Predictors of learning mathematics in 5th-8th graded students

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Abstract

Purpose of the current study was to determine the relative contribution of the student's motivation, fluid reasoning and working memory to the prediction of mathematics learning. The study was conducted at Durgapur KC high School in West Bengal. Out of all 5th-8th graded students, only two hundred late childhood students aged 10-13 years were taken as sample by simple random sampling. In the 2019 academic session, three tools named Muthee achievement motivation inventory (Muthee & Thomas, 2009), Raven's Standard Progressive Matrices (SPM), and Working Memory Index - Digit Span and Letter-Number Sequencing (WISC-IV) were administered to measure students' motivation, fluid reasoning and working memory respectively. The test score for the whole year, including the final exam of the 2019 academic session, were taken as a dependent variable to measure how much students were learning math. The result showed that student's motivation, fluid reasoning and working memory were predictor of math learning statistically significant. Cognitive abilities such as fluid & crystallized intelligence, working memory and mathematical memory (LTM), intrinsic motivation all work together when solving problems, resulting in math learning.

Key words: Fluid intelligence, Math learning, Motivation, Working memory.

1. Introduction-

Mathematics-

Mathematics is one kind of abstract science in which numbers, quantities, space as well as their relationships are accessed by mathematical operation. It is the practice based subjects which deal with definition, fundamental concept, mathematical formula, method, rule and relation, theorem, proof etc. The central component of mathematics is problem solving by logical mathematical thinking or critical thinking using mathematical knowledge of learner. Fundamental process of mathematical problem solving is basically depends on numerical operation i.e. addition, subtraction, multiplication and division by the help of number tables in working memory of problem solver. The school level mathematics syllabus are divided some branches like arithmetic, algebra, geometry (theorems & construction), trigonometry, statistics and mensuration. These branches are eventually studied by different aged students in their classes are shown as follows:

Classes	Developmental Stage(Age)	Piaget's Cognitive Stage	Branches
Nursery/ Pre-Primary	Early Childhood (2-6 yrs)	Pre Operational Stage	Elementary Mathematics
Primary (Grade 1 to 5)	Middle Childhood to Late Childhood (6-11 yrs)	Concrete Operational Stage	Arithmetic
Upper Primary (Grade 6 to 8)	Early Adolescence (12-14 yrs)	Formal Operational Stage	Arithmetic Algebra Geometry
Secondary (Grade 9 to 10)	Middle Adolescence (14-17 yrs)		Arithmetic, Algebra, Geometry (Theorems & Construction), Statistics, Mensuration

Reasoning in Mathematics-
The mathematics subject is consist of different types of reasoning like numerical reasoning, quantitative reasoning, fluid reasoning, logical reasoning, inductive & deductive reasoning, analogical reasoning, relation reasoning, verbal reasoning, abstract reasoning, visual spatial reasoning and geometric reasoning. Reasoning is to discover a rule or principal in a sensible way to reach a conclusion.

In nursery class, numerical reasoning and quantitative reasoning are learnt in elementary mathematics by early childhood learners who are at their pre operational stage. Those students when reaches to primary class have already enter in their middle & late childhood age which known as concrete operational stage, learn basic arithmetic in school. They develop verbal reasoning & logical reasoning ability and four fundamental arithmetic skills i.e. addition, subtraction, multiplication and division by using rapid recall of number tables up to ten. In upper primary class, the early adolescent student from the primary level now starts to learn algebra in addition to their arithmetic and geometry course. In formal operational stage, when their abstract thinking is going to start, they begin to explore more abstract concepts of numeric relationships, representation and symbolism. Algebra course is consists of several reasoning like fluid reasoning and abstract reasoning which promote deeper critical thinking in them as well as inductive reasoning, deductive reasoning, relation reasoning. They already start to explore visuo-spatial reasoning which is also known as geometric reasoning in geometry course. In secondary class these middle adolescent students study pure mathematics including arithmetic, algebra, geometry (theorems & construction), trigonometry, statistics, and mensuration in their formal operational stage. In this study, the researcher was taken 5th-8th graded upper primary students aged 10-13 years.

Predictors of learning mathematics –

Fluid reasoning (FR) or fluid Intelligence (G*f*)-
Fluid reasoning is the best cognitive predictors for learning mathematics (Primi.R., Ferrão.M.E., Almeida.L.S. 2010) and significant predictor of future math achievement (Green et al, 2018). According to Cattell, *"Fluid reasoning (FR) is the capacity to think logically and solve problems in novel situations, independent of acquired knowledge"* (Cattell, 1987). According to Cattell-Horn theory, one of the cognitive theories or process oriented theories of intelligence, there are two types of intelligence namely fluid intelligence (Gf) and crystallized intelligence (Gc). Fluid intelligence is what Cattell referred to as innate ability to flexibly and deliberately solve novel problems independent of acquired knowledge by using different reasoning like abstract reasoning, inductive & logical reasoning and analogical reasoning. Cattell believed that Fluid intelligence is a function of biological or genetical and neurological factors and not influenced by education, acculturation or training. Gf is also the mental capacity for adaptability in novel or strange situation. On the other hand, crystallized intelligence is a knowledge based ability to solve problem by using acquired information or knowledge & skill. So Gc is highly dependent on education, acculturation & training (Horn & Cattell 1966, 1967). More specifically, the Gf can be regarded as *"ability that involves the process of perceiving relation, educing correlates and maintaining span of immediate awareness in concept formation and attainment, reasoning and abstracting"* (Cattell, 1963). Thus fluid intelligence is the perceptual ability of those who identify patterns and find complex relationships to analyze novel problems that result in formation of new concepts, where logic and higher order thinking are used.

When a student solves new math, his brain's fluid intelligence (G*f*) helps him understand how to do it. Because he has not learned the novel math problem before and he does not know how to do it and what rules to apply. That is why G*f* is the most important predictors than other predictors. This leads the student to find out the relation and properties of the new math problem in an inductive or deductive way and logical way. On the other hand, the crystalline intelligence (Gc) of his brain works when the students do a similar new math calculation according to the rules he knows and the method he knows. Gc also drives the student in the case of old math practice.

Mathematical working memory (WM)-
Working memory is the active area in Short term memory where mathematics contextual information (i.e. number, sign, symbol, formula, diagram, mathematical relation, method etc.) are coming through *phonological loop* (for retaining verbal information) and *visual spatial sketchpad* (for visual spatial information) from mathematics material as well as retrieval process of the person's existing knowledge from mathematical memory (LTM), hold and manipulate to synthesize new knowledge in *central executive system* by math problem solver in time of calculation and interpretation (Baddeley and logie, 1999). WM is the vital cognitive factor for learning mathematics (Anderson.U.,2006; Munro.J.,2011; Tzoneva.I., 2015; Weijer-Bergsma E.V. and others, 2014).

From neuroscience perspective, working memory has an innate retrieval (i.e. re-production) power and innate memorization (i.e. retention) capacity to store knowledge to mathematical memory (LTM). In contrast, it has trainable nature itself which can be strengthened by intensive practice and training through several ways from early childhood through adulthood (St. Clair-Thompson, 2010; Morrison & Chein, 2011) as these two abilities share neural networks in the prefrontal and parietal cortices (Gray et al., 2003). Working memory can be strengthened by training and fluid intelligence can also be improved significantly with it depending upon the amount of training (Jaeggi et. al, 2008; Peng et.al, 2017).

Motivation for learning mathematics-
Motivation is a kind of driving force, a desire, a drive that direct students to participate in the activity to achieve their individual's goal. Motivation is the most evident learning factor that directly affects the success of the students in mathematics subject (Gottfried, Fleming, & Gottfried, 2001). Motivation is of two types: one is intrinsic motivation and another is extrinsic motivation. Intrinsic motivation is spontaneous impulse that can be aroused through inner rewards or incentives. It can also be referred as innate desire which challenge student to get pure fun by inherent involvement in sort of enjoyable activity to pursue goals for their own satisfaction. This type of motivation can help the student to gain some state of pleasure by their own sake and that can set a higher internal standard without any external reward or under any pressure (Deci and Ryan, 1985). On the other hand, extrinsic motivation means that external reward which student gets for doing activity related to some separable contingent

outcome that motivates them or makes them avoid negative consequences i.e. punishment or criticism. This type of motivation fulfils the purpose of gaining benefits from the societal norms. (Ryan, Connell, & Grolnick, 1992).Certain behavioral changes come from the use of external rewards or acclaiming an external goal which is initiated by extrinsic motivation.

Intrinsically motivated students generally can solve less or more difficult maths problem by their own interest which drive them to get pure enjoyment by solving the sums in an innovative way (Gottfried, et al, 2001). They are not governed by any external reward or incentives. Those students are more curious about any kind of mathematical problem. They enjoy exploring any new pattern and relationship in math problem. On the other side, extrinsically motivated students who reinforced by external reward like approval from their teachers, parents, peers and society, solve maths problem to avoid getting a poor score in mathematics. Several abstract rewards such as fame, anticipation of receiving praise, public recognition etc. can boost up the student to accomplish the math tasks attentively.
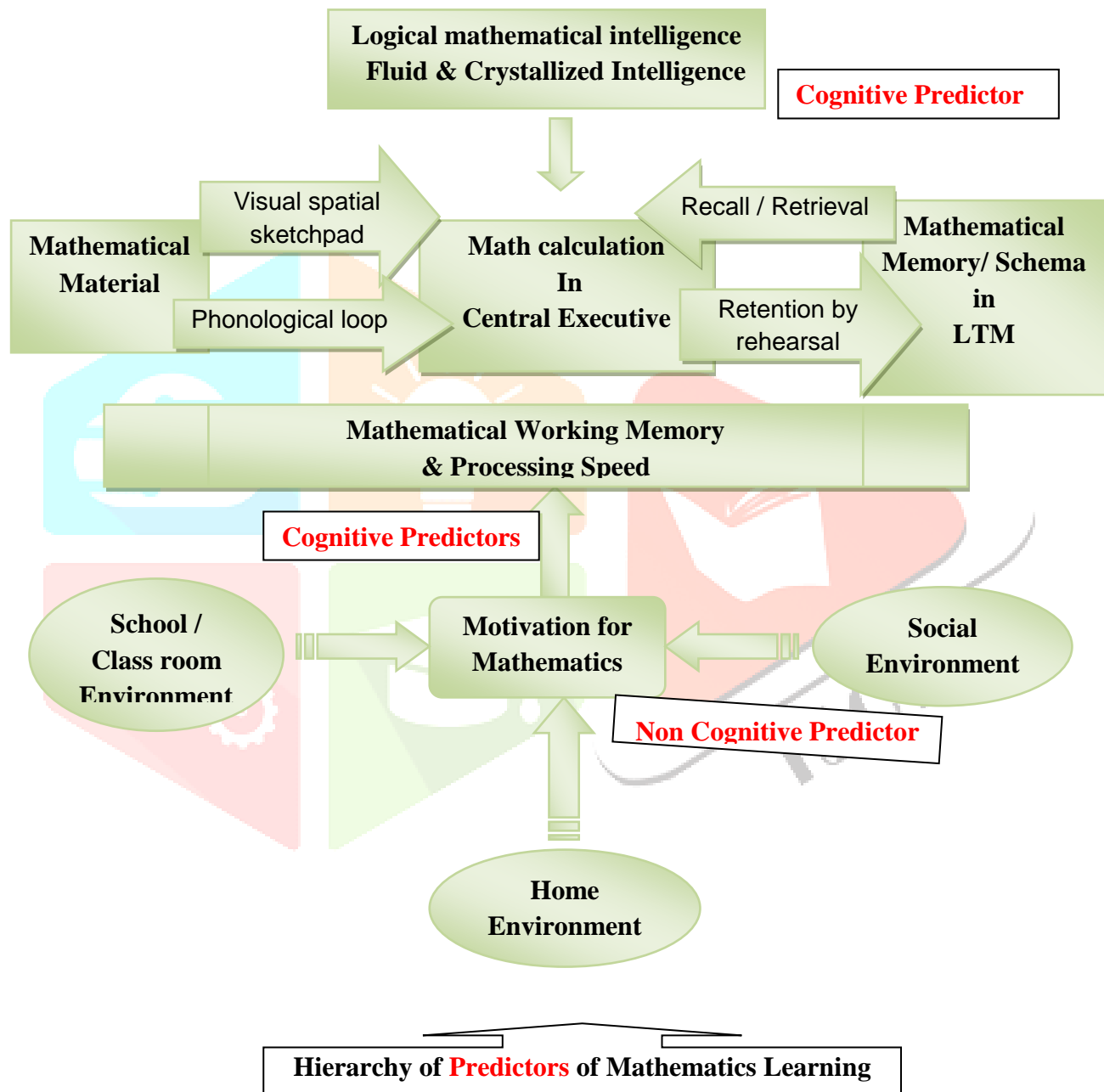

Figure 1: predictors of learning mathematics

The purpose of this study is to find out the contribution of predictors of learning mathematics to learn math in 5th-8th graded school students.

2. Objective of the study:

Objective 1: To determine the relative contribution of the student's motivation, fluid reasoning and working memory to the prediction of mathematics learning.
(H_0 :- There is no significant prediction of mathematics learning with motivation, fluid reasoning and working memory.

3. Method

Participants

All the 5th-8th graded school students of the 2019 academic session of Durgapur KC High School in south 24 parganas were the population of this study. Of these, two hundred late childhood students aged 10-13 years were selected as samples through simple random sampling method.

Materials

In the *2019* academic year, three tools were administered as following.

The Raven's standard progressive matrices (SPM; Raven.J.C, 1938) was used as a measure of fluid reasoning. SPM is a test of a person's capacity at the time of the test to apprehend meaningless geometrical figures presented for his observation and to see the relations between them to complete each matrix is to be conceived. By identifying an underlying characteristic (e.g. rule or trend) that governs the existing pattern, student develops a systematic method of reasoning. The scale consists of 60 problem divided 5 sets of 2×2 , 3×3 matrices as Set-A,B,C,D,E with 12 matrices in each set which progressively become more difficult. The sets contain diagrammatic puzzles exhibiting serial change in two dimensions simultaneously: pattern and shape. Each puzzle has a part missing which is finding out by student from the options provided.

Working memory index (WMI) -digit span & letter number sequencing (WISC-IV) was used as a measure of working memory. It assess student's ability to memorize numeric & alphabet stimulus, hold it in WM, concentrate (to be attentive), and manipulate that stimulus to reproduce immediately in different form. Digit span (DS) subtest divided in two parts- digit forward and digit backward. After listening to the verbal number series, student repeats the number in same sequence or reverse sequence at the time of digit forward and digit backward subtest conduction respectively. At time of letter number sequencing (LN) subtest conduction, the student is presented a mixed series of numbers and letters verbally and rearranges them such that numbers come first, from lowest to highest; then letters are next, in alphabetical order. Both series is consists of 8-10 items and three trials.

Muthee achievement motivation inventory (Muthee & Thomas, 2009) was used as a measure of the motivation of all students to learn mathematics. This scale made based on the important dimensions of achievement motivation as follows: (i) Motivation for achievement (evidenced by competitiveness and goal orientation); (ii) inner resources (evidenced by relaxed style, happiness, patience and self confidence); (iii) inter personal strengths (evidenced by assertiveness, personal diplomacy, extraversion and co-cooperativeness); and (iv) work habits (evidenced by planning and organization, initiatives, and team spirit). The scale has 32 items in total where 18 items were positively worded and 14 are negatively worded. The responses to the items were marked at a five point likert format.

Procedure

In the *2019 academic session*, above three tools were administered to measure students' motivation, fluid reasoning and working memory. Those three independent variable that were measured are the predictor of math learning. The test score for the whole year, including the final exam of the 2019 academic session, were taken as a dependent variable to measure how much students were learning math.

Statistical analysis

Gathered data was analyzed by using correlation, multiple regression method by SPSS.

4. Results

H₀ - There is no significant prediction of mathematics learning with student's motivation, fluid reasoning and working memory.

Table 1: correlations coefficient, Mean and Standard deviation

	Correlation	Partial correlation		
	Mathematics learning (A)	Mathematics learning (A)	Mean	Std. Deviation
Mathematics learning (A)	1.00		27.11	24.39
Motivation (B)	$r_{AB} = .505*$	$r_{AB . CD} = .364*$	101.93	27.875
Fluid Reasoning FR(C)	$r_{AC} = .687*$	$r_{AC. BD} = .499*$	29.80	28.424
Working Memory WM (D)	$r_{AD} = .661*$	$r_{AD. BC} = .417*$	24.66	9.157
N= 200 *- 0.01 level of significance				

Results from Table 1, indicated that there are positive and significant simple correlation between motivation, fluid reasoning and working memory with mathematics learning. The partial correlation coefficient of mathematics learning with motivation is **.364** while controlling the effects of FR and WM. The partial correlation coefficient of mathematics learning with fluid reasoning is

.499 while controlling the effects of motivation and WM. The partial correlation coefficient of mathematics learning with working memory is .417 while controlling the effects of motivation and FR.

Table 2: Predicting the mathematics learning on the base of motivation, fluid reasoning and working memory:

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	**.808ᵃ**	.653	.646	14.514

a. Predictors: (Constant), Motivation, Fluid reasoning, Working memory.

ANOVAᵃ

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	58647.811	3	19549.270	**92.798**	.000ᵇ
Residual	31178.505	196	210.666		
Total	89826.316	199			

a. Dependent Variable: Mathematics learning

b. Predictors: (Constant): Motivation, Fluid reasoning, Working memory

Coefficientsᵃ

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
(Constant)	-28.435	4.942		-5.754	.000
Motivation	.218	.046	**.249**	4.756	.000
Fluid reasoning	.361	.052	**.421**	6.998	.000
Working memory	.914	.164	**.343**	5.575	.000

a. Dependent Variable: Mathematics learning

Multiple correlations (R) between mathematics learning and the linear combination of motivation, fluid reasoning and working memory is .808. The multiple R^2 is .653. The linear combination of motivation, FR and WM explain percent 65% variance in math learning. Adjusted R square value is .646. The calculated value of the F is 92.789 that is significant at .01 level. This clearly means that motivation, FR and WM can linearly predict the math learning. So, we can actually test the significance of each of the **β** (slope of the regression line or regression co-efficient) separately. The **β** coefficient of motivation is .249 and it is significant (t = 4.756, p<.01). The beta coefficient of fluid reasoning is .421 and it is also significant (t = 6.998, p<.01). The beta coefficient of working memory is .343 and it is also significant (t = 5.575, p<.01). This means that if the score on motivation, FR, WM scale increase individually by one unit, the mathematics learning will change by the value of .249, .421,.343 respectively. This indicates that Fluid reasoning is most important, WM is next important, motivation is least important predictor among the three predictors. Model of math learning and its predictors:

Math learning = - 28.435 +.218Motivation +.361Fluid Reasoning+.914Working Memory + e

5. Discussion

Fluid intelligence, crystallized intelligence, working memory, long term memory, motivation all work together when students learn math. For example, a math problem:

In one test, 34% of students failed in arithmetic and 42% in algebra. If 20% of students failed in both subjects then what percentage passed in both subjects?

The math problem will go to the WM through the visual spatial sketchpad where the information will be processed with the help of G*f* and Gc. Whoever has more G*f* can easily understand the logical reasoning of the problem (Twenty percent failed students are in both 34% and 42% of students). This is also agreement with that of *Green et al. (2018).* If the problem is previously known to the problem solver, then working memory will retrieve retent knowledge from LTM, in which case Gc will help. Then there will be information manipulation and calculation in the working memory when the problem solver will understand how to do the problem.

$$[100\% - \{20\% + (34\% - 20\%) + (42\% -20\%)\}]$$
$$= [100 - \{20 +14 +22\}] \%$$
$$= [100 - 56] \%$$
$$= 44 \%$$

Those who have improved working memory will be able to do calculation easily that is in agreement with that of *Osei-Boadi. B. (2016).* Then the mathematical generalization obtained from the problem will be stored in long term memory and the mathematical memory will be expanded. All of those abilities improve with math practice. Again, in order to practice or solve

math, the student needs motivation. Intrinsic motived students are task oriented, always try to solve hard problem until getting right answer that is in agreement with that of *Duda & Nicholls (1992).*

The intrinsically motivated learner who has genetically high fluid intelligence and has improved working memory and developed mathematical memory foundation will be able to learn math more easily. From early childhood to adolescence when developing mathematical abilities (i.e. WM, Gf & Gc) and intrinsic motivation, teachers, parents and stakeholders must take care of them.

Those who have no intrinsic motivation, teacher should arouse desire for mathematical learning in them to produce inherent reward and make them avoid over justification effect i.e. incentive caused decreased motivation. This could be done by making the mathematics teaching process more curious, attention-grabbing and interesting so that student get pleasure and enjoy learning mathematics.

If children practice elementary basic calculations more from early age and practice numbers tables well, then the foundation of their mathematical working memory will be strong. Playing math kids and playing mathematical games on the computer from early age increase their motivation towards learning mathematics and also increase working memory. Since the number system and basic calculations have less attributes to remember, working memory and procedural memory increase as learner practice math over and over again. This strengthens the learner's mathematical skill and increases processing speed, thus taking less time to perform calculation. This is also supported by previous researchers (*Brenda et. al., 2016*). Those whose working memory is well developed can easily recall the number table, can do oral calculation in their mind, and do not have to count fingers. And those who have poor working memory do calculations in the exercise book and do finger counting because they can hold and manipulate a small amount of stimulus in their central executive part of WM. Processing speed and working memory will not improve if learners don't practice calculations in their mind as they get older. Later, when doing complex calculations they will not able to hold and manipulation the stimuli. Then they will do the math by doing calculation in the exercise book and it will take more time to finish the math. So those who are average achievers should practice math more than high achievers because if they practice, their math skill will improve.

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