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CONSTRUCTION THEORY OF MEASURES OF TOPOLOGICAL STRUCTURE

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Abstract : The main purpose in the paper is to investigate some characteristic properties of theory of measures based on construction model. It is being shown here that this approach is most appropriate in probability and functional analysis where classical approaches advanced by Gardner seems to be inadequate. The topological structure of measure theory is being critically examined in the light of its different applications.

Keywords : topological structure, Hausdorff space, Radon measure.

1. Introduction

In recent years, much work has been done on real-valued measures. The abstract theory due to Bourbaki comprises topological structure. It is being shown here that construction theory of measures of topological structure which is more extensive than classical theories.

Theorem 1. (Kisynski's Theorem). Let $\lambda : K(X) \to [0, \infty]$ be a Radon content on the Hausdorff space X. Then λ extends to a Radon measure μ which implies

$$\mu A = \sup\{\lambda k | k \subseteq A\}, A \text{ Borel} \qquad (1)$$

Theorem 2. (Riesztype Representation Theorem). Let X be a completely regular Hausdorff space and let C be a point-separating cane of non-negative continuous real valued functions on X which is closed under the operations "Δ" such that

$$f \wedge g = \min\{f, g\} \text{ and "\textbackslash" } f \setminus g = f - f \wedge g \qquad (2)$$

satisfying the stone's condition $f \Delta 1 \in C \forall f \in C$

Proof. Let the functional $T : C \to [0, \infty]$ satisfy the following conditions

$$T(f+g) = Tf + Tg \forall f, g \in C \quad (3)$$

$$Tf = \sup\{Tg | g \leq f, g \text{ bounded}, Tg < \infty\} \forall f \in C$$

$$\forall x \in X \exists f \in C : f(x) > 0, Tf < \infty$$

We obtain the necessary and sufficient conditions than there exists a Radon measure μ representing T.

$$\int f d\mu = Tf \forall f \in C$$

holds, provided

$$\forall Tf < \infty, \varepsilon > 0 \ \exists k \text{ compact}, \forall g \in C, g \leq f, g = 0 \text{ on } K.$$

The Radon measure μ is unique, when it exists it is locally finite.

(1) Sufficient Part : It is shown here that the set of function λ defined by

$$\lambda k = \inf\{Th | h \geq I_k, K \text{ compact}\}$$

is a Radon content.

The extension μ to a Radon measure is obtained by suitable application of Kisynski's theorem which satisfies the condition

$$\int f d\mu = Tf \text{ for all } f \in C$$

We show here that classical results are mere its corollaries. In particular, a positive linear functional defined on the continuous functions with compact support on a locally compact Hausdorff space is represented by a Radon measure. It is being derived here that a positive linear functional defined on all continuous functions of σ-compact. Locally Hausdorff space is represented by a Radon measure with compact support.

Theorem 3. Consider $M_+(X)$ provided with the topology of weak convergence. Let (μ_α) be a net on $M_+(X)$ and let $X_0 \subseteq X$. Then $\mu_\alpha(X) < \infty$ and the following conditions holds

(i) $\forall (G_k)_{k \subseteq X_0} \forall \in > 0, \exists k_1, k_2, ..., k_n$

(ii) $\min \mu_\alpha (X \setminus G_k) < \in$

Proof. Let $(G_k)_{k \subseteq X_0}$ denotes a family of open sets indexed by the compact subsets of X_0 such that $G_k \supseteq k$ for every compact subset k of X. Topose [5] derived some interesting results on compactness in spaces of measures. We extend using Prohorov's criteria for components on probability measures.

Let P is a set of probability measures on the Hausdorff space X and if P is uniformly tight $(\forall \varepsilon\ \exists K \text{ compact } \forall u \in P : \mu_k > 1 - \varepsilon)$ then P is relatively compact. Similarly if $P \supseteq M_+(X, t)$ with subset of X, then the set of restrictions of measures in P to F is relatively compact.

We consider here an usco-correspondence $\phi : X \rightarrow \gamma$ with X and γ Hausdorff spaces. Let us define $\eta \in \phi(u)$ such that $\eta(\gamma) = \mu(x)$ and $\mu(k) \leq \eta(\phi k)$ for every compact set $k \subseteq X$. We obtain a correspondence from $M_+(X, t)$ to $M_+(\gamma, t)$. It is being shown here than this correspondence has closed graph and preserves compact nets. If $\eta \in \phi(u)$, η is an image measure of μ under ϕ. We assume here that ϕ is usco and find that a given $\eta \in M_+(\gamma, t)$ is an image measure of a measure in $M_+(X, t)$ if and only if

$\sup\{\eta(\phi k)|k \text{ compact} = \eta(\gamma)$.

We further extend here the construction method developed by Gardner [1] in the context of Radon content in view of regularity of Berel Measures.

Since $\lim \mu_\alpha G \geq \mu G$ for all open G and $\lim \mu_\alpha k \geq \mu k$ for all compact k, it provides $M_+(X)$ with topology of weak convergence. It implies that the weakest topology rendering all maps $\mu_\alpha \leq \mu k$ upper semi-continuous for compact k', and all maps $\mu_\alpha \geq \mu G$ lower semi-continuous for open G's.

Theorem 4. For the topology of weak convergence every subset $P \subseteq M_+(X)$ with $\sup\ \{\mu X \mid \mu \leq P\} < \infty$ is compact relative to $M_+(X, t)$.

Proof. Since $M_+(X)$ is non-Hausdorff, it implies that every constant next μ with $\mu \in M(X)$ converge to a Radon measure $\tilde{\mu}$, though $M_+(X, t)$ may be non-Hausdorff. We thus conclude that $M_+(X, t)$ is a Hausdorff topology is and only if X is locally compact. This technique may be applied to projective limits of measure spaces. Hence, the theorem is proved.

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