Then the $\gcd\{f(p) | p \in N(v)\} = 1$

Now $N(x) \supset \{f(u), f(v), f(y)\}$

Here $f(u) = 2n + 2m + 11,$ $\quad$ $f(v) = 2n + 2m + 7$ $\quad$ $f(y) = 2n + 3$

Then the $\gcd\{f(p) | p \in N(x)\} = 1$

Now $N(y) \supset \{f(u), f(v), f(x)\}$

Here $f(u) = 2n + 2m + 11,$ $\quad$ $f(v) = 2n + 2m + 7$ $\quad$ $f(x) = 2n + 1$

Then the $\gcd\{f(p) | p \in N(y)\} = 1$

So, The Jelly fish graph $J(n, m)$ is NPL. Hence it is Neighborhood prime graph.

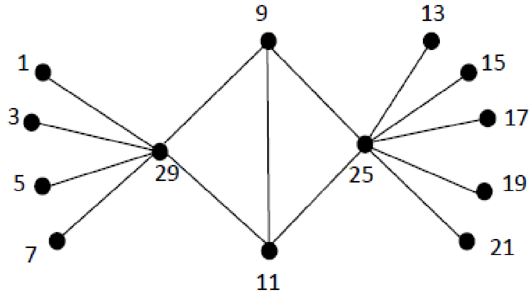

Fig 5:NPL of The Jelly fish graph $J(n,m)$.

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