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Moments Inequalities of some Ageing Classes

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Abstract

In this paper, we consider some classes of lifetime distributions, namely, decreasing mean residual life (DMRL), increasing mean residual life (IMRL), increasing failure rate average (IFRA), new better than used in convex ordering (NBUC), new renewal better than used (NRBU) and its dual new renewal worse used (NRWU) and new better than used failure rate (NBUFR). We derived the moment inequalities of these classes.

Key Words: Lifetime distributions, Aging Classes, Moment inequalities.

1 Introduction

In reliability theory, the concept of aging play an important role in engineering, maintenance, biometrics and many areas of science and technology. The aging concept describe how a component or system improves or deteriorates with age also helps to get an idea in the replacement models. Ever since the work of Barlow et al. (1963) and Bryson and Siddiqui (1969), various classes of life distributions have been introduced in reliability. Classes of life distributions are defined to classify the life distributions according to their aging properties.

Many researchers dedicates their research towards the identification and investigation of some class of lifetime distributions having physical importance. In the past few decades, the classification of life distributions based on some aspects of aging have been studied by statisticians and reliability analysts. Also the attention have been made to derive their moment inequalities.

Among these aspects various forms, we focus on the aging classes which have enormous applications in the areas of biomedical, engineering and statistical studies, namely, decreasing mean residual life (DMRL), increasing mean residual life (IMRL), increasing failure rate average (IFRA), new better than used in convex ordering (NBUC), new renewal better than used (NRBU) and its dual new renewal worse used (NRWU) and new better than used failure rate (NBUFR). We derived the moment inequalities of these classes.

2 Preliminaries

In this section, we give the basic definitions of some aging classes.

Let F be a continuous life distribution with survival function $\overline{F} = 1 - F$ and finite mean $\mu = \int_0^\infty \bar{F}(x)dx$.

Definition 2.1 *Mean Residual Life (MRL) Function*

The Mean Residual Life (MRL) function is defined as

$$m(x) = EX - x|X \geq x = \frac{\int_x^\infty \bar{F}(u)du}{\bar{F}(x)}.$$

Definition 2.2 *Decreasing Mean Residual Life (DMRL)*

A life distribution F of a non-negative aging random variable X is said to be a decreasing mean residual life (DMRL) distribution if $m(x)$ is non-increasing.

Definition 2.3 *Increasing Mean Residual Life (IMRL)*

A life distribution F of a non-negative aging random variable X is said to be a increasing mean residual life (DMRL) distribution if $m(x)$ is non-decreasing.

Definition 2.4 *Increasing Failure Rate Average (IFRA)*

A life distribution F of a non-negative aging random variable X is said to be a increasing failure rate average (IFRA) distribution if for any $0 \leq \alpha \leq 1$

$$\bar{F}(\alpha x) \leq \bar{F}^{\alpha}(x) x \geq 0.$$

Definition 2.5 *New Better than Used in Convex ordering (NBUC)*

A life distribution F of a non-negative aging random variable X is said to be a new better than used in convex ordering (NBUC) distribution if

$$\int_x^\infty \bar{F}(t+u)du \leq \bar{t} \int_x^\infty \bar{F}(u)du, \qquad x, t \geq 0.$$

Definition 2.6 *New Renewal Better than Used NRBU*

A life distribution F with $F(0) = 0$ of a non-negative aging random variable X is said to be a new renewal better than used (NRBU) distribution if

$$T_t \leq_{st} T_w \text{ i. e } T_t \text{ is stochastically less then } T_w.$$

In otherwords, a life distribution F with $F(0) = 0$ of a non-negative aging random variable X is said to be a new renewal better than used (NRBU) distribution if

$$\bar{F}_t(x) \leq m(x).$$

Definition 2.7 *New Renewal Worse than Used NRWU*

A life distribution F with $F(0) = 0$ of a non-negative aging random variable X is said to be a new renewal worse than used (NRWU) distribution if

$$T_t \geq_{st} T_w \text{ i. e } T_t \text{ is stochastically greater then } T_w.$$

In otherwords, a life distribution F with $F(0) = 0$ of a non-negative aging random variable X is said to be a new renewal worse than used (NRWU) distribution if

$$\bar{F}_t(x) \geq m(x).$$

Let T be a random variable represents life time of a device (system or component) with a continuous life distribution F(t). Upon arising the failure of the device, it can be substituted by a sequence of mutually independent devices which are identically distributed with the same life distribution F(t). The following stationary renewal distribution constitutes the remaining life distribution of the device under operation at time t.

$$W_F(t) = \mu_F^{-1} \int_0^t \overline{F}(u)du, \qquad t \geq 0,$$

where $\mu_F = \mu = \int_0^\infty \overline{F}(u)du$.

It is easy to show that

$$\overline{W_F}(t) = \mu_F^{-1} \int_t^\infty \overline{F}(u)du, \qquad t \geq 0.$$

Definition 2.8 *If X is a random variable with survival function $\overline{F}(x)$, then X is said to have new better (worse) than renewal used property, denoted by NBRU (NWRU), if*

$$\overline{W_F}(x|t) \leq (\geq)\overline{F}(x|0), \qquad x \leq 0, t \geq\geq 0,$$

or

$$\overline{W_F}(x+t) \leq (\geq)\overline{W_F}(t)\overline{F}(x), \qquad x \leq 0, t \geq\geq 0,$$

Depending on the above definition, (Mahmoud, EL-Sagheer, and Etman 2016) defined a new class which is called new better (worse) than renewal used in Laplace transform order NBRUL (NWRUL) as follows.

Definition 2.9 *X is said to be NBRUL (NWRUL) if*

$$\int_0^\infty e^{-sx}\overline{W_F}(x+t)dx \leq (\geq)\overline{W_F}(t)\int_0^\infty e^{-sx}\overline{F}(x)dx \qquad x, t, s \geq 0.$$

Any random lifetime may be modelled by a nonnegative random variable $X > 0$ with cumulative distribution function F and survival function $\bar{F} = 1 - F$. For any random variable X, let

$$X_t = [X - t|X > t] \quad for\ t \in \{x: F(x) < 1\}$$

denote the conditional random variable X on condition $X > t$. When X is the lifetime of a device, X_t represents the residual lifetime of the device at time t, given that the device has survived up to time t. Its survival function is

$$\bar{F}_t(x) = \frac{\bar{F}(t+x)}{\bar{F}(t)} for \bar{F}(t) > 0,$$

where $\bar{F}(x)$ is the survival function of X.

Let X be a non-negative random variable representing the lifetime of an equipment with absolutely continuous distribution function $\mathrm{F(x) = P(X < x)}$, survival function $\mathrm{\bar{F}(x) = 1 - F(x)}$ and density function $\mathrm{f(x) = \frac{dF(x)}{dx}}$. The corresponding failure rate r(t) is defined by

$$\mathrm{r(t) = \frac{f(t)}{\bar{F}(t)}}$$

Then the following nonparametric classes of lifetime distributions have been introduced for $\mathrm{t \geq 0}$.

Definition 2.10 *New Better than Used in Failure Rate (NBUFR)*

A life distribution F of a non-negative aging random variable X is said to be a new better than used in failure rate (NBUFR) distribution if

$$\lim_{x\to 0^+}\frac{F(x)}{x} \text{ exists and } \lim_{t\to 0^+} t^{-1}\int_0^t r(x)dx \leq r(t).$$

3 Moment Inequalities

In this section, the moment inequalities for the above considered aging class are established.

Theorem 3.1 *If F is DMRL (IMRL), then*

$$\mu_{(2)} \geq (\leq)\frac{\mu^2}{2},$$

where $\mu_{(r)} = E[\min(X_1, X_2)]^r$.

Proof: Since F is DMRL, $m(x) = \gamma(x)/\overline{F}(x)$ is $\downarrow$ and so $(d/dx)m(x) \leq 0$, i.e.,

$$\overline{F}^2(x) \geq \gamma(x)f(x), \tag{1}$$

where $\gamma_{(r)} = \int_x^\infty \overline{F}(u)du$.

Multiplying both sides in (1) by x and integrating over $(0, \infty)$, w. r. t. x

$$\int_0^\infty x\overline{F}^2(x)dx \geq \int_0^\infty x\gamma(x)dF(x). \tag{2}$$

But the right-hand side of (1) is given by

$$\int_0^\infty x\gamma(x)dF(x) = -\int_0^\infty x\bar{F}^2(x)dx - \int_0^\infty \gamma(x)\cdot\gamma'(x)dx$$
$$= -\int_0^\infty x\bar{F}^2(x)dx + \frac{\gamma^2(0)}{2}. \quad (3)$$

Substituting $\gamma(0) = \int_0^\infty \bar{F}(x)dx = \mu$ in (3), we obtain

$$\int_0^\infty x\gamma(x)dF(x) = -\int_0^\infty x\bar{F}^2(x)dx + \frac{\gamma^2(0)}{2}. \quad (4)$$

Using (4), the inequality in (2) becomes

$$2\int_0^\infty x\bar{F}^2(x)dx \geq \frac{\mu^2}{2}. \quad (5)$$

Since $\int_0^\infty x\bar{F}^2(x)dx = E\min(X_1, X_2)^{r+1}/(r+1)$, cf. Ahmad (2001), inequality (5) becomes $\mu_{(2)} \geq \mu^2/2$. The result follows.

Theorem 3.2 *If F is DMRL, then for all integers $r \geq 0$,*

$$(r+1)E[X_1\{\min(X_1, X_2)\}^r] \geq (r+2)E\{\min(X_1, X_2)\}^{r+1}.$$

Corollary 3.1 *Let $r = 0$, then*

$$\mu \geq 2E\{\min(X_1, X_2)\}.$$

Theorem 3.3 *If F is IFRA, then for all integers $r \geq 0$ and any $0 < \alpha < 1$,*

$$E(X_1^{r+1}) \geq E\left\{\min\left(\frac{X_1}{\alpha}, \frac{X_2}{1-\alpha}\right)^{r+1}\right\},$$

where X_1 and X_2 are two nonnegative independent copies of random variables with distribution F.

Corollary 3.2 *Let $r = 0$, then*

$$\mu \geq E\{\min(X_1/\alpha, X_2/(1-\alpha))\},$$

where $\mu = E(X)$.

Theorem 3.4 *If F is NBUC, then for all integers r and $s \geq 0$,*

$$(r+1)!\,(s+1)!\,E(X^{r+s+3}) \leq (r+s+3)!\,E(X^{r+2})E(X^{s+1}).$$

Corollary 3.3 *Let $r = 0$, then*

$$2((s+1)!)E(X^{s+3}) \leq (s+3)!\,E(X^2)E(X^{s+1}).$$

Corollary 3.4 *Let $s = 0$, then*

$$(r+2)!\,E(X^{r+3}) \leq (r+3)!\,E(X)E(X^{r+2}).$$

Corollary 3.5 *Let $r = s = 0$, then*

$$E(X^3) \leq 3E(X)E(X^2).$$

Theorem 3.5 *If F is NRBU (NRWU), then*

$$\frac{1}{2(r+2)!}\mu\mu_{(2r+2)} \leq (\geq) \frac{1}{(r+1)!(r+2)!}\mu_{(r+1)}\mu_{(r+2)}, \qquad r \leq 1.$$

Theorem 3.6 *Let F be NBRUL life distribution such that all moments exist and finite then for integers $r \geq 0$ and $s \geq 0$. Then*

$$\frac{\mu_{(r+2)}}{s(r+1)(r+2)}[1-\zeta(s)] \geq \frac{-(-1)^r r!}{s^{r+2}}\left[\mu_F - \frac{1}{s}(1-\zeta(s))\right] + \frac{r!}{s^{r+1}}\sum_r^{i=0}(-1)^i\frac{s^{r-i}}{(r-i+2)!}\mu_{(r-i+2)},$$

where $\mu_{(r)} = E(X^r), \zeta(s) = Ee^{-sX}$.

Corollary 3.6 *For $r = 1$, then equation (23) will be reduced to*

$$\frac{\mu_3}{6s}[1-\zeta(s)] \leq \frac{1}{s^3}\left[\mu - \frac{1}{s}(1-\zeta(s))\right] + \frac{1}{s^2}\left[\frac{s}{6}\mu_{(3)} - \frac{1}{2}\mu_{(2)}\right],$$

where $\mu_{(r)} = \int_0^\infty x^r dF(x)$.

Theorem 3.7 *If F is NBUFR (NWUFR), then for some integers $r, s \geq 0$ the inequality*

$$\lambda_{(r+s+2)} \leq (\geq) \lambda_{(1+r)} \left(\frac{1}{r(0)}\right)^{s+1}$$

holds.

Corollary 3.7

- If $r = s = 0$, then $\mu_{(2)} \leq \frac{2\mu_{(1)}}{r(0)}$

- If $r = 0$, then $\mu_{(s+2)} \leq \frac{(s+2)!}{(r(0))^{s+1}}$

- If $s = 0$, then $\mu_{(r+2)} \leq \frac{(r+2)\mu_{(r+1)}}{r(0)}$.

4 Conclusion

In this paper, we studied some classes of lifetime distributions, namely, decreasing mean residual life (DMRL), increasing mean residual life (IMRL), increasing failure rate average (IFRA), new better than used in convex ordering (NBUC), new renewal better than used (NRBU) and its dual new renewal worse used (NRWU) and new better than used failure rate (NBUFR). We derived the moment inequalities of these classes.

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