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Novel an analytic hierarchy process for multiple criteria decision-making: A logarithmic type-2 fuzzy preference programming methodology

Sheikh Ahmed Hossain

Assistant Professor
Department of Mathematics
Brahmananda Keshab Chandra College, Kolkata, India

Abstract: The type-2 fuzzy analytic hierarchy process (T2AHP) proves to be a very useful methodology for multiple criteria decision-making in type-2 fuzzy environments, which have found substantial applications in recent years. The vast majority of the applications use a crisp point estimate method such as the extent analysis or the type-2 fuzzy preference programming (FPP) based nonlinear method for T2AHP priority derivation. The extent analysis has been revealed to be invalid and the weights derived by this method do not represent the relative importance of decision criteria or alternatives. This is a general procedure by which the special model has been derived from this proposed model. The FPP-based nonlinear priority method also turns out to be subject to significant advantages. The numerical examples are tested to show the advantages of the proposed methodology and its potential applications in T2AHP decision-making.

***Index Terms* - Supply chain; Fuzzy Type-2 Variables; Expected value; Fuzzy analytic hierarchy process Multiple criteria decision-making; Type-2 fuzzy preference programming.**

I. INTRODUCTION

The MCDM (multiple criteria decision making) have been developed to help decision marker in taking decision with respect to the subjective observation to the activities and performance. The subjective observations can be classified in terms of several criterion attributing different characteristics of the system under consideration. The study and the theory of MCDM able to solve different problems in different area like energy, pollution, factory, production, service sectors, financial sectors, environment, sustainability. The approaches and use of MCDM techniques proposed and developed by using fuzzy logic, operations research and methods of soft computing. In the fast growing world data mining, date clustering, machine learning, we always give importance on the public perception and the perception can be effectively managed by the MCDM techniques. There are many useful techniques like AHP, CORPUS, VICOR, TOPSIS to manage MCDM.

A real problem and its variables are not finite in nature. The views or judgements are versatile in nature and may be considered a fuzzy variable. To estimate a crisp membership function from a type 1 fuzzy characteristic function for demand and source because inherent imprecision. Type-2 fuzzy set which is an extension of type -1 fuzzy set introduced by Zadeh (1975) is an effective tool to deal with such variables where uncertainly is much higher (Liu and Liu 2007). The credibility theory which is different from probability theory also very useful theory to deal with uncertain linguistic variables. Qin et al. (2011) further developed the type-2 fuzzy set and proposed several methods to reduce type-2 fuzzy set into type-1 fuzzy set. Type -2 fuzzy has important applications in transportation theory Pramanik at al. (2016) and Kundu et al. (2015). Fagad et al. (2011) have used triangular fuzzy number and interval fuzzy number to solve transportation problem. For a type-1 fuzzy set, the belongingness of a member in the set has a value, called membership value which ranges from 0 to 1. One we have assigned a membership value to a member of the set, it is clear that the value bears some level of uncertainty that is the presence of membership is again a fuzzy variable that lies in the interval (0,1), Such a fuzzy set are defined as type-2 fuzzy set (T2FS). A geometrical defuzzyfication of T2FS is developed by Coupland (2007) that convert a general T2FS into a geometrical T2FS. Three kinds of methods for reduction was introduced by Qin et al. (2011). The reduction method is known as critical value(CV) reduction, optimistic CV reduction, pessimistic CV reduction for a regular type - 2 fuzzy variables. A transportation problem is solved by using internal type -2 fuzzy sets for both demand and supply (Figueroa- Garce and Hernandez 2012). Kundu et al. (2014) solved a transportation problem with fixed charge considering continues type-2 variables and internal approximation. Type-2 fuzzy sets are used for the analytical hierarchy process to deal uncertainly in criterion (cf. [9]-[14],[8],[15]).

To deal with fuzziness and uncertainly in MCDM, FAHP type - 2 fuzzy set are used in recent years. Use of type-2 fuzzy numbers in judgement given by experts is very simple and more appropriate than crisp judgments. Thus for complex linguistic data analysis and thereby making decisions the use of type-1 and type-2 fuzzy AHP will find in near future in several real life applications. The weight for each criterion in type-1 and type-2 environment may be classified into two categories, calculations of pairwise matrix that is obtained from pairwise comparison and the elements are taken as type-1 or type-2 fuzzy variable and their crisp weight are obtained by suitable reduction process. There are many research who worked on fuzzy type-1 AHP (cf.[1], Wang and Chin[2], Nguyen and Nahavandi [16].

In the last years, we are witnessing an increase of real-world applications of type-2 fuzzy sets a sustainable consumptions and productions (SCP). The SCP initiatives in supply chain may be a foremost challenge creating numerous barriers. This work, therefore, aims to identify and calculate barriers for adoptions SCP. We have developed a T2FPP technique from the T1FPP model can be derived. A numeral example has been given to support of this technique.

II. Preliminaries ideas on type-2 fuzzy sets

In this section, we have discussed type-2 fuzzy set and the inference of type-2 fuzzy logic. This will lead to type-2 fuzzy data envelopment analysis (T2FDEA). To consider uncertainty in data, fuzzy logic system in the form of type-1 is adopted, where membership function assumes a crisp data. In type-2 fuzzy set the membership function of member of the set is itself a type-1 fuzzy set. The output inference of a type-2 fuzzy set that needs to be type reduced before defuzzification. Type reduction is usually achieved using iterative algorithm. A type-2 fuzzy logic can be structurally put in the form in Fig 1.

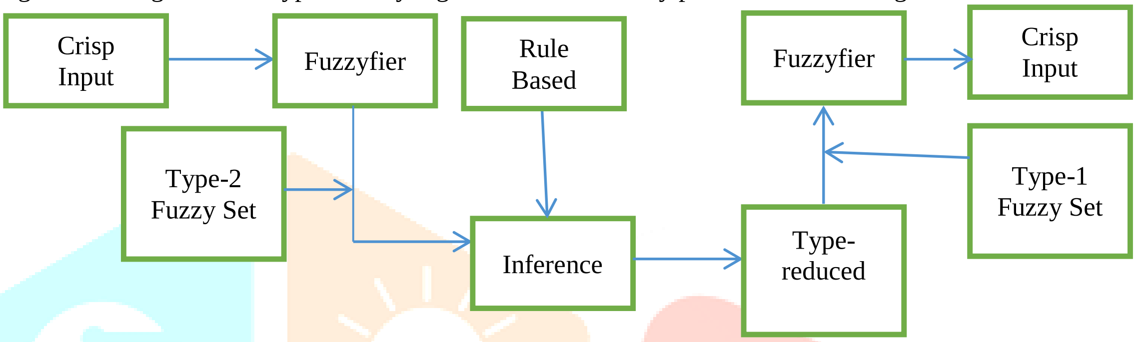

Fig.1- Structure of type-2 fuzzy set

A type-2 fuzzy set in universal set X is denoted as $\tilde{\xi}$ is defined by a type-2 membership function $u_{\tilde{A}}(x)$ given by $\tilde{\xi} = \int_{x\in X} u_{\tilde{\xi}}(x)/x = \int_{x\in X}\left[\int_{u\in J_x} f_x(u)/u\right]/x,\ J_x \subset [0,1]$, where J_x is the primary membership function of x and u, a fuzzy set, belongs to J_x and lying in open interval *(0,1)* for $\forall x \in X$.

Let us consider a fuzzy possibility space $(\Gamma, \mathrm{A}, \tilde{P}os)$, where Γ is the universe of discourse and A be an ample field which is a class of subsets of Γ that is closed under arbitrary union, intersection and complements in Γ. Bai *et al.* [2] employed possibility value at risk (VaR) reduction method to reduce type-2 fuzzy variable (T2FV) to type-1 fuzzy variable (T2FV). It is seen that the critical values (CV) of VaR-based reduced fuzzy variables and the CV-based reduction method developed by Qin *et al.* [26] give similar result. In this section we developed VaR-based critical values for type-2 triangular fuzzy variable which may be considered to be interval span of each member of membership function of type-1 triangular fuzzy number(TFN) (r_1,r_2,r_3). The membership function of TFN (r_1,r_2,r_3) is given by.

$$J_x = \begin{cases} J_x^L, & x\in[r_1,r_2] \\ J_x^U, & x\in[r_2,r_3] \end{cases},\quad J_x^L = \frac{x-r_1}{r_2-r_1},\quad J_x^U = \frac{r_3-x}{r_3-r_2}$$

Considering J_x to be the primary membership function, the secondary membership function $\tilde{\mu}_{\tilde{\xi}}(x)$ of triangular type-2 fuzzy set is defined by $\tilde{\mu}_{\tilde{\xi}}(x) = \left(J_x^L-\Theta_l^L, J_x^L, J_x^L+\Theta_r^L\right), x\in[r_1,r_2]$ and $\tilde{\mu}_{\tilde{\xi}}(x) = \left(J_x^U-\Theta_l^U, J_x^U, J_x^U+\Theta_r^U\right), x\in(r_2,r_3]$, where $\Theta_l^L, \Theta_r^L, \Theta_l^U, \Theta_r^U \in[0,1]$ such that $\tilde{\mu}_{\tilde{\xi}}(x) \in[0,1]\times[0,1]\times[0,1]$. Let us take the vales of $\Theta_l^L, \Theta_r^L, \Theta_l^U, \Theta_r^U$, as they depend on r_1, r_2, and r_3, as

$$\Theta_l^L = \theta_l \min\left\{\frac{x-r_1}{r_2-r_1}, \frac{r_2-x}{r_2-r_1}\right\},\ \Theta_r^L = \theta_r \min\left\{\frac{x-r_1}{r_2-r_1}, \frac{r_2-x}{r_2-r_1}\right\},\ \Theta_l^U = \theta_l \min\left\{\frac{x-r_2}{r_3-r_2}, \frac{r_3-x}{r_3-r_2}\right\},$$

$$\Theta_r^U = \theta_r \min\left\{\frac{x-r_2}{r_3-r_2}, \frac{r_3-x}{r_3-r_2}\right\},$$

where $\theta_l, \theta_r \in[0,1]$ are two parameters characterizing the degree of uncertainty of T2FS $\tilde{\xi}$ while taking the value of x. We denote a triangular T2FS by $(\tilde{r}_1,\tilde{r}_2,\tilde{r}_3;\theta_l,\theta_r)$.

III. Novel Type-2 Fuzzy FPP-Based Nonlinear Priority Method

Let us consider a set of decision makers giving judgements on an issues. The judgements contain certain level of uncertainties and we meet these uncertainties by assuming that the judgements are type-2 fuzzy variables. A decision maker can judge on criteria i within the range l_{ij} and u_{ij} times as important as criterion j. Since the values caries some uncertainties we lake left and right spreads σl_{ij} and σr_{ij}. Then, type-2 fuzzy comparison matrix can be written as

$$\tilde{A} = \begin{bmatrix} 1 & (l_{12}, m_{12}, u_{12}, \sigma l_{12}, \sigma r_{12}) & \cdots & (l_{12}, m_{12}, u_{12}, \sigma l_{12}, \sigma r_{12}) \\ (l_{21}, m_{21}, u_{21}, \sigma l_{21}, \sigma r_{21}) & 1 & \cdots & (l_{21}, m_{21}, u_{21}, \sigma l_{21}, \sigma r_{21}) \\ \cdots & \cdots & \cdots & \cdots \\ (l_{n1}, m_{n1}, u_{n1}, \sigma l_{n1}, \sigma r_{n1}) & (l_{n2}, m_{n2}, u_{n2}, \sigma l_{n2}, \sigma r_{n2}) & \cdots & 1 \end{bmatrix} \quad (1)$$

where $l_{ij} = 1/l_{ji}$, $m_{ij} = 1/m_{ji}$ $u_{ij} = 1/u_{ji}$ and $l_{ij} < m_{ij} < u_{ij}$ for all $i, j = 1,2,\cdots n, i \neq j$. Mikhailov [3] introduces the following membership function for each fuzzy judgement in $\tilde{A}$:

$$\mu_{ij}\left(\frac{w_i}{w_j}\right) = \begin{cases} \frac{2-\lambda\sigma r_{ij}-(1-\lambda)\sigma l_{ij}}{2}\left(\frac{(w_i/w_j)-l_{ij}}{m_{ij}-l_{ij}}\right) + \frac{\lambda\sigma r_{ij}}{2}, \ w_i/w_j \leq m_{ij} \\ \frac{2-\lambda\sigma r_{ij}-(1-\lambda)\sigma l_{ij}}{2}\left(\frac{u_{ij}-(w_i/w_j)}{u_{ij}-m_{ij}}\right) + \frac{\lambda\sigma r_{ij}}{2}, \ w_i/w_j \geq m_{ij} \end{cases} \quad (2)$$

where $(w_1, w_2, \cdots, w_n)$ is crisp priority vector with $w_1 + w_2 + \cdots + w_n = 1$ and $w_i \geq 0$.

Define $\eta = \min\{\mu_{ij}(w_i/w_j) | i = 1(1)n, j > i\}$. Then η defines minimum membership degree. Michailov [3] developed FPP-based model for priority on FPP based in type-2 extension:

$$\max \eta$$

$$s.t. \begin{cases} \mu_{ij}\left(\frac{w_j}{w_j}\right) \geq \eta, i = 1(1)n, j > i \\ \Sigma_1^n w_i = 1 \\ w_i \geq 0 \end{cases} \quad (3)$$

which can be equivalently expressed as

$$\max \eta$$

$$st \begin{cases} \left(\frac{2-\lambda\sigma r_{ij}-(1-\lambda)\sigma l_{ij}}{2}\right) w_i + \left(\frac{2-\lambda\sigma r_{ij}-(1-\lambda)\sigma l_{ij}}{2}\right) l_{ij} w_j + \left(\eta - \frac{\lambda\sigma r_{ij}}{2}\right)(m_{ij} - l_{ij}) w_j \leq 0 \\ \left(\frac{2-\lambda\sigma r_{ij}-(1-\lambda)\sigma l_{ij}}{2}\right) w_i + \left(\frac{2-\lambda\sigma r_{ij}-(1-\lambda)\sigma l_{ij}}{2}\right) u_{ij} w_j + \left(\eta - \frac{\lambda\sigma r_{ij}}{2}\right)(u_{ij} - m_{ij}) w_j \leq 0 \end{cases} \quad (4)$$

IV. An Illustrative Example

To illustrate the overall numerical optimization process we take following situation. Suppose a manufacturing company selects four plants to manufacture four types of items. The following matrix provides the views in type 2 fuzzy variable and is expressed type 2 pairwise comparison matrix M. The priorities $w_1, w_2, \cdots, w_3$ can be calculated by applying the type-2 fuzzy preference programming (T2FPP). From (4), the optimistic, normal and pessimistic values of different weights are given by:

$$-\left(\frac{2-\lambda\sigma r_{12}-(1-\lambda)\sigma l_{12}}{2}\right) w_1 + \left(\frac{2-\lambda\sigma r_{12}-(1-\lambda)\sigma l_{12}}{2}\right) l_{12} w_2 + \left(\eta - \frac{\lambda\sigma r_{12}}{2}\right)(m_{12} - l_{12}) w_2 \leq 0$$

$$\left(\frac{2-\lambda\sigma r_{12}-(1-\lambda)\sigma l_{12}}{2}\right) w_1 - \left(\frac{2-\lambda\sigma r_{12}-(1-\lambda)\sigma l_{12}}{2}\right) u_{12} w_2 + \left(\eta - \frac{\lambda\sigma r_{12}}{2}\right)(u_{12} - m_{12}) w_2 \leq 0$$

$$-\left(\frac{2-\lambda\sigma r_{13}-(1-\lambda)\sigma l_{13}}{2}\right) w_1 + \left(\frac{2-\lambda\sigma r_{13}-(1-\lambda)\sigma l_{13}}{2}\right) l_{13} w_3 + \left(\eta - \frac{\lambda\sigma r_{13}}{2}\right)(m_{13} - l_{13}) w_3 \leq 0$$

$$\left(\frac{2-\lambda\sigma r_{13}-(1-\lambda)\sigma l_{13}}{2}\right) w_1 - \left(\frac{2-\lambda\sigma r_{13}-(1-\lambda)\sigma l_{13}}{2}\right) u_{13} w_3 + \left(\eta - \frac{\lambda\sigma r_{13}}{2}\right)(u_{13} - m_{13}) w_3 \leq 0$$

$$-\left(\frac{2-\lambda\sigma r_{14}-(1-\lambda)\sigma l_{14}}{2}\right) w_1 + \left(\frac{2-\lambda\sigma r_{14}-(1-\lambda)\sigma l_{14}}{2}\right) l_{14} w_4 + \left(\eta - \frac{\lambda\sigma r_{14}}{2}\right)(m_{14} - l_{14}) w_4 \leq 0$$

$$\left(\frac{2-\lambda\sigma r_{14}-(1-\lambda)\sigma l_{14}}{2}\right) w_1 - \left(\frac{2-\lambda\sigma r_{14}-(1-\lambda)\sigma l_{14}}{2}\right) u_{14} w_4 + \left(\eta - \frac{\lambda\sigma r_{14}}{2}\right)(u_{14} - m_{14}) w_4 \leq 0$$

$$-\left(\frac{2-\lambda\sigma r_{23}-(1-\lambda)\sigma l_{23}}{2}\right) w_2 + \left(\frac{2-\lambda\sigma r_{23}-(1-\lambda)\sigma l_{23}}{2}\right) l_{23} w_3 + \left(\eta - \frac{\lambda\sigma r_{23}}{2}\right)(m_{23} - l_{23}) w_3 \leq 0$$

$$\left(\frac{2-\lambda\sigma r_{23}-(1-\lambda)\sigma l_{23}}{2}\right) w_2 - \left(\frac{2-\lambda\sigma r_{23}-(1-\lambda)\sigma l_{23}}{2}\right) l_{23} w_3 + \left(\eta - \frac{\lambda\sigma r_{23}}{2}\right)(m_{23} - l_{23}) w_3 \leq 0$$

$$-\left(\frac{2-\lambda\sigma r_{24}-(1-\lambda)\sigma l_{24}}{2}\right) w_2 + \left(\frac{2-\lambda\sigma r_{24}-(1-\lambda)\sigma l_{24}}{2}\right) l_{24} w_4 + \left(\eta - \frac{\lambda\sigma r_{24}}{2}\right)(m_{24} - l_{24}) w_4 \leq 0$$

$$\left(\frac{2-\lambda\sigma r_{24}-(1-\lambda)\sigma l_{24}}{2}\right) w_2 + \left(\frac{2-\lambda\sigma r_{24}-(1-\lambda)\sigma l_{24}}{2}\right) u_{24} w_4 + \left(\eta - \frac{\lambda\sigma r_{24}}{2}\right)(u_{24} - m_{24}) w_4 \leq 0$$

$$-\left(\frac{2-\lambda\sigma r_{34}-(1-\lambda)\sigma l_{34}}{2}\right) w_3 + \left(\frac{2-\lambda\sigma r_{34}-(1-\lambda)\sigma l_{34}}{2}\right) l_{34} w_4 + \left(\eta - \frac{\lambda\sigma r_{34}}{2}\right)(m_{34} - l_{34}) w_4 \leq 0$$

$$\left(\frac{2-\lambda\sigma r_{34}-(1-\lambda)\sigma l_{34}}{2}\right) w_2 + \left(\frac{2-\lambda\sigma r_{34}-(1-\lambda)\sigma l_{34}}{2}\right) u_{34} w_4 + \left(\eta - \frac{\lambda\sigma r_{34}}{2}\right)(u_{34} - m_{34}) w_4 \leq 0$$

The optimistic values for different weights w_1, w_2, w_3, w_4 are given by ($\lambda = 1$)

$\max \eta$ (5)

subject to

$$-\left(\frac{2-\sigma r_{12}}{2}\right) w_1 + \left(\frac{2-\sigma r_{12}}{2}\right) l_{12} w_2 + \left(\eta - \frac{\sigma r_{12}}{2}\right)(m_{12} - l_{12}) w_2 \leq 0$$

$$\left(\frac{2-\sigma r_{12}}{2}\right) w_1 - \left(\frac{2-\sigma r_{12}}{2}\right) u_{12} w_2 + \left(\eta - \frac{\sigma r_{12}}{2}\right)(u_{12} - m_{12}) w_2 \leq 0$$

$$-\left(\frac{2-\sigma r_{13}}{2}\right) w_1 + \left(\frac{2-\sigma r_{13}}{2}\right) l_{13} w_3 + \left(\eta - \frac{\sigma r_{13}}{2}\right)(m_{13} - l_{13}) w_3 \leq 0$$

$$\left(\frac{2-\sigma r_{13}}{2}\right) w_1 - \left(\frac{2-\sigma r_{13}}{2}\right) u_{13} w_3 + \left(\eta - \frac{\sigma r_{13}}{2}\right)(u_{13} - m_{13}) w_3 \leq 0$$

$$-\left(\frac{2-\sigma r_{14}}{2}\right) w_1 + \left(\frac{2-\sigma r_{14}}{2}\right) l_{14} w_4 + \left(\eta - \frac{\sigma r_{14}}{2}\right)(m_{14} - l_{14}) w_4 \leq 0$$

$$\left(\frac{2-\sigma r_{14}}{2}\right) w_1 - \left(\frac{2-\sigma r_{14}}{2}\right) u_{14} w_4 + \left(\eta - \frac{\sigma r_{14}}{2}\right)(u_{14} - m_{14}) w_4 \leq 0$$

$$-\left(\frac{2-\sigma r_{23}}{2}\right) w_2 + \left(\frac{2-\sigma r_{23}}{2}\right) l_{23} w_3 + \left(\eta - \frac{\sigma r_{23}}{2}\right)(m_{23} - l_{23}) w_3 \leq 0$$

$\left(\frac{2-\sigma r_{23}}{2}\right)w_2-\left(\frac{2-\sigma r_{23}}{2}\right)l_{23}w_3+\left(\eta-\frac{\sigma r_{23}}{2}\right)(m_{23}-l_{23})w_3\le 0$

$-\left(\frac{2-\sigma r_{24}}{2}\right)w_2+\left(\frac{2-\sigma r_{24}}{2}\right)l_{24}w_4+\left(\eta-\frac{\sigma r_{24}}{2}\right)(m_{24}-l_{24})w_4\le 0$

$\left(\frac{2-\sigma r_{24}}{2}\right)w_2+\left(\frac{2-\sigma r_{24}}{2}\right)u_{24}w_4+\left(\eta-\frac{\sigma r_{24}}{2}\right)(u_{24}-m_{24})w_4\le 0$

$-\left(\frac{2-\sigma r_{34}}{2}\right)w_3+\left(\frac{2-\sigma r_{34}}{2}\right)l_{34}w_4+\left(\eta-\frac{\sigma r_{34}}{2}\right)(m_{34}-l_{34})w_4\le 0$

$\left(\frac{2-\sigma r_{34}}{2}\right)w_2+\left(\frac{2-\sigma r_{34}}{2}\right)u_{34}w_4+\left(\eta-\frac{\sigma r_{34}}{2}\right)(u_{34}-m_{34})w_4\le 0$

$w_1+w_2+w_3+w_4=1$

The normal values for different weights w_1, w_2, w_3, w_4 are given by $(\lambda = 0.5)$

$\max\eta$ (6)

subject to

$-\left(\frac{2-0.5\sigma r_{12}-0.5\sigma l_{12}}{2}\right)w_1+\left(\frac{2-0.5\sigma r_{12}-0.5\sigma l_{12}}{2}\right)l_{12}w_2+\left(\eta-\frac{0.5\sigma r_{12}}{2}\right)(m_{12}-l_{12})w_2\le 0$

$\left(\frac{2-0.5\sigma r_{12}-0.5\sigma l_{12}}{2}\right)w_1-\left(\frac{2-0.5\sigma r_{12}-0.5\sigma l_{12}}{2}\right)u_{12}w_2+\left(\eta-\frac{0.5\sigma r_{12}}{2}\right)(u_{12}-m_{12})w_2\le 0$

$-\left(\frac{2-0.5\sigma r_{13}-0.5\sigma l_{13}}{2}\right)w_1+\left(\frac{2-0.5\sigma r_{13}-0.5\sigma l_{13}}{2}\right)l_{13}w_3+\left(\eta-\frac{0.5\sigma r_{13}}{2}\right)(m_{13}-l_{13})w_3\le 0$

$\left(\frac{2-0.5\sigma r_{13}-0.5\sigma l_{13}}{2}\right)w_1-\left(\frac{2-0.5\sigma r_{13}-0.5\sigma l_{13}}{2}\right)u_{13}w_3+\left(\eta-\frac{0.5\sigma r_{13}}{2}\right)(u_{13}-m_{13})w_3\le 0$

$-\left(\frac{2-0.5\sigma r_{14}-0.5\sigma l_{14}}{2}\right)w_1+\left(\frac{2-0.5\sigma r_{14}-0.5\sigma l_{14}}{2}\right)l_{14}w_4+\left(\eta-\frac{0.5\sigma r_{14}}{2}\right)(m_{14}-l_{14})w_4\le 0$

$\left(\frac{2-0.5\sigma r_{14}-0.5\sigma l_{14}}{2}\right)w_1-\left(\frac{2-0.5\sigma r_{14}-0.5\sigma l_{14}}{2}\right)u_{14}w_4+\left(\eta-\frac{0.5\sigma r_{14}}{2}\right)(u_{14}-m_{14})w_4\le 0$

$-\left(\frac{2-0.5\sigma r_{23}-0.5\sigma l_{23}}{2}\right)w_2+\left(\frac{2-0.5\sigma r_{23}-0.5\sigma l_{23}}{2}\right)l_{23}w_3+\left(\eta-\frac{0.5\sigma r_{23}}{2}\right)(m_{23}-l_{23})w_3\le 0$

$\left(\frac{2-0.5\sigma r_{23}-0.5\sigma l_{23}}{2}\right)w_2-\left(\frac{2-0.5\sigma r_{23}-0.5\sigma l_{23}}{2}\right)l_{23}w_3+\left(\eta-\frac{0.5\sigma r_{23}}{2}\right)(m_{23}-l_{23})w_3\le 0$

$-\left(\frac{2-0.5\sigma r_{24}-0.5\sigma l_{24}}{2}\right)w_2+\left(\frac{2-0.5\sigma r_{24}-0.5\sigma l_{24}}{2}\right)l_{24}w_4+\left(\eta-\frac{0.5\sigma r_{24}}{2}\right)(m_{24}-l_{24})w_4\le 0$

$\left(\frac{2-0.5\sigma r_{24}-0.5\sigma l_{24}}{2}\right)w_2+\left(\frac{2-0.5\sigma r_{24}-0.5\sigma l_{24}}{2}\right)u_{24}w_4+\left(\eta-\frac{0.5\sigma r_{24}}{2}\right)(u_{24}-m_{24})w_4\le 0$

$-\left(\frac{2-0.5\sigma r_{34}-0.5\sigma l_{34}}{2}\right)w_3+\left(\frac{2-0.5\sigma r_{34}-0.5\sigma l_{34}}{2}\right)l_{34}w_4+\left(\eta-\frac{0.5\sigma r_{34}}{2}\right)(m_{34}-l_{34})w_4\le 0$

$\left(\frac{2-0.5\sigma r_{34}-0.5\sigma l_{34}}{2}\right)w_2+\left(\frac{2-0.5\sigma r_{34}-0.5\sigma l_{34}}{2}\right)u_{34}w_4+\left(\eta-\frac{0.5\sigma r_{34}}{2}\right)(u_{34}-m_{34})w_4\le 0$

$w_1+w_2+w_3+w_4=1$

The optimistic values for different weights w_1, w_2, w_3, w_4 are given by $(\lambda = 0)$

$\max\eta$ (7)

subject to

$-\left(\frac{2-\sigma l_{12}}{2}\right)w_1+\left(\frac{2-\sigma l_{12}}{2}\right)l_{12}w_2+\eta(m_{12}-l_{12})w_2\le 0$

$\left(\frac{2-\sigma l_{12}}{2}\right)w_1-\left(\frac{2-\sigma l_{12}}{2}\right)u_{12}w_2+\eta(u_{12}-m_{12})w_2\le 0$

$-\left(\frac{2-\sigma l_{13}}{2}\right)w_1+\left(\frac{2-\sigma l_{13}}{2}\right)l_{13}w_3+\eta(m_{13}-l_{13})w_3\le 0$

$\left(\frac{2-\sigma l_{13}}{2}\right)w_1-\left(\frac{2-\sigma l_{13}}{2}\right)u_{13}w_3+\eta(u_{13}-m_{13})w_3\le 0$

$-\left(\frac{2-\sigma l_{14}}{2}\right)w_1+\left(\frac{2-\sigma l_{14}}{2}\right)l_{14}w_4+\eta(m_{14}-l_{14})w_4\le 0$

$\left(\frac{2-\sigma l_{14}}{2}\right)w_1-\left(\frac{2-\sigma l_{14}}{2}\right)u_{14}w_4+\eta(u_{14}-m_{14})w_4\le 0$

$-\left(\frac{2-\sigma l_{23}}{2}\right)w_2+\left(\frac{2-\sigma l_{23}}{2}\right)l_{23}w_3+\eta(m_{23}-l_{23})w_3\le 0$

$\left(\frac{2-\sigma l_{23}}{2}\right)w_2-\left(\frac{2-\sigma l_{23}}{2}\right)l_{23}w_3+\eta(m_{23}-l_{23})w_3\le 0$

$-\left(\frac{2-l_{24}}{2}\right)w_2+\left(\frac{2-\sigma l_{24}}{2}\right)l_{24}w_4+\eta(m_{24}-l_{24})w_4\le 0$

$\left(\frac{2-\sigma l_{24}}{2}\right)w_2+\left(\frac{2-\sigma l_{24}}{2}\right)u_{24}w_4+\eta(u_{24}-m_{24})w_4\le 0$

$-\left(\frac{2-\sigma l_{34}}{2}\right)w_3+\left(\frac{2-\sigma l_{34}}{2}\right)l_{34}w_4+\eta(m_{34}-l_{34})w_4\le 0$

$\left(\frac{2-\sigma l_{34}}{2}\right)w_2+\left(\frac{2-\sigma l_{34}}{2}\right)u_{34}w_4+\eta(u_{34}-m_{34})w_4\le 0$

$w_1+w_2+w_3+w_4=1$

Let us consider the matrix M as

$$M=\begin{bmatrix}(1,1,1,0.9,0.5) & (1.5,2,2.5,0.9,0.5) & (1.5,2,2.5,0.9,0.5) & (.66,1,1.5,0.9,0.5)\\ (0.4,0.5,0.66,0.9,0.5) & (1,1,1,0.9,0.5) & (0.5,1,1.5,0.9,0.5) & (0.15,0.4,0.66,0.9,0.5)\\ (0.4,0.5,0.66,0.9,0.5) & (1,1,1,0.9,0.5) & (1,1,1,0.9,0.5) & (0.4,0.5,0.66,0.9,0.5)\\ (.66,1,1.5,0.9,0.5) & (0.4,0.5,1.5,0.9,0.5) & (1.5,2,2.5,0.9,0.5) & (1,1,1,0.9,0.5)\end{bmatrix}$$

We solve the problems (5), (6) and (7) in LINGO software based on the matrix M and optimistic, normal and pessimistic values are obtained as follows:

Table 1. Pessimistic value of AHP model

Optimistic value	Normal value	Pessimistic value
0.8985227	0.3269402	0.3259205
0.3258325	0.1562623	0.1557749
0.1557329	0.1679752	0.1705702
0.1707943	0.3488223	0.3477343

Table 2. Optimistic value of AHP model

	Optimistic value	Normal value	Pessimistic value
\eta	0.3794149	0.2544149	0.1294149
w_1	0.3590558	0.3590558	0.3590558
w_2	0.2263513	0.2263513	0.2263513
w_3	0.1606141	0.160614	0.16614
w_4	0.25339789	0.2539788	0.2539788

Table 3. Pessimistic value of AHP model

	Optimistic value	Normal value	Pessimistic value
\eta	0.5035319	0.3035319	0.1035319
w_1	0.3591729	0.3590557	0.3590558
w_2	0.2264251	0.2263513	0.2263513
w_3	0.1603404	0.1606141	0.160614
w_4	0.2540616	0.2539789	0.2539788

Table 4. Natural value of AHP model

	Optimistic value	Normal value	Pessimistic value
\eta	0.2552979	0.2096117	0.1639255
w_1	0.3590558	0.3590558	0.3590558
w_2	0.2263513	0.2263513	0.2263513
w_3	0.160614	0.160614	0.160614
w_4	0.2539788	0.2539788	0.2539788

Table 5. Natural value of AHP model

	TI
\eta	0.1725529
w_1	0.3590644
w_2	0.2263566
w_3	0.1605942
w_4	0.2539849

IV. RESULTS AND DISCUSSION

From experiments, we determined compromise solutions using type-2 fuzzy preference programming (T2FPP) for MCDM and type-2 fuzzy analytic hierarchy process (AHP) problems for different optimistic levels with different degrees. In order to validate the results, we obtained a sensitivity analysis with different spreads 5_l, 5_r which are tabulated from Table 2 to Table 4. In these tabulated values, it is observed that the AHP decision making rank should be different for different values of spreads selection. In this Table-5, we are reported that optimistic results for T1 fuzzy variables, all expected results ar to lead some insights of managers.

The use of fuzzy AHP for MCDM requires scientific weight derivation from type-2 fuzzy (T2F) pair wise comparison matrices. Existing approaches for deriving T2F weights from T2F pair wise comparison matrices turn out to be too sophisticated and rare to be applied, while the approaches for deriving crisp weights from T2F pair wise comparison matrices prove to be either invalid or subject to significant drawbacks such as producing multiple even conflict priority vectors for a T2F pair wise comparison matrix, leading to distinct conclusions.

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