



A METHOD FOR SOLVING GENERALIZED OCTAGONAL FUZZY TRANSPORTATION PROBLEM-HARMONIC MEAN METHOD

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ABSTRACT

The main objective of this paper work is to find the transportation cost by Harmonic Mean Method for generalized octagonal fuzzy number. The Ranking procedure is used to defuzzificate the data. Further, a above procedure is illustrated by a numerical example.

Keywords: Fuzzy Transportation Problem (FTP); Generalized Octagonal Fuzzy Number (GOFN); Ranking function; Harmonic Mean Method (HMM).

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1.Introduction

Transportation problem, an important network that structured in LPP, it arises in several contexts. The central concept of the problem is to find the least cost of total transportation in order to satisfy the demand of destinations using supplies at origin. Transportation problem can be used for variety of situations such as scheduling, productions, investment, plant location, inventory control, employment scheduling, etc.,

The transportation problem was originally introduced by Hitchcock [7]. A standard linear programming problem modeled the transportation problem, which can solve the simplex method (or) simplex algorithm. However, it was recognized early, that the transportation problem can be solved by simplex method it be quite efficient in terms of how to evaluate the necessary method information. A stepping stone method was developed by Charnes and Cooper[5], it provides the alternative method of determining the simplex method. The primal simplex transportation method was derived from transportation method of simplex method and it was used by Dantzig and Thapa [12].

In Transportation problem, the solutions are generally solved by the assumptions that the transportation costs and value of supply and demand are determined by the crisp environment. However, in real life, there are many situations due to uncertainty in judgements, etc., it is not possible to get correct data for the cost parameter. In this case, the corresponding elements of the problem can be formulated by the means of fuzzy sets and its approaches the fuzzy transportation problem in natural way.

Zimmermann [15] shows that solutions are obtained by fuzzy linear programming method are always efficient. In this, they developed some fuzzy optimization methods to solve the transportation problem. A Hatami-Marbini [14] has developed an extension for fuzzy parameters of the linear programming methods. Saad the transportation problem in fuzzy environment. Gani and Razak [10] discussed a two stage of fuzzy

transportation problem for cost minimizing in which supplies and demands are used with trapezoidal fuzzy numbers. A.A.Ebrahimnejad [17] proposed a new approach for solving fuzzy transportation problem with trapezoidal fuzzy numbers. K.Dhurai [8], introduced a new ranking function for an octagonal fuzzy number to solve the transportation. Pandian and Natarajan [17] introduced a new algorithm for finding optimal solution for transportation problem, namely fuzzy Zero Point method, where transportation cost, demand and supply are represented by trapezoidal fuzzy numbers. FeblinC.Kennedy [19], proposed an new approach for solving fuzzy transportation problem using octagonal fuzzy numbers. In this, they discussed a ranking for the octagonal fuzzy numbers, and it is possible to compare and this used for convert the fuzzy transportation problem. SrinivasaraoThota [1], introduced a new method to solve generalized fuzzy transportation problem.

In this paper, a procedure, namely the harmonic mean method, is used to solve a form of fuzzy transport problem by assuming that the decision-maker is only correct in terms of transport costs due to uncertainty. In this proposed method, the transportation costs are represented by generalized octagonal fuzzy numbers. To illustrate, a numeral example can be solved by HMM. The proposed method of HMM can be easily to understand and it can be applied for real life transportation problem used by decision makers.

2. Preliminaries

In this section, we see some definitions and an existing method for octagonal generalized fuzzy numbers are defined.

Definition (FUZZY SET):

Fuzzy set is defined by membership function and it takes values from the domain, space or universe of discourse. It maps into the unit interval $[0,1]$. A fuzzy set A in the universal set X is defined as $A = \{x, \mu(x) : x \in X\}$ here $\mu_A : A \rightarrow [0,1]$ is the grade of the membership function and $\mu_A(x)$ is the grade value of $x \in X$ in the fuzzy set A .

Definition (FUZZY NUMBER):

A fuzzy number A is a subset of real line R , with the membership function μ_A from the conditions:

1. $\mu_A(x)$ is piecewise continuous in its domain.
2. A is normal, i.e., there is $x_0 \in A$ such that $\mu_A(x_0) = 1$.
3. A is convex, i.e., $\mu_A(\lambda x_1 + (1-\lambda)x_2) = \min(\mu_A(x_1), \mu_A(x_2))$. $\forall x_1, x_2$ in X .

Definition (OCTAGONAL FUZZY NUMBER):

This method presents for ranking fuzzy numbers. It calculates the ranking scores of the octagonal fuzzy number. Fuzzy numbers (A, B, C) are an octagonal fuzzy number and is denoted by $(A, B, C) = (a, b, c, d, e, f, g, h)$ where (a, b, c, d, e, f, g, h) are real numbers. Its membership functions are $\mu_A(x)$, $\mu_B(x)$, and $\mu_C(x)$ are given below

$$\mu_A(x) = \begin{cases} \frac{x-a}{d-a}, & a \leq x \leq d \\ 1, & d \leq x \leq e \\ \frac{x-h}{e-h}, & e \leq x \leq h \\ 0, & \text{otherwise,} \end{cases}$$

$$\mu_B(x) = \begin{cases} \frac{x-a}{c-a} & a \leq x \leq c \\ 0.8, & c \leq x \leq f \\ \frac{x-h}{f-h} & f \leq x \leq h \\ 0 & \text{otherwise,} \end{cases}$$

$$\mu_C(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ 0.6, & b \leq x \leq g \\ \frac{x-g}{g-h} & g \leq x \leq h \\ 0, & \text{otherwise.} \end{cases}$$

Arithmetic Operations:

Consider A_1 and A_2 be two octagonal fuzzy numbers characterized by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$, respectively. Let us consider simplified fuzzy number arithmetic operations between the fuzzy numbers A_1 and A_2 are as follows:

Fuzzy numbers addition:

$$+ : \{ [a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8] + [b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8] \} = [a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7, a_8+b_8].$$

Fuzzy numbers subtraction:

$$\ominus : (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \ominus (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) = (\max(0, a_1-b_8), \max(0, a_2-b_7), \max(0, a_3-b_6), \max(0, a_4-b_5), \max(0, a_5-b_4), \max(0, a_6-b_3), \max(0, a_7-b_2), \max(0, a_8-b_1)).$$

Octagonal Formula:

Let we define the real number $j, j=1, 2, \dots, n$ for given octagonal fuzzy numbers. We find the value for all rows and columns with this formula. In this, we find the maximum or minimum value for the particular row and column. Then we find Harmonic Mean for the fuzzy numbers. The formula is,

$$\frac{1}{\sum_{j=1}^1 \frac{w_j}{a_j}} + \frac{1}{\sum_{j=1}^2 \frac{w_j}{a_j}} + \dots + \frac{1}{\sum_{j=1}^8 \frac{w_j}{a_j}}$$

Ranking Function:

The powerful approach used to compare fuzzy numbers is to use the rating function [7,17,19], $R: F(R) \rightarrow R$, where $F(R)$ is a collection of fuzzy numbers specified in the set of real numbers, which maps each fuzzy number to the actual line. i.e., if

- (i) $\tilde{A} >_R \tilde{B}$ if and only if $R(\tilde{A}) > R(\tilde{B})$
- (ii) $\tilde{A} <_R \tilde{B}$ if and only if $R(\tilde{A}) < R(\tilde{B})$
- (iii) $\tilde{A} =_R \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2, e_2, f_2, g_2, h_2; \omega_2)$ be two generalized trapezoidal fuzzy numbers and $\omega = \min(\omega_1, \omega_2)$.

$$R(\tilde{A}_1) = \frac{\omega(a_1+b_1+c_1+d_1+e_1+f_1+g_1+h_1)}{4} \text{ and}$$

$$R(\tilde{A}_2) = \frac{\omega(a_2+b_2+c_2+d_2+e_2+f_2+g_2+h_2)}{4}.$$

3. Proposed Method

The methodology of HMM is followed as

Step 1: First we have to check whether the given transportation is balanced or not. If not balanced, we have to add dummy row or column with costs are fuzzy zero. Then we go for step 2.

Step 2: Then we have to find the harmonic mean for each row and each value. Then find the maximum value in this

Step 3: Allocate the maximum supply or demand at the place value of maximum value of the corresponding row and column.

Step 4: Repeat the step 2 and step 3 until the values of supplies are exhausted and the demands are satisfied.

Step 5: Total maximum fuzzy cost = sum of product of fuzzy cost and the corresponding values are allocated for supply and demand.

4.Numerical Example

To illustrate this Harmonic Mean Method (HMM), the following Fuzzy transportation problem is solved,

Example 1: Table 1 gives the availability of the product that are available at three sources and demands at three destinations, and the approximate unit transportation cost of the product is represented by generalized octagonal fuzzy number. Determine the optimal solution for the fuzzy optimal products such that the transportation cost is minimum.

Table 1

| | D ₁ | D ₂ | D ₃ | Supply(a _i) |
|----------------|------------------------------|------------------------------|------------------------------|-------------------------|
| S ₁ | (11,13,14,18,19,20,22,23;.5) | (20,21,24,27,29,30,31,32;.7) | (14,15,16,17,19,20,22,24;.4) | 13 |
| S ₂ | (6,7,8,11,12,15,17,18;.2) | (9,11,12,13,15,16,18,20;.2) | (20,21,24,27,30,33,34,36;.7) | 20 |
| S ₃ | (14,15,17,18,20,21,23,25;.4) | (15,16,18,19,20,22,23,25;.5) | (10,11,12,13,15,16,19,21;.6) | 5 |

Demand (b_j) 12

15

11

Using step1, $\sum_{i=1}^3 a_i = \sum_{j=1}^3 b_j = 38$, so the given problem is a balanced FTP. Using this formula, $\frac{1}{\sum_{j=1}^8 \frac{w_j}{a_j}}$

We get

| | D ₁ | D ₂ | D ₃ | Supply(a _i) |
|----------------|--|--|--|-------------------------|
| S ₁ | (11,13,14,18,19,20,22,23;.5) 7 | (20,21,24,27,29,30,31,32;.7) | (14,15,16,17,19,20,22,24;.4) 6 | * |
| S ₂ | (6,7,8,11,12,15,17,18;.2) 5 | (9,11,12,13,15,16,18,20;.2) 15 | (20,21,24,27,30,33,34,36;.7) | * |
| S ₃ | (14,15,17,18,20,21,23,25;.4) | (15,16,18,19,20,22,23,25;.5) | (10,11,12,13,15,16,19,21;.6) 5 | * |

Demand

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(b_j)

The minimum fuzzy transportation cost for given problem is equivalent to $7(11,13,14,18,19,20,22,23;.5) + 6(14,15,16,17,19,20,22,24;.4) + 5(6,7,8,11,12,15,17,18;.2)$

+ 15(9,11,12,13,15,16,18,20;.2) + 5(10,11,12,13,15,16,19,21;.6) = (376,436,474,543,607,655,736,800;.2).
Therefore, the ranking function is equivalent to $R(A) = 115.68$.

Results with normalization process:

If all the values used in problem. 1, the parameters are first normalized and the problem is solved by using the HMM method, then the fuzzy optimal value is $x_0 = (376,436,474,543,607,655,736,800;.1)$.

Results Without Normalization Process:

In the same problem, if all the values of parameters are not normalized and then the problem is solved by HMM, then the optimal value of fuzzy is $x_0 = (376,436,474,543,607,655,736,800;.2)$.

Example 2:

| | D ₁ | D ₂ | D ₃ | Supply(a _i) |
|----------------|------------------------------|------------------------------|----------------------------|-------------------------|
| S ₁ | (1,4,6,9,12,15,19,22;.5) | (11,12,16,20,23,27,30,33;.4) | (2,5,8,12,16,18,21,25;.5) | 8 |
| S ₂ | (8,9,12,17,20,24,26,31;.6) | (3,5,8,11,15,19,23,24;.2) | (7,9,13,17,19,21,26,29;.4) | 10 |
| S ₃ | (11,12,16,18,20,25,29,32;.5) | (5,9,10,14,18,23,26,29;.2) | (4,5,8,11,15,17,21,24;.6) | 11 |

(b_j) 10 7 12

5.Comparative and Result Analysis

It is clear that HMM is better and the results are compared from the investigations and it is given in Table 2. It is better than NWCR, MMM, VAM for solving fuzzy transportation problem and the solution of the HMM Problem is an optimal solution.

Table 2

| S. No | ROW | COLUMN | NWCR | MMM | VAM | HMM |
|-------|-----|--------|--------|--------|--------|--------|
| 1. | 3 | 3 | 135.63 | 125.13 | 121.88 | 115.68 |
| 2. | 3 | 3 | 79.50 | 79.5 | 79.50 | 76.80 |

Table 2 represents the solutions obtained by NWCR, MMM, VAM, and HMM. This data says the better performance for the fuzzy transportation problem of the Harmonic Mean Method.

6.Conclusion

From the comparison of Table 2, we can observe that the optimum solution given by the HMM is less than that of other methods. The existing method is also very easy to understand and less computation works. If we use the Harmonic Mean Method to solve a Fuzzy transportation problem, we can get the best solution in a smaller point.

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