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STEADY PLANE COUETTE FLOW OF VISCOUS INCOMPRESSIBLE FLUID BETWEEN TWO POROUS PARALLEL PLATES THROUGH POROUS MEDIUM WITH MAGNETIC FIELD

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ABSTRACT: *In this paper, we have investigated the Steady Plane Couette Flow of Viscous Incompressible Fluid between two Porous Parallel Plates through Porous Medium with Magnetic Field. We have studied the velocity, average Velocity, Shear stress, Skin frictions, volumetric flow, Drag Coefficients & Streamlines.*

KEY WORDS: *Steady Couette Flow, Viscous Parallel Plates, Incompressible Fluid, Porous Medium & Magnetic Field*

NOMENCLATURE

u = Velocity component along x -axis

v = Velocity component along y -axis

t = The Time

ρ = The Density of Fluid

p = The Fluid Pressure

k = The Thermal Conductivity

μ = Coefficient of Viscosity

ν = Kinematic Viscosity

Q = The Volumetric Flow

INTRODUCTION

We have investigated the Steady Plane Couette Flow of Viscous Incompressible Fluid between two Porous Parallel Plates through Porous Medium with Magnetic Field. Attempts have been made by several researchers i.e. O. R. Burggraf [1] investigated Analytical and Numerical Studies of the Structure of Steady Separated Flows. O. R. Burggraf [2] investigated Computational Study of Supersonic Flow over Backward-Facing Steps at High Reynolds Number. G. I. Busws [3] investigated the Construction of Special Explicit Solution of the Boundary Layer Equations Steady Flows. K. Butler & B. F. Farrell [4] investigated Three-Dimensional Optimal Perturbations in Viscous Shear Flow. Canadam & T. Mulnke [5] investigated Forced Vibrations of a Piezoelectric Layer of Six (mm) Crystal Class. V. C. Carey & J. C. Mollendorf [6] investigated Variable Viscosity Effects in Several Natural Convection Flows. W. Cazemier, R.W.C.P. Verstappen, & A. E. P. Veldman [7] investigated Proper Orthogonal Decomposition and Low-Dimensional Models for the Driven Cavity Flows. T. Cebeci & P. Bradshaw [8] investigated Momentum Transfer in Boundary Layers. T. Cebeci & K. C. Chang [9] investigated a General Method for Calculating Momentum and Heat Transfer in Laminar and Turbulent Duct Flows. T. Cebeci, F. Thiele, P. G. Williams & K. Stewartson [10] investigated on the Calculation of Symmetric Waves-I, Two-Dimensional Flow. I. Celik & Z. Wei-Mung [11] investigated Calculation of Numerical Uncertainty Using Richardson Extrapolating Application to Some Simple Turbulent Flow Calculations. Chamkha and H. Al-Naser [12] investigated Hydro Magnetic Double-Diffusive Convection in a Rectangular Enclosure with Uniform Side Heat and Mass Fluxes and Opposing Temperature and Concentration

Gradients. O. P. Chandna & E. O. Oku-Ukpong [13] investigated Some Solutions of Second Grade Fluid Flow Von Misses Co-ordinates Transformations. D. S. Chauhan & R. Agarwal [14] investigated MHD through a Porous Medium Adjacent to a Stretching Sheet Numerical and an Approximation Solution. G. Chavepeyer, J. K. Plat Ten, & M. B. Bada [15] investigated Laminar Thermal Convection in a Vertical Slot. A. Gulhan, T. Thiele, F. Siebe, B. Kronen & T. Schleitker [16] investigated Aero Thermal Measurements from the ExoMars Schiaparelli Capsule Entry. I. Hashem & M. H. Mohamed [17] investigated Aerodynamic Performance Enhancements of H-Rotor Darrieus Wind Turbine. M. Jafari, A. Razavi & M. Mirhosseini [18] investigated Affect of Airfoil Profile on Aerodynamic Performance and Economic Assessment of H-Rotor Vertical Axis Wind Turbines. In this paper, we have investigated the Velocity, average Velocity, Shear stress, Skin frictions, volumetric flow, Drag Coefficients & Streamlines.

FORMULATION OF THE PROBLEM

Let us consider two infinite Porous plates AB & CD separated by a distance 2h. The fluid enters in y-direction. The velocity component along x-axis is a function of y only. The motion of incompressible fluid is in two dimension and is steady then

$$u = u(y), w = 0 \text{ \& } \frac{\partial}{\partial t} \equiv 0$$

The equation of continuity for incompressible fluid

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \text{ put } w = 0 \text{ \& } \frac{\partial u}{\partial x} = 0 \Rightarrow \frac{\partial v}{\partial y} = 0$$

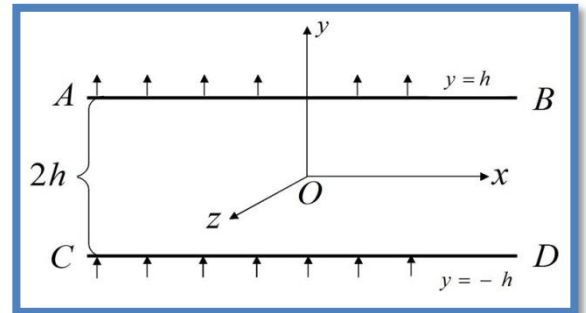


Figure-1

v is independent of y but motion is along y-axis. So we can say that v is constant velocity i.e. $v = v_0$ or the fluid enters in flow region through one plate at the same constant velocity v_0 .

Also Navier-Stoke's equations for incompressible fluid in the absence of body force when flow is steady

$$v_0 \frac{du}{dy} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{d^2u}{dy^2} + \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right) \nu u \dots\dots (1) \text{ \& } -\frac{1}{\rho} \frac{\partial p}{\partial y} = 0 \dots\dots (2)$$

SOLUTION OF THE PROBLEM

Equation (2) shows that the pressure does not depend on y hence p is a function of x only & so equation (1) reduces to

$$\frac{dp}{dx} = \rho \left(\nu \frac{d^2u}{dy^2} - v_0 \frac{du}{dy} + \frac{\nu u}{k} + \frac{\sigma B_0^2 \nu u}{\mu} \right) \Rightarrow \frac{d^2u}{dy^2} - \frac{v_0}{\nu} \frac{du}{dy} + \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right) u = -\frac{P}{\rho \nu} \text{ where } \frac{dp}{dx} = -P$$

$$\text{A.E. } m^2 - \frac{v_0}{\nu} m + \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right) = 0 \Rightarrow m = \frac{\frac{v_0}{\nu} \pm \sqrt{\left(\frac{v_0}{\nu} \right)^2 - 4 \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right)}}{2} = \frac{v_0}{2\nu} \pm \sqrt{\left(\frac{v_0}{2\nu} \right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right)}$$

$$\text{Let } \sqrt{\left(\frac{v_0}{2\nu} \right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right)} = A, \quad \frac{1}{k} + \frac{\sigma B_0^2}{\mu} = B \quad \& \quad \left(\frac{v_0}{2\nu} \right)^2 > \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right)$$

$$\therefore \text{C.F.} = e^{\frac{v_0}{2\nu} y} [C_1 \text{Cosh } Ay + C_2 \text{Sinh } Ay] \text{ \& } \text{P.I.} = -\frac{P}{\mu B} \Rightarrow u(y) = e^{\frac{v_0}{2\nu} y} [C_1 \text{Cosh } Ay + C_2 \text{Sinh } Ay] - \frac{P}{\mu B}$$

Using boundary conditions $u = 0$ at $y = -h$ & $u = U$ at $y = h$

$$e^{-\frac{v_0}{2\nu} h} [C_1 \text{Cosh } Ah - C_2 \text{Sinh } Ah] - \frac{P}{\mu B} = 0 \dots\dots (3) \text{ \& } U = e^{\frac{v_0}{2\nu} h} [C_1 \text{Cosh } Ah + C_2 \text{Sinh } Ah] - \frac{P}{\mu B} \dots\dots (4)$$

$$\Rightarrow \frac{P}{\mu B} e^{\frac{v_0}{2\nu} h} = C_1 \text{Cosh } Ah - C_2 \text{Sinh } Ah \text{ \& } \left(U + \frac{P}{\mu B} \right) e^{-\frac{v_0}{2\nu} h} = C_1 \text{Cosh } Ah + C_2 \text{Sinh } Ah$$

$$C_1 = \frac{1}{2 \text{Cosh } Ah} \left[\left(U + \frac{P}{\mu B} \right) e^{-\frac{v_0 h}{2v}} + \frac{P}{\mu B} e^{\frac{v_0 h}{2v}} \right] \quad \& \quad C_2 = \frac{1}{2 \text{Sinh } Ah} \left[\left(U + \frac{P}{\mu B} \right) e^{-\frac{v_0 h}{2v}} - \frac{P}{\mu B} e^{\frac{v_0 h}{2v}} \right]$$

$$u(y) = \frac{e^{\frac{v_0 y}{2v}} \text{Cosh } Ay}{2 \text{Cosh } Ah} \left\{ \left(U + \frac{P}{\mu B} \right) e^{-\frac{v_0 h}{2v}} + \frac{P}{\mu B} e^{\frac{v_0 h}{2v}} \right\} + \frac{e^{\frac{v_0 y}{2v}} \text{Sinh } Ay}{2 \text{Sinh } Ah} \left\{ \left(U + \frac{P}{\mu B} \right) e^{-\frac{v_0 h}{2v}} - \frac{P}{\mu B} e^{\frac{v_0 h}{2v}} \right\} - \frac{P}{\mu B}$$

$$u(y) = \left(U + \frac{P}{\mu B} \right) \frac{e^{\frac{v_0 (y-h)}{2v}} \text{Sinh } A(y+h)}{2 \text{Sinh } Ah \text{Cosh } Ah} - \frac{P}{\mu B} \frac{e^{\frac{v_0 (y+h)}{2v}} \text{Sinh } A(y-h)}{2 \text{Sinh } Ah \text{Cosh } Ah} - \frac{P}{\mu B}$$

$$u(y) = \frac{1}{\text{Sinh } 2Ah} \left[\left(U + \frac{P}{\mu B} \right) e^{\frac{v_0 (y-h)}{2v}} \text{Sinh } A(y+h) - \frac{P}{\mu B} e^{\frac{v_0 (y+h)}{2v}} \text{Sinh } A(y-h) \right] - \frac{P}{\mu B} \dots\dots\dots (5)$$

FOR PLANE COUETTE FLOW In this case put $P = 0$ in equation (5)

$$u(y) = \frac{1}{\text{Sinh } 2Ah} \left[U e^{\frac{v_0 (y-h)}{2v}} \text{Sinh } A(y+h) \right] \dots\dots\dots (6)$$

THE SHEAR STRESS AT ANY POINT

$$\sigma_{xy} = \mu \frac{du}{dy} = \frac{\mu U}{\text{Sinh } 2Ah} \left[\frac{v_0}{2v} e^{\frac{v_0 (y-h)}{2v}} \text{Sinh } A(y+h) + A e^{\frac{v_0 (y-h)}{2v}} \text{Cosh } A(y+h) \right]$$

$$\sigma_{xy} = \frac{\mu U e^{\frac{v_0 (y-h)}{2v}}}{\text{Sinh } 2Ah} \left[\frac{v_0}{2v} \text{Sinh } A(y+h) + A \text{Cosh } A(y+h) \right] \dots\dots\dots (7)$$

THE SKIN FRICTIONS AT LOWER AND UPPER PLATE

$$(\sigma_{xy})_{y=h} = \frac{\mu U}{\text{Sinh } 2Ah} \left[\frac{v_0}{2v} \text{Sinh } 2Ah + A \text{Cosh } 2Ah \right] = \mu U \left[\frac{v_0}{2v} + A \text{Coth } 2Ah \right] \dots\dots\dots (8)$$

$$(\sigma_{xy})_{y=-h} = \frac{\mu U e^{-\frac{v_0 h}{2v}}}{\text{Sinh } 2Ah} A = \frac{\mu U A e^{-\frac{v_0 h}{2v}}}{\text{Sinh } 2Ah} \dots\dots\dots (9)$$

THE AVERAGE VELOCITY DISTRIBUTION IN PLANE COUETTE FLOW

$$(u)_{av} = \frac{1}{2h} \int_{-h}^h \frac{U}{\text{Sinh } 2Ah} e^{\frac{v_0 (y-h)}{2v}} \text{Sinh } A(y+h) dy = \frac{U}{2h \text{Sinh } 2Ah} \int_{-h}^h e^{\frac{v_0 (y-h)}{2v}} \left\{ \frac{e^{A(y+h)} - e^{-A(y+h)}}{2} \right\} dy$$

$$(u)_{av} = \frac{U}{4h \text{Sinh } 2Ah} \left[\frac{\left(\frac{v_0}{2v} - A \right) \left(e^{2Ah} - e^{-\frac{v_0 h}{v}} \right) - \left(\frac{v_0}{2v} + A \right) \left(e^{-2Ah} - e^{-\frac{v_0 h}{v}} \right)}{\left\{ \left(\frac{v_0}{2v} \right)^2 - A^2 \right\}} \right]$$

Since $\sqrt{\left(\frac{v_0}{2v} \right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right)} = A, \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu} \right) = B \Rightarrow \left(\frac{v_0}{2v} \right)^2 - A^2 = B$

$$(u)_{av} = \frac{U}{4Bh \text{Sinh } 2Ah} \left[\frac{v_0}{2v} \left\{ e^{2Ah} - e^{-\frac{v_0 h}{v}} - e^{-2Ah} + e^{-\frac{v_0 h}{v}} \right\} - A \left\{ e^{2Ah} - e^{-\frac{v_0 h}{v}} + e^{-2Ah} - e^{-\frac{v_0 h}{v}} \right\} \right]$$

$$(u)_{av} = \frac{U}{2Bh \text{Sinh } 2Ah} \left[\frac{v_0}{2v} \text{Sinh } 2Ah - A \left(\text{Cosh } 2Ah - e^{-\frac{v_0 h}{v}} \right) \right] \dots\dots\dots (10)$$

THE VOLUMETRIC FLOW

$$Q = 2h u_{av} = \frac{U}{B \sinh 2Ah} \left[\frac{v_0}{2\nu} \sinh 2Ah - A \left(\cosh 2Ah - e^{-\frac{v_0 h}{\nu}} \right) \right] \dots\dots\dots(11)$$

THE DRAG COEFFICIENTS

$$(C_f)_{y=h} = \frac{(\sigma_{xy})_{y=h}}{\frac{1}{2} \rho u_{av}^2} = \frac{\frac{\mu U}{\sinh 2Ah} \left(\frac{v_0}{2\nu} \sinh 2Ah + A \cosh 2Ah \right)}{\frac{1}{2} \rho \frac{U^2}{4B^2 h^2 \sinh^2 2Ah} \left(\frac{v_0}{2\nu} \sinh 2Ah - A \cosh 2Ah + A e^{-\frac{v_0 h}{\nu}} \right)^2}$$

$$(C_f)_{y=h} = \frac{8B^2 h^2 \mu \sinh 2Ah \left(\frac{v_0}{2\nu} \sinh 2Ah + A \cosh 2Ah \right)}{\rho U \left(\frac{v_0}{2\nu} \sinh 2Ah - A \cosh 2Ah + A e^{-\frac{v_0 h}{\nu}} \right)^2} \dots\dots\dots(12)$$

$$(C_f)_{y=-h} = \frac{\mu U A e^{-\frac{v_0 h}{\nu}}}{\sinh 2Ah} \frac{8B^2 h^2 \sinh^2 2Ah}{\rho U^2 \left(\frac{v_0}{2\nu} \sinh 2Ah - A \cosh 2Ah + A e^{-\frac{v_0 h}{\nu}} \right)^2}$$

$$(C_f)_{y=-h} = \frac{8\mu B^2 h^2 A e^{-\frac{v_0 h}{\nu}} \sinh 2Ah}{\rho U \left(\frac{v_0}{2\nu} \sinh 2Ah - A \cosh 2Ah + A e^{-\frac{v_0 h}{\nu}} \right)^2} \dots\dots\dots(13)$$

THE STREAM LINE IN THE PLANE COUETTE FLOW

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \text{ where } \vec{q} = u\hat{i} + v\hat{j} + w\hat{k} \Rightarrow \frac{dx}{\frac{U}{\sinh 2Ah} e^{\frac{v_0}{2\nu}(y-h)} \sinh A(y+h)} = \frac{dy}{v_0} = \frac{dz}{0}$$

Taking first two equations $\frac{v_0 \sinh 2Ah}{U} \int dx = \int e^{\frac{v_0}{2\nu}(y-h)} \sinh A(y+h) dy + C_1$

$$\frac{v_0 \sinh 2Ah}{U} x - \int e^{\frac{v_0}{2\nu}(y-h)} \left\{ \frac{e^{A(y+h)} - e^{-A(y+h)}}{2} \right\} dy = C_1$$

$$\Rightarrow \frac{v_0}{U} \sinh 2Ah x - \frac{1}{2} \int \left\{ e^{\frac{v_0}{2\nu}(y-h)+A(y+h)} - e^{\frac{v_0}{2\nu}(y-h)-A(y+h)} \right\} dy = C_1$$

$$\Rightarrow \frac{v_0}{U} \sinh 2Ah x - \frac{1}{2} \left[\frac{e^{\frac{v_0}{2\nu}(y-h)+A(y+h)}}{\left(\frac{v_0}{2\nu} + A\right)} - \frac{e^{\frac{v_0}{2\nu}(y-h)-A(y+h)}}{\left(\frac{v_0}{2\nu} - A\right)} \right] = C_1$$

$$\Rightarrow \frac{v_0}{U} \text{Sinh } 2Ah \ x - \frac{1}{2B} \left[\left(\frac{v_0}{2v} - A \right) e^{\frac{v_0}{2v}(y-h)} e^{A(y+h)} - \left(\frac{v_0}{2v} + A \right) e^{\frac{v_0}{2v}(y-h)} e^{-A(y+h)} \right] = C_1$$

$$\Rightarrow \frac{v_0}{U} \text{Sinh } 2Ah \ x - \frac{e^{\frac{v_0}{2v}(y-h)}}{2B} \left\{ \frac{v_0}{2v} \left\{ e^{A(y+h)} - e^{-A(y+h)} \right\} - A \left\{ e^{A(y+h)} + e^{-A(y+h)} \right\} \right\} = C_1$$

$$\Rightarrow \frac{v_0}{U} \text{Sinh } 2Ah \ x - \frac{e^{\frac{v_0}{2v}(y-h)}}{2B} \left\{ \frac{v_0}{v} \text{Sinh } A(y+h) - 2A \text{Cosh } A(y+h) \right\} = C_1$$

$$\Rightarrow \frac{v_0}{U} \text{Sinh } 2Ah \ x - \frac{e^{\frac{v_0}{2v}(y-h)}}{B} \left\{ \frac{v_0}{2v} \text{Sinh } A(y+h) - A \text{Cosh } A(y+h) \right\} = C_1 \dots \dots \dots (14)$$

& Second stream line is given by $z = C_2 \dots \dots \dots (15)$

Now $\text{Curl } \vec{q} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{pmatrix} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{U e^{\frac{v_0}{2v}(y-h)} \text{Sinh } A(y+h)}{\text{Sinh } 2Ah} & v_0 & 0 \end{pmatrix}$

$$\text{Curl } \vec{q} = - \frac{U e^{\frac{v_0}{2v}(y-h)}}{\text{Sinh } 2Ah} \left[\frac{v_0}{2v} \text{Sinh } A(y+h) + A \text{Cosh } A(y+h) \right] \hat{k} \neq \vec{0}$$

\Rightarrow Motion of the Fluid is Rotational.

Comparison between Porous medium, Magnetic Field & Porous medium with Magnetic Field

Tables for velocity and skin friction

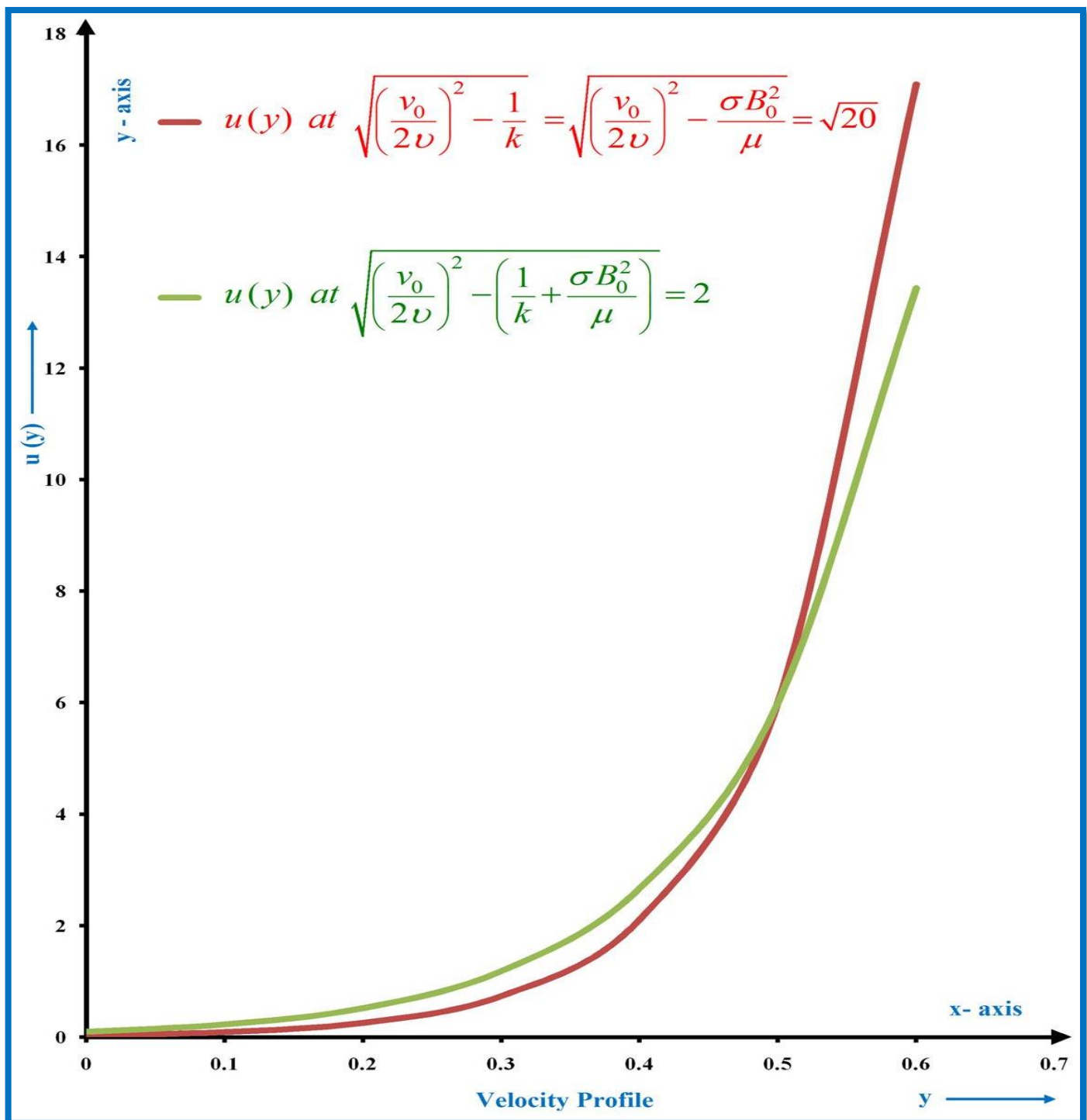
Let $U=6, \mu=h=.5, \frac{v_0}{2v}=6$ & $A = \sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ all are fixed

Let $\frac{1}{k}$ & $\frac{\sigma B_0^2}{\mu}$ are vary $\Rightarrow \sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{1}{k}}$ & $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{\sigma B_0^2}{\mu}}$ are also vary.

Case(1): Let $\frac{1}{k} = \frac{\sigma B_0^2}{\mu} = 16 \Rightarrow \sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{1}{k}} = \sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{20} \therefore \sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$

Table-1 (for velocity)

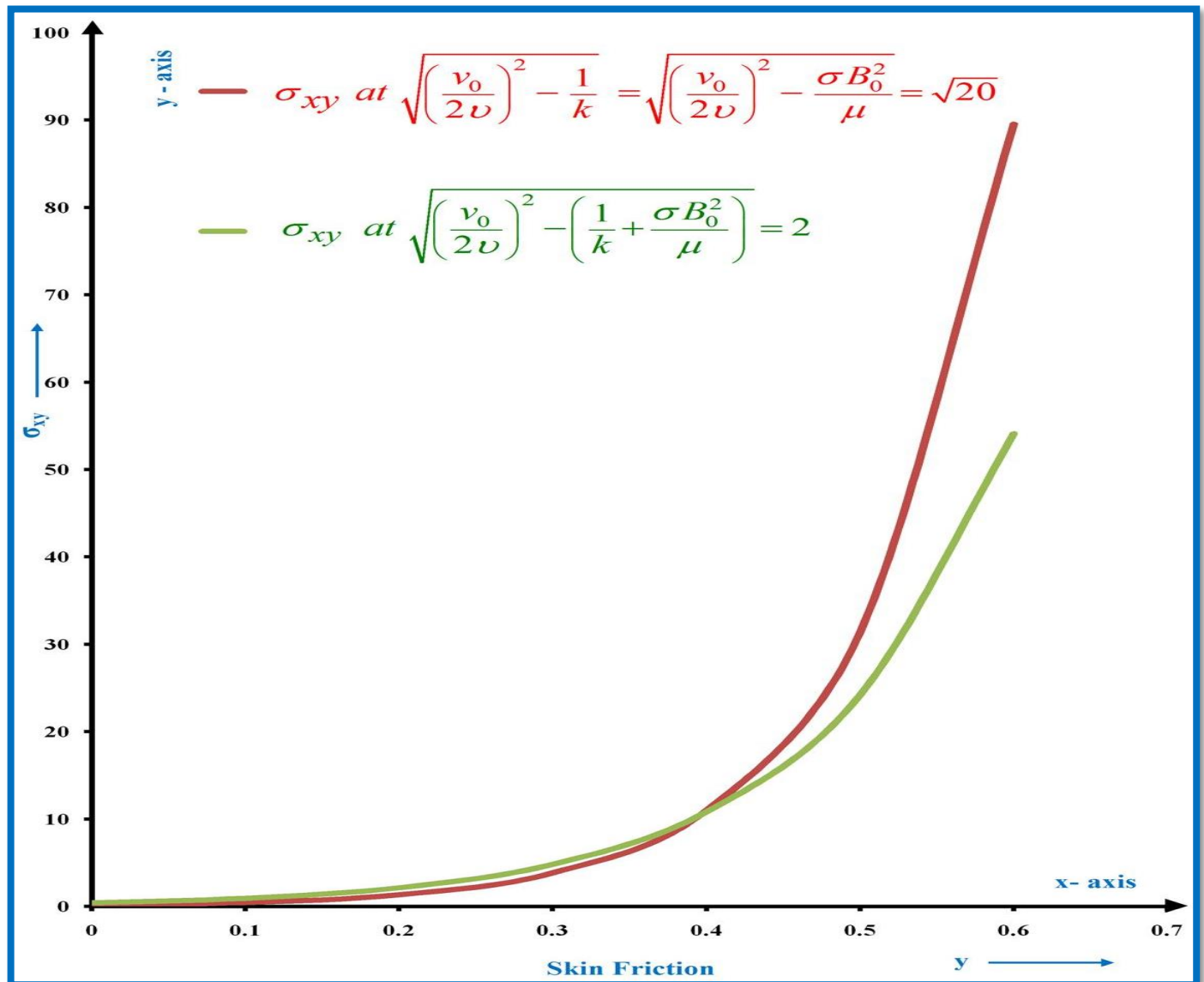
	y	0	.1	.2	.3	.4	.5	.6
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = \sqrt{20}$	u(y)	.032	.091	.259	.738	2.11	6	17.09
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{20}$	u(y)	.032	.091	.259	.738	2.11	6	17.09
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$	u(y)	.097	.227	.52	1.184	2.67	6	13.44



Graph of table-1

Table-2 (for skin friction)

	y	0	.1	.2	.3	.4	.5	.6
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = \sqrt{20}$	σ_{xy}	.167	.476	1.36	3.87	11.02	31.42	89.54
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{20}$	σ_{xy}	.167	.476	1.36	3.87	11.02	31.42	89.54
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$	σ_{xy}	.417	.95	2.15	4.835	10.84	24.22	54.08



Graph of table-2

Tables for velocity and skin friction

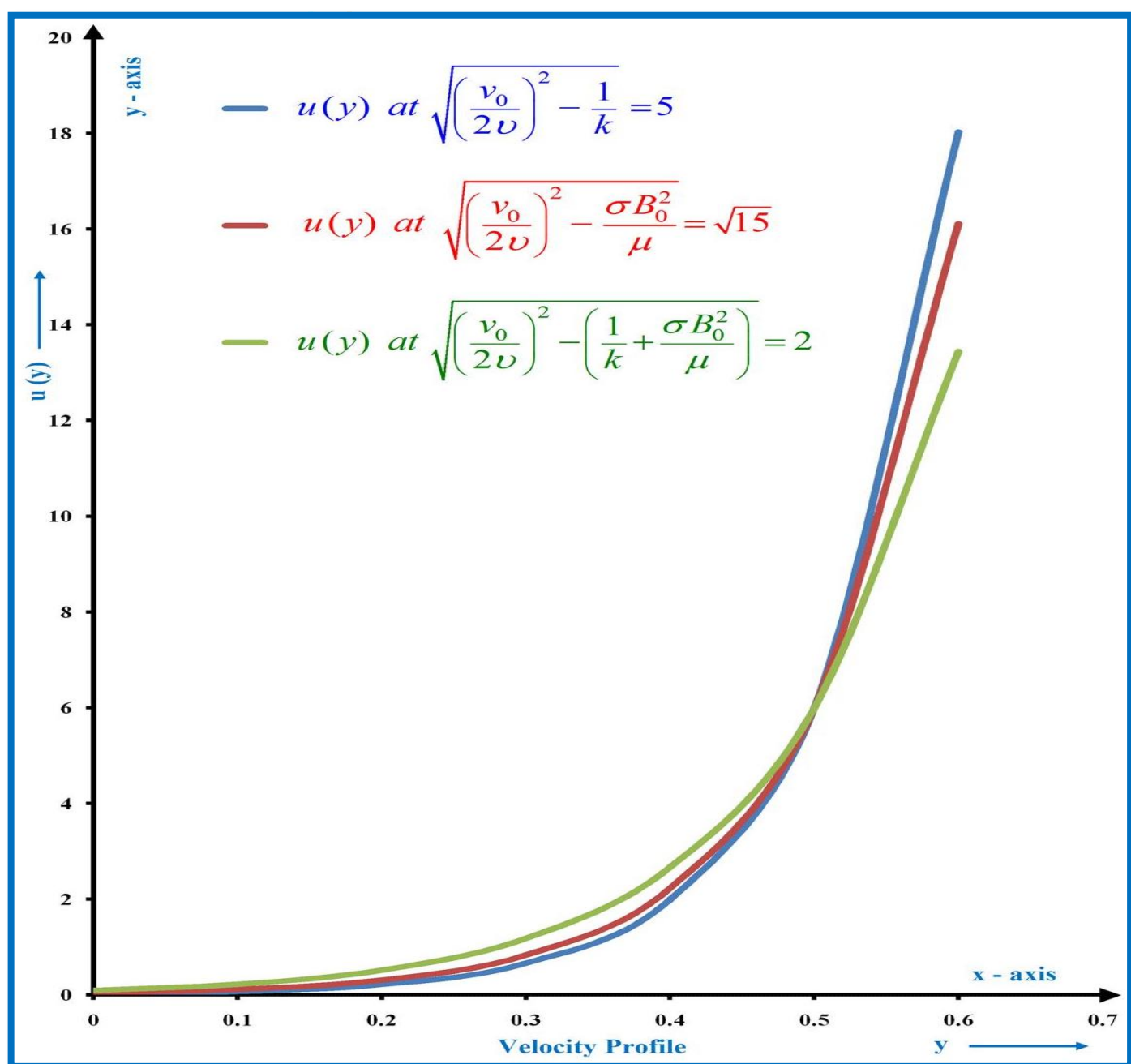
Let $U=6, \mu=h=.5, \frac{v_0}{2\nu}=6$ & $A = \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ all are fixed

Let $\frac{1}{k}$ & $\frac{\sigma B_0^2}{\mu}$ are vary $\Rightarrow \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}}$ & $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}}$ are also vary.

Case(2): $\frac{1}{k} < \frac{\sigma B_0^2}{\mu}$ Let $\frac{1}{k} = 11, \frac{\sigma B_0^2}{\mu} = 21 \Rightarrow \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = 5$ & $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{15}$

Table-3 (for velocity)

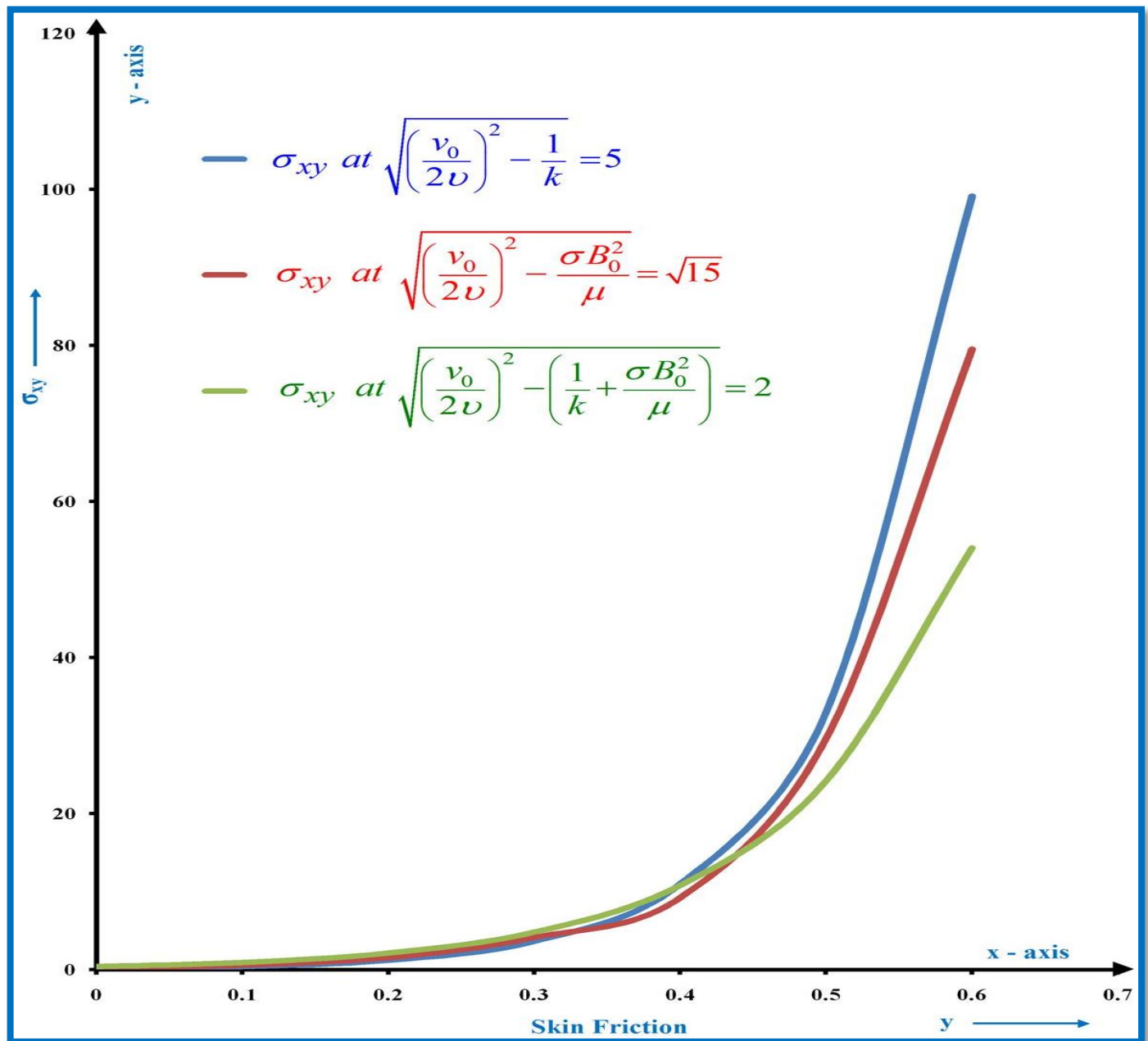
	y	0	.1	.2	.3	.4	.5	.6
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = 5$	u(y)	.024	.073	.221	.665	1.997	6	18.025
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{15}$	u(y)	.042	.115	.309	.832	2.23	6	16.11
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$	u(y)	.097	.227	.52	1.184	2.67	6	13.44



Graph of table-3

Table-4 (for skin friction)

	y	0	.1	.2	.3	.4	.5	.6
$\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{1}{k}} = 5$	σ_{xy}	.135	.405	1.22	3.66	10.98	33	99.14
$\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{15}$	σ_{xy}	.212	.57	1.53	4.112	9.21	29.63	79.53
$\sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$	σ_{xy}	.417	.95	2.15	4.835	10.84	24.22	54.08



Graph of table-4

Tables for velocity and skin friction

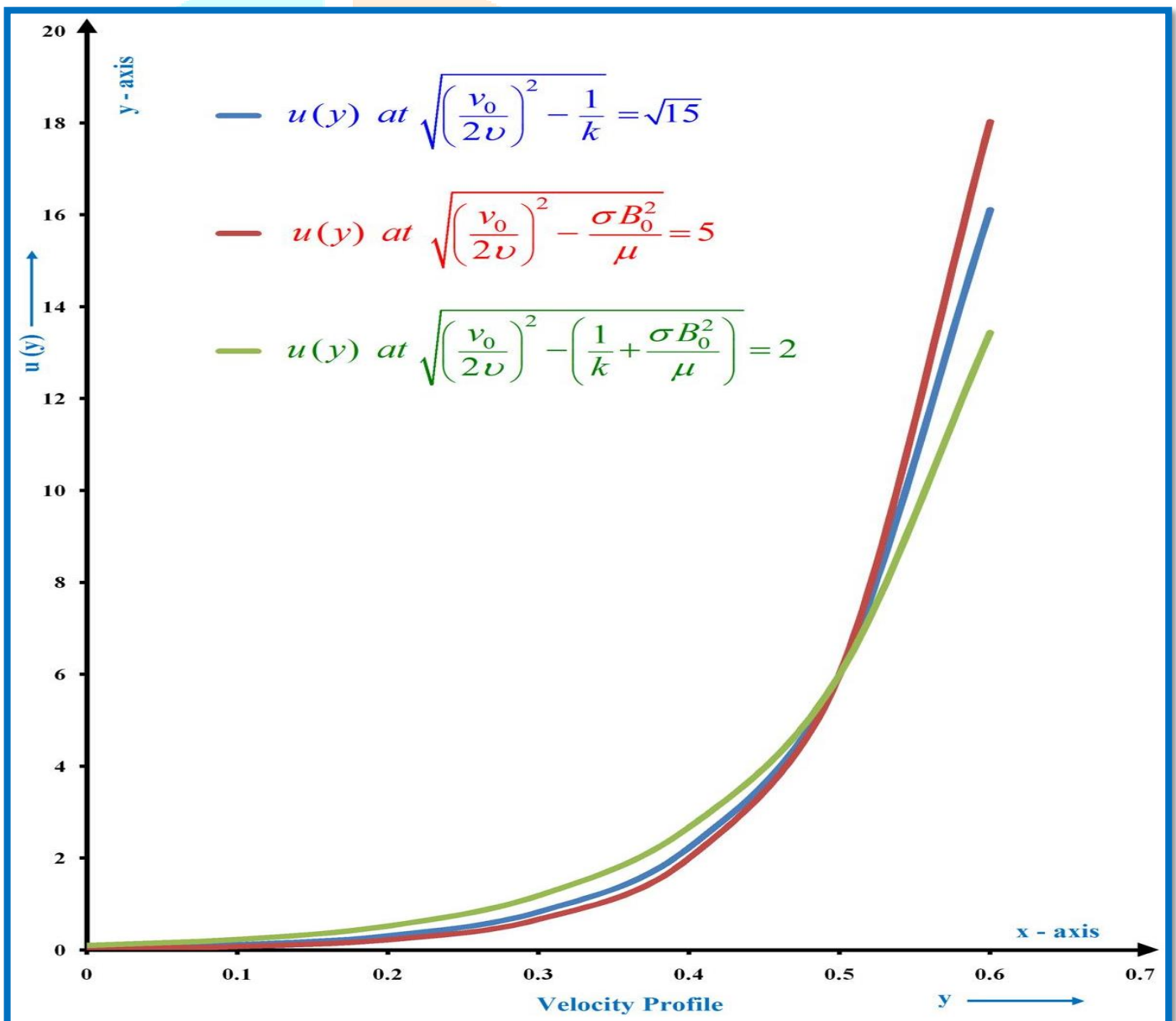
Let $U = 6, \mu = h = .5, \frac{v_0}{2v} = 6$ & $A = \sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ all are fixed

Let $\frac{1}{k}$ & $\frac{\sigma B_0^2}{\mu}$ are vary $\Rightarrow \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}}$ & $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}}$ are also vary.

Case(3): $\frac{1}{k} > \frac{\sigma B_0^2}{\mu}$ Let $\frac{1}{k} = 21$, $\frac{\sigma B_0^2}{\mu} = 11 \Rightarrow \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = \sqrt{15}$ & $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = 5$

Table-5 (for velocity)

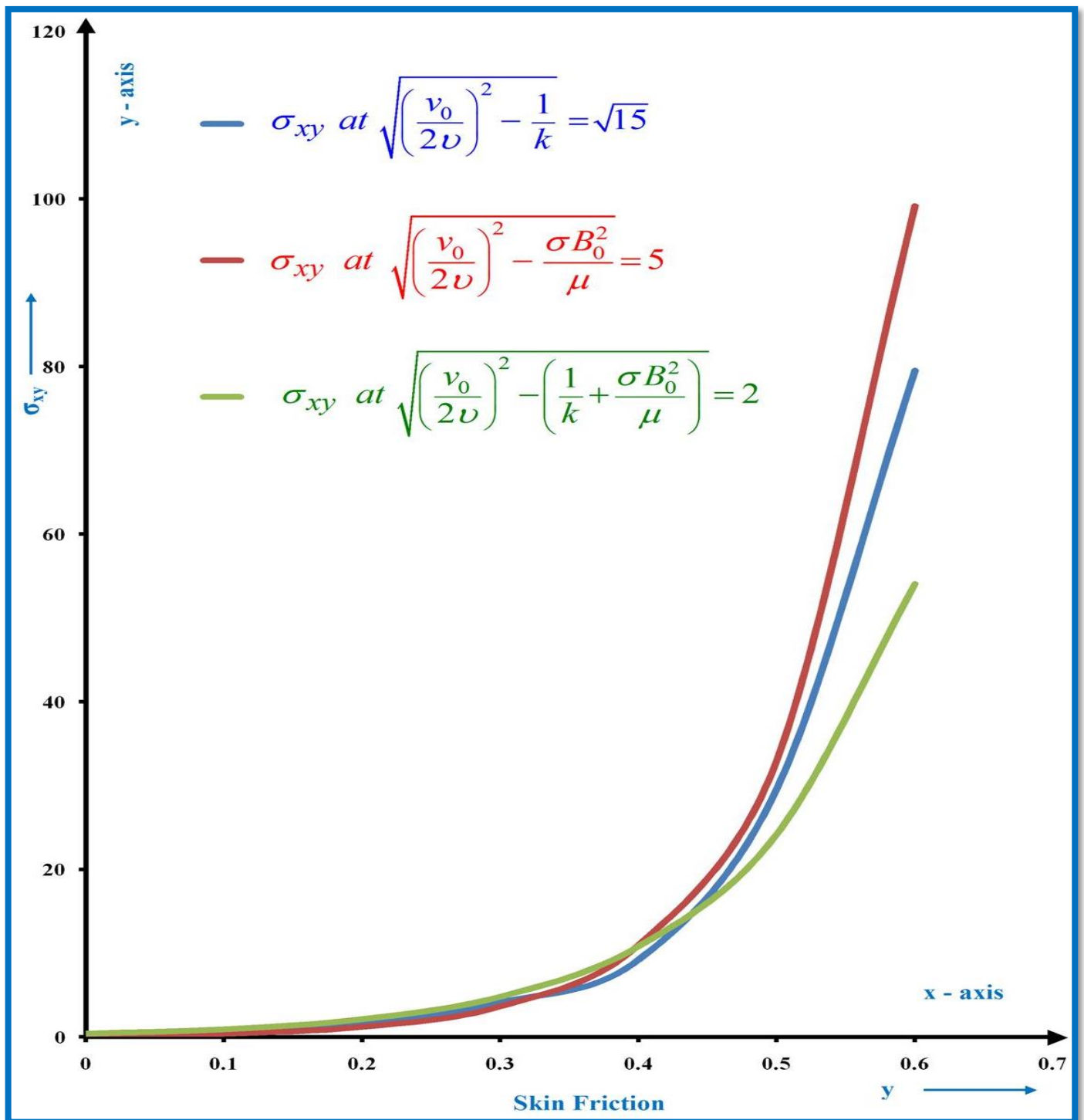
	y	0	.1	.2	.3	.4	.5	.6
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = \sqrt{15}$	u(y)	.042	.115	.309	.832	2.23	6	16.11
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = 5$	u(y)	.024	.073	.221	.665	1.997	6	18.025
$\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$	u(y)	.097	.227	.52	1.184	2.67	6	13.44



Graph of table-5

Table-6 (for skin friction)

	y	0	.1	.2	.3	.4	.5	.6
$\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{1}{k}} = \sqrt{15}$	σ_{xy}	.212	.57	1.53	4.112	9.21	29.63	79.53
$\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{\sigma B_0^2}{\mu}} = 5$	σ_{xy}	.135	.405	1.22	3.66	10.98	33	99.14
$\sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$	σ_{xy}	.417	.95	2.15	4.835	10.84	24.22	54.08



Graph of table-6

CONCLUSION AND DISCUSSION

In this paper, we have investigated the velocity by the table-1 of equation (6). The velocity in Porous

medium and Magnetic field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{20}$ is less than the corresponding

value of velocity in Porous with Magnetic field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ in the interval $0 \leq y \leq .4$

and equal $\{u(y)=6\}$ in all medium at $y=.5$. But the value of velocity in Porous medium and Magnetic

field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{20}$ is greater than the corresponding value of velocity in

Porous with Magnetic field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ at $y=.6$.

Again by the table-3, the value of the velocity in Porous medium at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = 5$ is less than the

corresponding value of velocity in Magnetic field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{15}$ and also is less than the

corresponding value of velocity in Porous medium with Magnetic field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ in

the interval $0 \leq y \leq .4$. The Velocity is equal $\{u(y)=6\}$ in all medium at $y=.5$ and is greater than the

corresponding value of velocity in Magnetic field and Porous with Magnetic field at $y=.6$.

Again by the table-5, the value of the velocity in Magnetic field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = 5$ is less than the

corresponding value of velocity in Porous medium at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = \sqrt{15}$ and also is less than the

corresponding value of velocity in Porous medium with Magnetic field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ in

the interval $0 \leq y \leq .4$. The velocity is equal $\{u(y)=6\}$ in all medium at $y=.5$ and is greater than the

corresponding value of velocity in Porous medium and Porous with Magnetic field at $y=.6$.

Again we have investigated the skin friction by the table-2, of equation (7). The skin friction in Porous

medium and Magnetic field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{k}} = \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{20}$ is less than the corresponding

value of Skin friction in Porous with Magnetic field at $\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ in the interval

$0 \leq y \leq .3$ and the Skin friction in Porous medium and Magnetic field at

$\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{1}{k}} = \sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{20}$ is greater than the corresponding value of Skin friction in

Porous with Magnetic field at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ in the interval $\cdot 4 \leq y \leq \cdot 6$.

Again by the table-4, the Skin friction in Porous medium at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{1}{k}} = 5$ is less than the

corresponding value of Skin friction in Magnetic field at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{15}$ and also is less than

the corresponding value of Skin friction in Porous with Magnetic field at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ in

the interval $0 \leq y \leq \cdot 3$ and Skin friction in Porous medium at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{1}{k}} = 5$ is greater than the

corresponding value of Skin friction in Magnetic field at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{15}$ and is also greater

than the corresponding value of Skin friction in Porous with Magnetic field at

$\sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ in the interval $\cdot 4 \leq y \leq \cdot 6$.

Again by the table-6, the Skin friction in Magnetic field at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{\sigma B_0^2}{\mu}} = 5$ is less than the

corresponding value of Skin friction in Porous medium at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{1}{k}} = \sqrt{15}$ and also is less than the

corresponding value of Skin friction in Porous with Magnetic field at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ in the

interval $0 \leq y \leq \cdot 3$ and Skin friction in Magnetic field at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{\sigma B_0^2}{\mu}} = 5$ is greater than the

corresponding value of Skin friction in Porous medium at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \frac{1}{k}} = \sqrt{15}$ and is also greater than

the corresponding value of Skin friction in Porous with Magnetic field at $\sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu}\right)} = 2$ in

the interval $\cdot 4 \leq y \leq \cdot 6$. Also we have investigated the Skin frictions, average Velocity, the Volumetric flow, Drag Coefficients & Stream lines by the equations (8), (9), (10), (11), (12), (13), (14) & (15) respectively.

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