



# A critical overview of Quantum Theory of Radiation

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**Abstract :** Prevost's theory, Wien's theory and Rayleigh – Jeans theory are reconsidered to assimilate with Planck's quantum theory of black body radiation. Discrepancies are explained in pedagogic way. Epistemological structure is established in theoretical reordering of the findings.

**Keywords :** Black body radiation, Prevost's theory, Wien's theory, Rayleigh -Jeans theory, Max Planck's theory.

## 1. Introduction

Radiation plays a crucial role in the development of quantum physics. Many theoretical stairs are available to understand the physics involved. In spite of that it is also possible to review the picture from a pedagogic point of view. Instead of serving the theory directly the emphasis is on the concept that why it would have been assumed. To accomplish such a picture, it might be essential to reorder various outcomes of the theoretical regime by various authorities. Our Submission might also include such reordering.

## 2. Prevost's Theory of Heat Exchange

Once we accept the emission of radiation because of temperature as a measure of internal property, it becomes inevitable to allow the emission at all temperature and by all bodies irrespective of their role as system or surrounding. It leads to dynamical equilibrium rather than static one. It connotes that in thermal equilibrium exchange must go on. The outcome of exchange exhibits that there must be quantization of the entity being exchanged. The continuity of the process of exchange leads to the quantization of the entity being exchanged and nowadays this entity is the photon. The continuity of exchanging entity might lead to equal potentials of both the bodies and it might stop the further exchange; it connotes that continuity of exchanged entities leads to the quantization or stopping of the exchange process.

## 3. Stefan-Boltzmann's Law

In accordance with the Prevost's theory some mathematical structure can be allowed to it so that quantitative understanding of the theory can be conceptualized. The corresponding author introduced the ansatz. The total energy radiated per unit area per second should depend upon the temperature of the system and surrounding.

$$E = \sigma(T^n - T_0^n) \quad \dots(1).$$

For the surrounding to be maintained at the same temperature  $T_0$  we may proceed as

$$E = \sigma T^n + Constant$$

or

$$E - Constant = \sigma T^n$$

or

$$\varepsilon = \sigma T^n \quad \dots (2).$$

Experimental observations must accomplish to find out  $\varepsilon$  at various values of  $T$ . Eq.(2) can be used to decide 'n' and ' $\sigma$ ' from experiments

$$\ln \epsilon = \ln \sigma + n \ln T$$

$$\Delta \ln \epsilon = 0 + n \Delta \ln T$$

$$n = \frac{\partial \ln \epsilon}{\partial \ln T} \quad \dots(3).$$

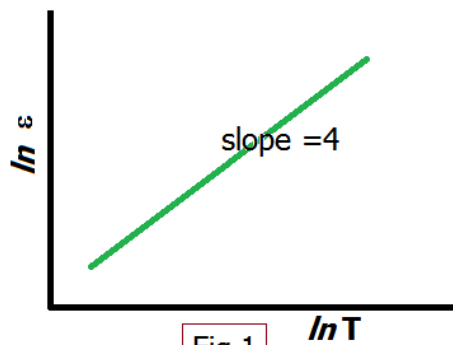


Fig.1

Experimentally 'n' will be found equal to '4',

$$\epsilon = \sigma T^4 \quad \dots(4).$$

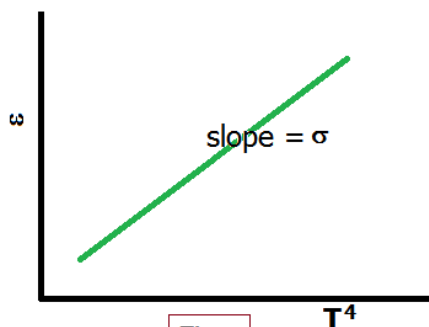


Fig.2

Eq. (4) can be used to decide Stefan's Constant 'σ'

$$\sigma = \frac{\partial \epsilon}{\partial T^4} \quad \dots(5).$$

#### 4. Wien's Radiation Law

The spectral energy density  $u_\lambda(\lambda, T)$  measurement is the fine probing into the further investigation of nature of radiation.

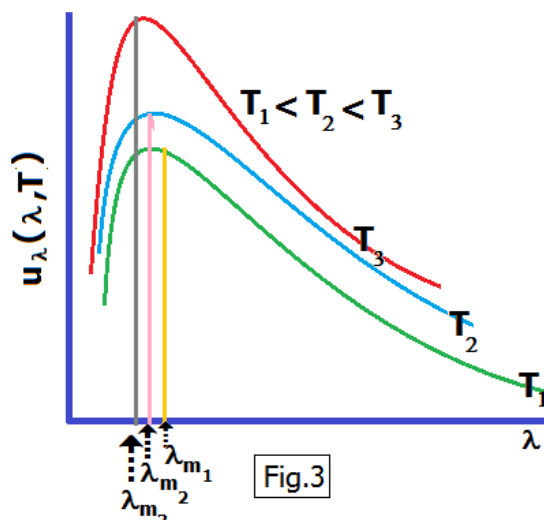


Fig.3

Mathematical considerations of  $\lambda_m T^n$  suggest that for  $n=1$ , the product attains a constant value and  $\lambda_m.T = b$  can be formulated as Wien's displacement law where 'b' is known as Wien's constant. . This is the Nobel observation first took by Wien.

Now it becomes essential to express the spectral energy density as a function of  $\lambda T$  as a new variable to determine further dependence on  $\lambda$ .

For systematic formulation we expect the extension of Ideas subjected for Stefan`s Law :

$$\int_0^\infty u_\lambda d\lambda = \sigma T^4 \Rightarrow \int_0^\infty \frac{\lambda^5 u_\lambda d(\lambda T)}{(\lambda T)^5} = \sigma$$

This expression suggests that if we plot  $\lambda^5 u_\lambda$  as a function of  $(\lambda T)$  we should get a unique curve and that is confirmed in experimental observations.

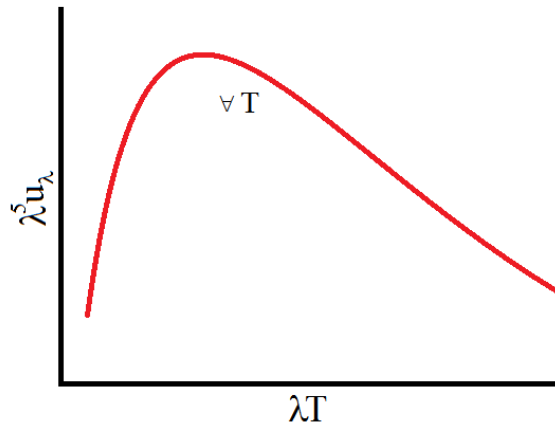


Fig.4

Wien's findings may be summarised as

- i.  $\lambda_m T = b$
- ii.  $u_\lambda (\lambda_{max}, T) \propto T^5$
- iii.  $u_\lambda \propto \frac{1}{\lambda^5 \exp(\frac{a}{\lambda T})}$

Wien's Radiation Law formulation is very good for short wavelength regime but not quite satisfactory at large wavelengths and we have to analyse the mathematical picture

$\lambda$  small :  $e^{a/\lambda T} \gg 1 \gg \lambda^5$

$\lambda$  large :  $e^{a/\lambda T} \ll \lambda^5$

For small value of  $\lambda$ , exponential factor is dominating and for large values of  $\lambda$ ,  $\lambda^5$  factor is dominating .

$$\lim_{\lambda \rightarrow 0} \frac{1}{\lambda^5 e^{a/\lambda T}} \rightarrow 0, \quad \lim_{\lambda \rightarrow \infty} \frac{1}{\lambda^5 e^{a/\lambda T}} \rightarrow 0$$

The discrepancies at large wave lengths suggest that we have to reconsider exponential factor and  $\lambda^5$  dependence. The exponential factor is developed in accordance with Boltzmann distribution. The only deviation is, in Boltzmann formulation Boltzmann constant appears naturally, while Wien's structure leaves it. Therefore, modification in structure is an obvious void in the formulation.

**5. Rayleigh- Jeans Radiation Law:**

Rayleigh proposed that the walls of cavity as an ensemble of atomic oscillator and standing wave modes of electromagnetic radiation. This classical approach with semi quantum mechanical ideas like Bohr's model of atomic structure got slight modification and deviations. The idea of standing modes is quite fruitful and the degree of freedom in frequency range  $[\nu, \nu + d\nu]$  is  $N_\nu d\nu$  given by

$$N_\nu d\nu = \frac{8\pi\nu^2}{c^3} d\nu \quad \dots(6)$$

This Formulation of degrees of freedom leads to good modification. Moreover, in Rayleigh jeans development the average energy per degree of freedom for an oscillator is taken as  $k_B T$ . Here inclusion of Boltzmann Constant is an initiative, but disappearance of exponential factor which is quite satisfactory in low wavelength regime introduces deviation in the theory. Thus, Rayleigh jeans Law is a well approach in high-wavelength domain but inadequate in low-wavelength domain. The absence of exponential factor allows the ultraviolet catastrophe.

### 6. Planck s Radiation Law <sup>[1]</sup>:

We must not ignore exponential factor and idea of standing wave modes. The corresponding author developed the ansatz. To include Boltzmann structure,  $k_B T$  must appear in the exponential factor.

Boltzmann distribution asks a factor  $e^{-E/k_B T}$  and Wien's formulation has a factor  $e^{-a/\lambda T}$ ; we should proceed in in the following manner:

$$e^{-a/\lambda T} = e^{-av/cT} = e^{-\left(\frac{ak_B}{c}\right)v/k_B T} \quad \dots(7)$$

Now the problem reduces to restructure the group  $\frac{ak_B}{c}$ . Here  $k_B$  and  $c$  are fundamental constant and only modification can be done is to redefine 'a' only. Planck's golden idea is to introduce a new fundamental constant in the following way

$$\frac{ak_B}{c} = nh \quad \dots(8)$$

$$\therefore e^{-a/\lambda T} = e^{-nhv/k_B T} = e^{-E_n/k_B T}, \quad \dots(9)$$

where  $E_n = nhv$ .

The number of oscillators with energy  $E_n$ , in accordance with Maxwell- Boltzmann Distribution Law is <sup>[2]</sup>:

$$\mathcal{N} \propto e^{-nhv/k_B T}$$

The energy is now not the constant  $k_B T$ , but it depends upon the frequency of the oscillator:

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} \mathcal{N} \cdot E_n}{\sum_{n=0}^{\infty} \mathcal{N}} = (hv) \cdot \left( \frac{1}{e^{hv/k_B T} - 1} \right)$$

The multiplicative factor here connotes the Bose Einstein nature of the oscillators.

The number of modes of oscillations of the electro-magnetic field in the frequency range  $[v, v + dv]$  is as proposed by Rayleigh-Jeans:

$$N_v dv = \frac{8\pi v^2}{c^3} dv$$

Hence the spectral energy density in the frequency range  $[v, v + dv]$  would be<sup>[3],[4]</sup>

$$u_v(v, T) dv = \frac{8\pi hv^3}{c^3} \cdot \frac{1}{e^{hv/k_B T} - 1} dv$$

Rayleigh – Jeans modes of electromagnetic field =  $N_v = N_{RJ}$

Maxwell Boltzmann probability factor =  $\mathcal{N} = f_{MB}$

Bose Einstein Distribution factor =  $f_{BE} = \frac{1}{e^{hv/k_B T} - 1}$

Thus the spectral energy density acquires the form

$$u_v(v, T) dv = hv \cdot N_{RJ} \cdot f_{BE} dv \quad \dots(10)$$

Here  $hv$  is the basic quantum of energy of Electromagnetic field.

**Conclusions :**

Experimental observations and old theoretical concepts are made to assimilate the quantum picture of the blackbody radiation. Our assertion is to remove the inadequacy instead of proving failure. Here we observe that Planck's Nobel picture of black body radiation is the golden extension of old theories. We found that catastrophe and other discrepancies are due to the omissions. This article might be helpful for scholars of quantum world.

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