IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

On Some Types of Fuzzy δ- separated Set in Fuzzy Topological Space on Fuzzy Set

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Abstract : In this paper we introduced and study some types of fuzzy separated set like(Ω-separated set**,** α − Ω- separated set, feebly - separated set, α- separated set, β- separated set, Sp-separated set, a- separated set) and the relationships between them and fuzzy δ- separated set in fuzzy topological space on fuzzy set. And We give counter examples if they are invalid And introduce Some theorems are included about this object.

Introduction :

The recent concept is introduced by Zadeh in (1965) [1], In (1968) Chang [2] introduced the definition of fuzzy topological spaces and extended in a straight forward manner some concepts of crisp topological spaces to fuzzy topological spaces. In (1973) wrong given The definition of fuzzy point such away that an ordinary point was not special case of fuzzy point.

In (1974) While Wong [3] discussed and generalized some properties of fuzzy topological spaces. In (1980) Ming, p.p. and Ming, L.Y. [4] used fuzzy topology to define the neighborhood structure of fuzzy point.

In (1982) Hdeib [7,13] introduced the concept of fuzzy Ω-open set in topological space, In (1982) Maheshwari S.N. and Jain P.G. [12] defined the notion of fuzzy feebly open and fuzzy feebly closed set in fuzzy topological space and studied their properties .

In (1986) Mashhour A.S. and others [9] introduced the notion of α-open sets in topological space. In (1987) Mashhour A.S. and others [8,15] introduced the concept fuzzy β-open set in general topology, In (1995) A.M.Zahran [10] introduced the notion of fuzzy δ-open set in fuzzy topological spaces, In (1996) Dontchev and Przemski have introduced the concept of Sp-open set in general topology [11]. In (1998) Bai Shi – Zhong and Wang Wan – Liang [5] have introduced The notion of fuzzy topology on fuzzy set and they defined the quasi-coincident in fuzzy topological space on fuzzy set. In (2003) Mahmoud, fath-Alla and Abd.Ellah [6] defined fuzzy interior and fuzzy closure in fuzzy topological space on fuzzy set and investigate their properties ,

In (2016) otchana and others introduced the concept of α-Ω open set in topological space [13].

Definition 1.1 [2] **:**

Let X be a nonempty set, a fuzzy set $\tilde{A}$ in X is characterized by a function

$\mu_{\tilde{A}} : X \to I$, where $I = [0,1]$ which is written as

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, 0 \le \mu_{\tilde{A}}(x) \le 1\}$, the collection of all fuzzy sets in X will be denoted by I^X,that is

$I^X = \{ \tilde{A} : \tilde{A}$ is a fuzzy sets in X$\}$ where $\mu_{\tilde{A}}$ is called the membership function

Proposition 1.2 [13] **:**

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets in X with membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ respectively, then for all $x \in X$: -

1. $\tilde{A} \subseteq \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(x)$.
2. $\tilde{A} = \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$.
3. $\tilde{C} = \tilde{A} \cap \tilde{B} \leftrightarrow C(x) = \min\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}$.
4. $\tilde{D} = \tilde{A} \cup \tilde{B} \leftrightarrow D(x) = \max\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$.
5. $\tilde{B}^c$ the complement of $\tilde{B}$ with membership function
   $\mu_{\tilde{B}^c}(x) = \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)$.

Definition 1.3 [2] **:**

A fuzzy point x_r is a fuzzy set such that :

$\mu_{x_r}(y) = r > 0$ if $x = y, \ \forall \ y \in X$ and

$\mu_{x_r}(y) = 0$ if $x \neq y, \forall \ y \in X$

The family of all fuzzy points of $\tilde{A}$ will be denoted by $FP(\tilde{A})$.

Definition 1.4 [2] **:**

A collection $\tilde{T}$ of a fuzzy subsets of $\tilde{A}$, such that $\tilde{T} \subseteq P(\tilde{A})$ is said to be fuzzy topology on $\tilde{A}$ if it satisfied the following conditions

1. $\tilde{A}$, $\tilde{\phi} \in \tilde{T}$
2. If $\tilde{B}, \tilde{C} \in \tilde{T}$, then $\tilde{B} \cap \tilde{C} \in \tilde{T}$
3. If $\tilde{B}_\alpha \in \tilde{T}$, then $\cup_\alpha \tilde{B}_\alpha \in \tilde{T}$, $\alpha \in \Lambda$

$(\tilde{A}, \tilde{T})$ is said to be Fuzzy topological space and every member of $\tilde{T}$ is called fuzzy open set in $\tilde{A}$ and its complement is a fuzzy closed set.

***Definition 1.5 [14]*:**

A fuzzy set $\tilde{B}$ in a fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be Fuzzy delta set if, $\mu_{Int(Cl(\tilde{B}))}(x) \leq \mu_{\tilde{B}}(x) \leq \mu_{Cl(Int(\tilde{B}))}(x)$

Such that,

- **Fuzzy $\boldsymbol{\delta}$-open set if $\boldsymbol{\mu_{Int(Cl(\tilde{B}))}(x) \leq \mu_{B}(x)}$** .
- **Fuzzy $\boldsymbol{\delta}$-closed set if $\boldsymbol{\mu_{B}(x) \leq \mu_{Cl(Int(\tilde{B}))}(x)}$** .
- **Fuzzy $\boldsymbol{\delta}$-closed set if A= δcl(A)** , where **$\boldsymbol{\mu_{\delta cl(\tilde{B})}(x) = \min\{ \mu_{F}(x) : \tilde{F}}$ is a fuzzy δ – closed set in $\boldsymbol{\tilde{A}}$, $\boldsymbol{\mu_{B}(x) \leq \mu_{F}(x)\}}$** .

The complement of fuzzy δ-closed set is fuzzy δ-open set

Some Types of Fuzzy Open Sets 1.6 :

In this section we study the properties and relations of various typess of fuzzy open set in fuzzy topological spaces on fuzzy set which will be needed later on

Definition 1.7:

A fuzzy set $\tilde{B}$ of a fuzzy topological space $(\tilde{A},\tilde{T})$ is said to be :-

1) **Fuzzy Ω-open (Fuzzy Ω-closed)** set if

$\mu_{Cl(\tilde{B})}(x) \leq \mu_{Cl(Int(\tilde{B}))}(x)$, $(\mu_{intCl(\tilde{B})}(x) \leq \mu_{cl(\tilde{B})}(x)$) , $\forall$ x ∈ X.

The family of all fuzzy Ω-open (fuzzy Ω-closed) sets

in $\tilde{A}$ will be denoted by FΩO($\tilde{A}$) (FΩC($\tilde{A}$)).

2) **Fuzzy $\boldsymbol{\alpha - \Omega}$ open** (**Fuzzy $\boldsymbol{\alpha - \Omega}$ closed**) set if

$\mu_{\tilde{B}}(x) \leq \mu_{Int_{\Omega}(Cl(Int_{\Omega}(\tilde{B})))}(x)$, $(\mu_{Cl_{\Omega}(Int(Cl_{\Omega}(\tilde{B})))} \leq \mu_{\tilde{B}}(x))$.

$\tilde{B}$ is called **(**Fuzzy $\alpha - \Omega$ closed **)** set if its complement is Fuzzy

$\alpha - \Omega$open sets

the family of all Fuzzy $\alpha - \Omega$ open (Fuzzy $\alpha - \Omega$ closed) sets

in $\tilde{A}$ will be denoted by F$\alpha - \Omega$O($\tilde{A}$) (F$\alpha - \Omega$C($\tilde{A}$)).

3) **Fuzzy $\boldsymbol{feebly - open}$ ($\boldsymbol{feebly - closed}$)** set if

$\mu_{\tilde{B}}(x) \leq \mu_{s}(Cl(Int(\tilde{B})))(x)$, $(\mu_{s}(Int(Cl(\tilde{B})))(x) \leq \mu_{\tilde{B}}(x)$, $\forall$ x ∈ X

The family of all fuzzy $feebly$ – open (fuzzy $feebly$ – closed) sets in $\tilde{A}$ will be denoted by F$feebly$O($\tilde{A}$) (F$feebly$C($\tilde{A}$)).

4) **Fuzzy $\boldsymbol{\alpha}$-open (fuzzy $\boldsymbol{\alpha}$-closed)** set if

$\mu_{\tilde{B}}(x) \leq \mu_{Int(Cl(Int(\tilde{B})))}(x)$, $(\mu_{Cl(Int(Cl(\tilde{B})))} \leq \mu_{\tilde{B}}(x))$.

The family of all fuzzy α-open (fuzzy α-closed) sets in $\tilde{A}$ will be denoted by FαO($\tilde{A}$) (FαC($\tilde{A}$)).

5) **Fuzzy ꞵ-open (fuzzy ꞵ-closed)** set if

$$\mu_{\tilde{B}}(x) \leq \mu_{Int\left(Cl(\tilde{B})\right)}(x)\ ,\ (\mu_{Cl\left(Int(\tilde{B})\right)} \leq \mu_{\tilde{B}}(x))\quad ,\forall\ x \in X$$

The family of all fuzzy ꞵ-open (fuzzy ꞵ-closed) sets in Ã will be denoted by FꞵO(Ã) (FꞵC(Ã)).

6) **Fuzzy Sp-open (fuzzy Sp-closed)** set if ,

$$\mu_{\tilde{B}}(x) \leq \max\{\mu_{Int\left(Cl(\tilde{B})\right)}(x), \mu_{Cl\left(Int(\tilde{B})\right)}(x)\}$$

$$\mu_{\tilde{B}}(x) \geq \min\{\mu_{Int\left(Cl(\tilde{B})\right)}(x), \mu_{Cl\left(Int(\tilde{B})\right)}(x)\}\ ,\ \forall\ x \in X$$

The family of all fuzzy Sp-open (fuzzy Sp-closed) sets in Ã will be denoted by FSpO(Ã) (FSpC(Ã)).

7) **Fuzzy a-open (fuzzy a-closed)** set if ,

$$\mu_{\tilde{B}}(x) \leq \mu_{Int(Cl(Int_s(\tilde{B})))}(x)\ ,\ (\mu_{Cl(Int(Cl_s(\tilde{B})))} \leq \mu_{\tilde{B}}(x))$$

The family of all fuzzy a-open (fuzzy a-closed) sets in Ã will be denoted by FaO(Ã) (FaC(Ã)).

Definition 1.8 :

Let B̃ is a fuzzy set in a fuzzy topological space (Ã ,T̃) then :

- **The Ω – closure** of B̃ is denoted by (Ωcl(B̃)) and defined by
  $\mu_{\Omega cl(\tilde{B})}(x) = \min\{\ \mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy Ω – closed set in Ã, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$
- **The $\boldsymbol{\alpha}$ – Ω– closure** of B̃ is denoted by ($\boldsymbol{\alpha}$ − Ωcl(B̃)) and defined by
  $\mu_{\boldsymbol{\alpha}-\Omega cl(\tilde{B})}(x) = \min\{\ \mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy $\boldsymbol{\alpha}$ − Ω closed set in Ã, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$
- **The $\boldsymbol{feebly}$– closure** of B̃ is denoted by ($feebly$cl(B̃)) and defined by
  $\mu_{feebly cl(\tilde{B})}(x) = \min\{\ \mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy $feebly$ – closed set in Ã, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$
- **The $\boldsymbol{\alpha}$ – closure** of B̃ is denoted by (αcl(B̃)) and defined by
  $\mu_{\alpha cl(\tilde{B})}(x) = \min\{\ \mu_{cl(\tilde{F})}(x) : \tilde{F}$ is a fuzzy open set in Ã,
  $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$

Proposition 1.9 :

Let (Ã , T̃) be a fuzzy topological space then :

1) The complement of fuzzy Ω-open set is fuzzy Ω-closed set .
2) The complement of fuzzy α-Ω -open set is fuzzy α-Ω -closed set
3) The complement of fuzzy feebly-open set is fuzzy feebly-closed set
4) The complement of fuzzy α-open set is fuzzy α-closed set
5) The complement of fuzzy ꞵ-open set is fuzzy ꞵ-closed set
6) The complement of fuzzy Sp -open set is fuzzy Sp -closed set
7) The complement of fuzzy a -open set is fuzzy a -closed set

Proof **:** Obvious .

Proposition 1.10 :

Let (Ã , T̃) be a fuzzy topological space then :

1) Every fuzzy δ-open set is fuzzy open set (fuzzy Ω-open set , fuzzy feebly-open set , fuzzy a-open set)
2) Every fuzzy open set is fuzzy Ω-open set (fuzzy feebly open set , fuzzy α-open set)
3) Every fuzzy Ω-open set is fuzzy α-Ω open set (fuzzy a-open set)
4) Every fuzzy α-open set is fuzzy α-Ω open set (fuzzy β-open set , fuzzy Sp-open set)
5) Every fuzzy β-open set is fuzzy Sp-open set.
6) Every fuzzy a-open set is fuzzy α-open set.

Remark 1.11 :

Figure - 1 – illustrates the relation between fuzzy δ-open set and some types of fuzzy open sets.

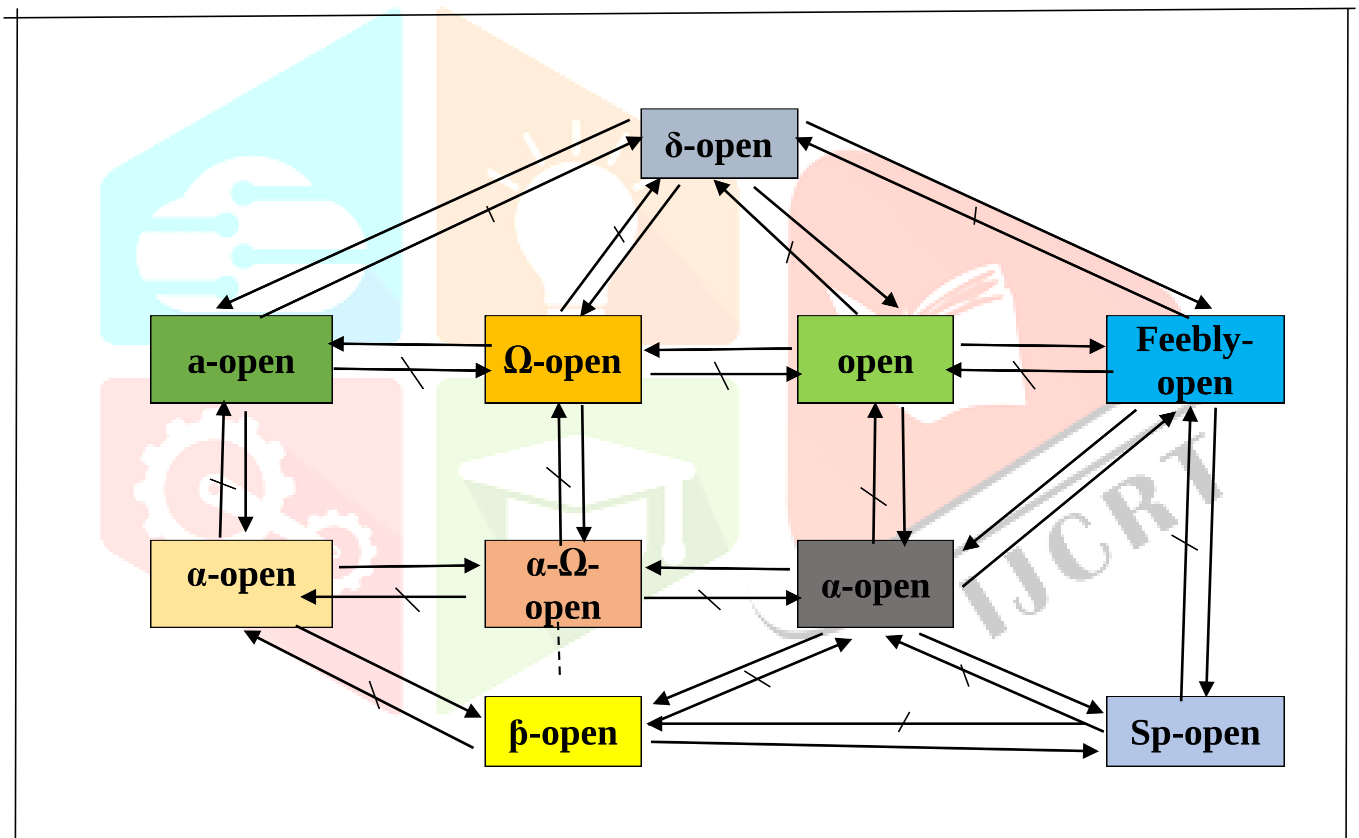

Figure -1-

Fuzzy δ-Separated Sets 2.0 :

In this section we introduce the definition of fuzzy δ-separated set and some theorems and remarks are included throughout this work.

Definition 2.1 [11]:

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space and $\tilde{B}$, $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are said to be **fuzzy δ-separated sets** if and only if

$\min\{\mu_{\delta cl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\delta cl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.

Theorem 2.2 [11]:

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$ and $\tilde{D}$ is a fuzzy set in $\tilde{A}$, then $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{D}}(x)\}$ and $\min\{\mu_{\tilde{D}}(x), \mu_{\tilde{C}}(x)\}$ are fuzzy δ-separated sets in $\tilde{A}$.

Proof : Obvious

Theorem 2.3 [11]:

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$, $\tilde{M}$ and $\tilde{N}$ are fuzzy sets in $\tilde{A}$ such that $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ and $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$, then $\tilde{M}$ and $\tilde{N}$ are fuzzy δ-separated sets in $\tilde{A}$.

Proof : Obvious

Theorem 2.4 [11]:

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$ if and only if there exist fuzzy δ-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$,

$\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\tilde{C}}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.

Proof : Obvious

Theorem 2.5 :

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$ if $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-closed sets in $\tilde{A}$ and $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.

Proof : Obvious

Some Types of Fuzzy Separated Sets 3.0:

In this section we introduce the definition of some types of fuzzy separated (Ω-separated, $\alpha - \Omega$-separated, feebly-separated, α-separated, β-separated, Sp-separated, a-separated) and the relationship between them and fuzzy δ-separated set and some theorems and remarks are included throughout this work

Definition 3.1 :

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space and $\tilde{B}$, $\tilde{C}$ are fuzzy set in $\tilde{A}$, then :-

1. $\tilde{B}$ and $\tilde{C}$ are said to be **fuzzy Ω-separated sets** if and only if
   $\min\{ \mu_{\Omega cl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and
   $\min\{ \mu_{\Omega cl(\tilde{C})}(x) , \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
2. $\tilde{B}$ and $\tilde{C}$ are said to be **fuzzy $\boldsymbol{\alpha} - \Omega$-separated sets** if and only if
   $\min\{ \mu_{\alpha-\Omega cl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and
   $\min\{ \mu_{\alpha-\Omega cl(\tilde{C})}(x) , \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.
3. $\tilde{B}$ and $\tilde{C}$ are said to be **fuzzy feebly-separated sets** if and only if
   $\min\{ \mu_{feeblycl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and
   $\min\{ \mu_{feeblycl(\tilde{C})}(x) , \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.
4. $\tilde{B}$ and $\tilde{C}$ are said to be **fuzzy $\boldsymbol{\alpha}$-separated sets** if and only if
   $\min\{ \mu_{\alpha cl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{ \mu_{\alpha cl(\tilde{C})}(x) , \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.
5. $\tilde{B}$ and $\tilde{C}$ are said to be **fuzzy ƥ-separated sets** if and only if
   $\min\{ \mu_{ƥcl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{ \mu_{ƥcl(\tilde{C})}(x) , \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.
6. $\tilde{B}$ and $\tilde{C}$ are said to be **fuzzy Sp-separated sets** if and only if
   $\min\{ \mu_{Spcl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{ \mu_{Spcl(\tilde{C})}(x) , \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.
7. $\tilde{B}$ and $\tilde{C}$ are said to be **fuzzy a-separated sets** if and only if
   $\min\{ \mu_{acl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{ \mu_{acl(\tilde{C})}(x) , \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.

Theorem 3.2 :

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy Ω-separated sets in $\tilde{A}$ and $\tilde{D}$ is a fuzzy set in $\tilde{A}$, then $\min\{ \mu_{\tilde{B}}(x) , \mu_{\tilde{D}}(x)\}$ and $\min\{ \mu_{\tilde{D}}(x) , \mu_{\tilde{C}}(x)\}$ are fuzzy Ω-separated sets in $\tilde{A}$.

Proof :

Since $\tilde{B}$ and $\tilde{C}$ are fuzzy Ω-separated sets in $\tilde{A}$

Then $\min\{ \mu_{\Omega cl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{\Omega cl(\tilde{C})}(x) , \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

To prove $\min\{ \mu_{\Omega cl(\min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{D}}(x)\})}(x) , \mu_{(\min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x)\})}(x) \} = \mu_{\tilde{\emptyset}}(x)$

and $\min\{ \mu_{\Omega cl(\min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x)\})}(x) , \mu_{(\min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{D}}(x)\})}(x) \} = \mu_{\tilde{\emptyset}}(x)$

Since, $\min\{ \mu_{\Omega cl(\min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{D}}(x)\})}(x) , \mu_{(\min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x)\})}(x) \} \leq \min\{\min\{ \mu_{\Omega cl(\tilde{B})}(x) , \mu_{\delta cl(\tilde{D})}(x) \}, \mu_{(\min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x)\})}(x) \} = \min\{\min\{ \mu_{\Omega cl(\tilde{B})}(x) , \mu_{\tilde{C}}(x) \}, \mu_{(\min\{\mu_{\Omega cl(\tilde{D})}(x),\ \mu_{\tilde{D}}(x)\})}(x) \} = \min\{ \mu_{\tilde{\emptyset}}(x) , \mu_{(\min\{\mu_{\Omega cl(\tilde{D})}(x),\ \mu_{\tilde{D}}(x)\})}(x) \} = \mu_{\tilde{\emptyset}}(x)$

then

min{ $\mu_{\Omega cl(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{D}}(x))}(x)$, $\mu_{(min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x))}(x)$ } $\leq \mu_{\tilde{\emptyset}}(x)$

Hence

min{ $\mu_{\Omega cl(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{D}}(x))}(x)$, $\mu_{(min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x))}(x)$ }$=\mu_{\tilde{\emptyset}}(x)$

Similarly

min { $\mu_{\Omega cl(min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x))}(x)$, $\mu_{(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{D}}(x))}(x)$ } = $\mu_{\tilde{\emptyset}}(x)$

Then min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{D}}(x)$} and min{ $\mu_{\tilde{D}}(x)$, $\mu_{\tilde{C}}(x)$} are fuzzy **Ω**-separated sets in Ã ■

Theorem 3.3 :

1) If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy α-**Ω**-separated sets in Ã and D̃ is a fuzzy set in Ã then min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{D}}(x)$} and min{ $\mu_{\tilde{D}}(x)$, $\mu_{\tilde{C}}(x)$} are fuzzy α-**Ω**-separated sets in Ã.
2) If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy feebly-separated sets in Ã and D̃ is a fuzzy set in Ã then min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{D}}(x)$} and min{ $\mu_{\tilde{D}}(x)$, $\mu_{\tilde{C}}(x)$} are fuzzy feebly-separated sets in Ã.
3) If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy α-separated sets in Ã and D̃ is a fuzzy set in Ã then min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{D}}(x)$} and min{ $\mu_{\tilde{D}}(x)$, $\mu_{\tilde{C}}(x)$} are fuzzy α-separated sets in Ã.
4) If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy β-separated sets in Ã and D̃ is a fuzzy set in Ã then min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{D}}(x)$} and min{ $\mu_{\tilde{D}}(x)$, $\mu_{\tilde{C}}(x)$} are fuzzy β-separated sets in Ã.
5) If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy Sp-separated sets in Ã and D̃ is a fuzzy set in Ã then

   min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{D}}(x)$} and min{ $\mu_{\tilde{D}}(x)$, $\mu_{\tilde{C}}(x)$} are fuzzy Sp-separated sets in Ã.
6) If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy a-separated sets in Ã and D̃ is a fuzzy set in Ã then

   min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{D}}(x)$} and min{ $\mu_{\tilde{D}}(x)$, $\mu_{\tilde{C}}(x)$} are fuzzy a-separated sets in Ã.

Proof : (1) (2) (3) (4) (5) and (6) :- Obvious

Theorem 3.4 :

If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy Ω-separated sets in Ã , M̃ and Ñ are fuzzy set in Ã such that $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ and $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$ then M̃ and Ñ are fuzzy Ω-separated sets in Ã

Proof :

Since $\tilde{B}$ and $\tilde{C}$ are fuzzy Ω-separated sets in $\tilde{A}$ then

$\min\{ \mu_{\Omega cl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{\Omega cl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ then $\mu_{\Omega cl(\tilde{M})}(x) \leq \mu_{\Omega cl(\tilde{B})}(x)$ then

$\min\{ \mu_{\Omega cl(\tilde{M})}(x), \mu_{\tilde{C}}(x)\} \leq \min\{ \mu_{\Omega cl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Hence, $\min\{ \mu_{\Omega cl(\tilde{M})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

$\min\{ \mu_{\Omega cl(\tilde{M})}(x), \mu_{(min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{N}}(x)\})}(x)\} =$

$\min\{ \min\{ \mu_{\Omega cl(\tilde{M})}(x), \mu_{\tilde{C}}(x)\}, \mu_{\tilde{N}}(x)\} = \min\{ \mu_{\tilde{\emptyset}}(x), \mu_{\tilde{N}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$ then $\min\{ \mu_{\tilde{N}}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{N}}(x)$

implies that $\min\{ \mu_{\Omega cl(\tilde{M})}(x), \mu_{\tilde{N}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Similarly $\min\{ \mu_{\Omega cl(\tilde{N})}(x), \mu_{\tilde{M}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Hence $\tilde{M}$ and $\tilde{N}$ are fuzzy Ω-separated sets in $\tilde{A}$ ■

Theorem 3.5 :

1) If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy $\alpha - \Omega$-separated sets in $\tilde{A}$, $\tilde{M}$ and $\tilde{N}$ are fuzzy sets in $\tilde{A}$ such that $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ and $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$, then $\tilde{M}$ and $\tilde{N}$ are fuzzy $\alpha - \Omega$-separated sets in $\tilde{A}$.
2) If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy feebly-separated sets in $\tilde{A}$, $\tilde{M}$ and $\tilde{N}$ are fuzzy sets in $\tilde{A}$ such that $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ and $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$, then $\tilde{M}$ and $\tilde{N}$ are fuzzy feebly-separated sets in $\tilde{A}$.
3) If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated sets in $\tilde{A}$, $\tilde{M}$ and $\tilde{N}$ are fuzzy sets in $\tilde{A}$ such that $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ and $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$, then $\tilde{M}$ and $\tilde{N}$ are fuzzy α-separated sets in $\tilde{A}$.
4) If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy β-separated sets in $\tilde{A}$, $\tilde{M}$ and $\tilde{N}$ are fuzzy sets in $\tilde{A}$ such that $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ and $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$, then $\tilde{M}$ and $\tilde{N}$ are fuzzy β-separated sets in $\tilde{A}$.
5) If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated sets in $\tilde{A}$, $\tilde{M}$ and $\tilde{N}$ are fuzzy sets in $\tilde{A}$ such that $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ and $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$, then $\tilde{M}$ and $\tilde{N}$ are fuzzy Sp-separated sets in $\tilde{A}$.
6) If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy a-separated sets in $\tilde{A}$, $\tilde{M}$ and $\tilde{N}$ are fuzzy sets in $\tilde{A}$ such that $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ and $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$, then $\tilde{M}$ and $\tilde{N}$ are fuzzy a-separated sets in $\tilde{A}$.

Proof : (1) (2) (3) (4) (5) and (6) :- Obvious

Theorem 3.6 :

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated sets in $\tilde{A}$ if and only if there exist fuzzy Sp-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$, $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\tilde{C}}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.

Proof

($\Longrightarrow$) Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated sets in $\tilde{A}$

Implies that $\min\{\mu_{Spcl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{\mu_{Spcl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$,

since $\mu_{\tilde{B}}(x) \leq \mu_{Spcl(\tilde{B})}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{Spcl(\tilde{C})}(x)$,

then $\mu_{\tilde{E}}(x) = \mu_{Spcl(\tilde{B})}(x)$ and $\mu_{\tilde{F}}(x) = \mu_{Spcl(\tilde{C})}(x)$

Hence, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$, $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$, $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

and $\min\{\mu_{\tilde{C}}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

($\Longleftarrow$) Since $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$,

then $\mu_{Spcl(\tilde{B})}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{Spcl(\tilde{C})}(x) \leq \mu_{\tilde{F}}(x)$

Implies that

$\min\{\mu_{Spcl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} \leq \min\{\mu_{\tilde{E}}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

And $\min\{\mu_{Spcl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} \leq \min\{\mu_{\tilde{F}}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Hence $\min\{\mu_{Sp(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{Spcl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ therefore $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated sets in $\tilde{A}$ ■

Theorem 3.7 :

1) If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy Ω-separated sets in $\tilde{A}$ if and only if there exist fuzzy Ω-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$, $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\tilde{C}}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
2) If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy $\alpha-\Omega$-separated sets in $\tilde{A}$ if and only if there exist fuzzy $\alpha-\Omega$-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and
$\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$,
$\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\tilde{C}}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
3) If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy $feebly$-separated sets in $\tilde{A}$ if and only if there exist fuzzy $feebly$-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$,
$\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\tilde{C}}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

4) If $(\tilde{A},\tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated sets in $\tilde{A}$ if and only if there exist fuzzy α-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$, $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\tilde{C}}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
5) If $(\tilde{A},\tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy ƥ-separated sets in $\tilde{A}$ if and only if there exist fuzzy ƥ-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$, $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\tilde{C}}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
6) If $(\tilde{A},\tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy a-separated sets in $\tilde{A}$ if and only if there exist fuzzy a-closed sets $\tilde{E}$ and $\tilde{F}$ in $\tilde{A}$ such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$, $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\tilde{C}}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Proof **:** (1) (2) (3) (4) (5) and (6):- Obvious

Theorem 3.8 :

If $(\tilde{A},\tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy Ω-separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$,

then $\min\{\mu_{\delta cl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{\mu_{\delta cl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\delta cl(\tilde{B})}(x)$ and $\mu_{\delta cl(\tilde{C})}(x)$ are fuzzy δ-closed sets in $\tilde{A}$

Implies that by proposition (1.3.3)

$\min\{\mu_{\Omega cl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{\mu_{\Omega cl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy Ω-separated in $\tilde{A}$. ■

Remark 3.9 :

The converse of theorem (3.8) is not true in general as following example shows:-

Example 3.10 :

Let X = { a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$, $\tilde{H}$ are fuzzy subset in $\tilde{A}$ where

$\tilde{A}$ = { (a , 0.6) , (b , 0.6) } , $\tilde{B}$ = { (a , 0.6) , (b , 0.0) }
$\tilde{C}$ = { (a , 0.4) , (b , 0.0) } , $\tilde{D}$ = { (a , 0.0) , (b , 0.5) }
$\tilde{E}$ = { (a , 0.0) , (b , 0.6) } , $\tilde{F}$ = { (a , 0.0) , (b , 0.4) }
$\tilde{G}$ = { (a , 0.2) , (b , 0.0) } , $\tilde{H}$ = { (a , 0.1) , (b , 0.0) }

The fuzzy topology defined on $\tilde{A}$ is $\tilde{T}$ = { $\emptyset$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ }

Then $\tilde{F}$ and $\tilde{H}$ are fuzzy Ω-separated sets in $\tilde{A}$ but not fuzzy δ-separated sets in $\tilde{A}$.

Since: $\mu_{\Omega cl(\tilde{F})}(x) = \tilde{E}$, $\mu_{\Omega cl(\tilde{H})}(x) = \tilde{B}$

min{ $\mu_{\Omega cl(\tilde{F})}(x)$, $\mu_{\tilde{H}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{\Omega cl(\tilde{H})}(x)$, $\mu_{\tilde{F}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Hence $\tilde{F}$ and $\tilde{H}$ are fuzzy Ω-separated in $\tilde{A}$.

But :

$\mu_{\boldsymbol{\delta} cl(\tilde{F})}(x) = \tilde{E}$, $\mu_{\boldsymbol{\delta} cl(\tilde{H})}(x) = \tilde{C}^c$

min{ $\mu_{\boldsymbol{\delta} cl(\tilde{F})}(x)$, $\mu_{\tilde{H}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$

but , min{ $\mu_{\boldsymbol{\delta} cl(\tilde{H})}(x)$, $\mu_{\tilde{F}}(x)$} ≠ $\mu_{\tilde{\emptyset}}(x)$

Then $\tilde{F}$ and $\tilde{H}$ are not fuzzy δ-separated sets in $\tilde{A}$.

Theorem 3.11 :

If ($\tilde{A}$,$\tilde{T}$) is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy $feebly$-separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$,

then min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{\delta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$, since $\mu_{\delta cl(\tilde{B})}(x)$ and $\mu_{\delta cl(\tilde{C})}(x)$ are fuzzy δ-closed sets in $\tilde{A}$

Implies that by proposition (1.3.3)

min { $\mu_{\boldsymbol{feebly} cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and

min { $\mu_{\boldsymbol{feebly} cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy $feebly$-separated in $\tilde{A}$.

Remark 3.12 :

The converse of theorem (3.11) is not true in general as following example shows:-

Example 3.13 :

Let X = { a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$, $\tilde{H}$, $\tilde{I}$, $\tilde{J}$, $\tilde{K}$, $\tilde{L}$, $\tilde{M}$, $\tilde{N}$ are fuzzy subset in $\tilde{A}$ where
$\tilde{A}$ = { (a , 0.6) , (b , 0.6) } , $\tilde{B}$ = { (a , 0.4) , (b , 0.1) } ,
$\tilde{C}$ = { (a , 0.1) , (b , 0.4) } , $\tilde{D}$ = { (a , 0.0) , (b , 0.4) } ,
$\tilde{E}$ = { (a , 0.3) , (b , 0.4) } , $\tilde{F}$ = { (a , 0.1) , (b , 0.1) } ,
$\tilde{G}$ = { (a , 0.4) , (b , 0.0) } , $\tilde{H}$ = { (a , 0.4) , (b , 0.4) } ,
$\tilde{I}$ = { (a , 0.0) , (b , 0.1) } , $\tilde{J}$ = { (a , 0.1) , (b , 0.0) } ,
$\tilde{K}$ = { (a , 0.0) , (b , 0.5) } , $\tilde{L}$ = { (a , 0.5) , (b , 0.0) } ,
$\tilde{M}$ = { (a , 0.2) , (b , 0.6) } , $\tilde{N}$ = { (a , 0.6) , (b , 0.2) } ,

The fuzzy $\tilde{D}$ and $\tilde{J}$ are fuzzy $feebly$-separated sets in $\tilde{A}$ but not fuzzy δ-separated sets in $\tilde{A}$.

Since: $\mu_{\boldsymbol{feebly}\mathrm{cl}(\tilde{D})}(x) = \tilde{D}$, $\mu_{\boldsymbol{feebly}\mathrm{cl}(\tilde{J})}(x) = \tilde{G}$

min{ $\mu_{\boldsymbol{feebly}\mathrm{cl}(\tilde{D})}(x)$, $\mu_{\tilde{J}}$ (x)}= $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{\boldsymbol{feebly}\mathrm{cl}(\tilde{J})}(x)$, $\mu_{\tilde{D}}$ (x)}= $\mu_{\tilde{\emptyset}}(x)$

Hence $\tilde{D}$ and $\tilde{J}$ are fuzzy $feebly$-separated sets in $\tilde{A}$.

But : $\mu_{\boldsymbol{\delta}\mathrm{cl}(\tilde{D})}(x) = \tilde{B}^{c}$, $\mu_{\boldsymbol{\delta}\mathrm{cl}(\tilde{J})}(x) = \tilde{C}^{c}$

min{ $\mu_{\boldsymbol{\delta}\mathrm{cl}(\tilde{D})}(x)$, $\mu_{\tilde{J}}(x)$} ≠ $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{\boldsymbol{\delta}\mathrm{cl}(\tilde{J})}(x)$, $\mu_{\tilde{D}}$ (x)} ≠ $\mu_{\tilde{\emptyset}}(x)$,

then $\tilde{E}$ and $\tilde{F}$ are not fuzzy δ-separated sets in $\tilde{A}$.

Theorem 3.14 :

If ($\tilde{A}$,$\tilde{T}$) is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy a-separated sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy a-separated sets in $\tilde{A}$,

then min{ $\mu_{a\mathrm{cl}(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{a\mathrm{cl}(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Since $\mu_{a\mathrm{cl}(\tilde{B})}(x)$ and $\mu_{a\mathrm{cl}(\tilde{C})}(x)$ are fuzzy a-closed sets in $\tilde{A}$

Implies that by proposition (1.3.3)

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated in $\tilde{A}$.

Remark 3.15 :

The converse of theorem (2.2.14) is not true in general as following example shows:-

Example 2.2.16 :

Let X = { a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$ are fuzzy subset in $\tilde{A}$ where

$\tilde{A}$ = { (a , 0.8) , (b , 0.8) }

$\tilde{B}$ = { (a , 0.8) , (b , 0.0) } , $\tilde{C}$ = { (a , 0.6) , (b , 0.0) }

$\tilde{D}$ = { (a , 0.5) , (b , 0.0) } , $\tilde{E}$ = { (a , 0.0) , (b , 0.8) }

$\tilde{F}$ = { (a , 0.3) , (b , 0.0) }

The fuzzy topology defined on $\tilde{A}$ is $\tilde{T}$ = { $\emptyset$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$}

Then $\tilde{C}$and $\tilde{D}$ are fuzzy α-separated sets in $\tilde{A}$ but not fuzzy α-separated sets in $\tilde{A}$.

Since: $\mu_{\alpha cl(\tilde{C})}(x) = \emptyset$, $\mu_{\alpha cl(\tilde{E})}(x) = \emptyset$

min{ $\mu_{\alpha cl(\tilde{D})}(x)$, $\mu_{\tilde{E}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{\alpha cl(\tilde{E})}(x)$, $\mu_{\tilde{D}}$ (x)}= $\mu_{\tilde{\emptyset}}(x)$

Hence $\tilde{C}$ and $\tilde{D}$ are fuzzy $\boldsymbol{\alpha}$-separated in $\tilde{A}$.

But :

$\mu_{\alpha cl(\tilde{C})}(x) = \tilde{B}$, $\mu_{\alpha cl(\tilde{D})}(x) = \tilde{B}$

min{ $\mu_{\alpha cl(\tilde{C})}(x)$, $\mu_{\tilde{D}}(x)$} $\neq \mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{\alpha cl(\tilde{D})}(x)$, $\mu_{\tilde{C}}$ (x)} $\neq \mu_{\tilde{\emptyset}}(x)$,

then $\tilde{C}$ and $\tilde{D}$ are not fuzzy α-separated sets in $\tilde{A}$.

Theorem 3.17 :

If ($\tilde{A}$,$\tilde{T}$) is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated sets in $\tilde{A}$,

then min{ $\mu_{\alpha cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{\alpha cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\alpha cl(\tilde{B})}(x)$ and $\mu_{\alpha cl(\tilde{C})}(x)$ are fuzzy δ-closed sets in $\tilde{A}$

Implies that by proposition (1.3.3)

min{ $\mu_{Spcl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and min{ $\mu_{Spcl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)\}= \mu_{\tilde{\emptyset}}(x)$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated in $\tilde{A}$. ■

Remark 3.18 :

The converse of theorem (3.17) is not true in general as following example shows:-

Example 2.2.19 :

In example (3.16) the fuzzy set $\tilde{C}$, $\tilde{D}$ are fuzzy Sp-separated sets in $\tilde{A}$ but not fuzzy α-separated sets in $\tilde{A}$

Theorem 3.20 :

If ($\tilde{A}$,$\tilde{T}$) is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy β-separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy α-separated sets in $\tilde{A}$,

then min{ $\mu_{\alpha cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)\}= \mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{\alpha cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)\}= \mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\alpha cl(\tilde{B})}(x)$ and $\mu_{\alpha(\tilde{C})}(x)$ are fuzzy α-closed sets in $\tilde{A}$

Implies that by proposition (1.3.3)

min{ $\mu_{\beta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)\}= \mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{\beta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)\}= \mu_{\tilde{\emptyset}}(x)$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy β-separated in $\tilde{A}$. ■

Remark 3.21 :

The converse of theorem (3.20) is not true in general as following example shows:-

Example 3.22 :

Let X = { a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$ are fuzzy subset in $\tilde{A}$ where

$\tilde{A}$ = { (a , 0.7) , (b , 0.7) } , $\tilde{B}$ = { (a , 0.5) , (b , 0.0) }
$\tilde{C}$ = { (a , 0.3) , (b , 0.0) } , $\tilde{D}$ = { (a , 0.2) , (b , 0.7) }

$\tilde{E} = \{ (a, 0.7), (b, 0.2) \}$, $\tilde{F} = \{ (a, 0.0), (b, 0.5) \}$

Then $\tilde{C}$ and $\tilde{F}$ are fuzzy β-separated sets in $\tilde{A}$ but not fuzzy α-separated sets in $\tilde{A}$.

Since: $\mu_{\beta cl(\tilde{C})}(x) = \tilde{B}$, $\mu_{\beta cl(\tilde{F})}(x) = \tilde{F}$

$\min\{ \mu_{\beta cl(\tilde{C})}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{\beta cl(\tilde{F})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Hence $\tilde{C}$ and $\tilde{F}$ are fuzzy β-separated in $\tilde{A}$.

But :

$\mu_{\alpha cl(\tilde{C})}(x) = \emptyset$, $\mu_{\alpha cl(\tilde{F})}(x) = \tilde{D}$

$\min\{ \mu_{\alpha cl(\tilde{C})}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

but , $\min\{ \mu_{\alpha cl(\tilde{F})}(x), \mu_{\tilde{C}}(x)\} \neq \mu_{\tilde{\emptyset}}(x)$

Then $\tilde{C}$ and $\tilde{F}$ are not fuzzy α-separated sets in $\tilde{A}$.

Theorem 3.23 :

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$,

then $\min\{ \mu_{\delta cl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{\delta cl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\delta cl(\tilde{B})}(x)$ and $\mu_{\delta cl(\tilde{C})}(x)$ are fuzzy δ-closed sets in $\tilde{A}$

Implies that by proposition (1.3.3)

$\min\{ \mu_{cl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{cl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy separated in $\tilde{A}$. ■

Remark 3.24 :

The converse of theorem (3.23) is not true in general as following example shows:-

Example 2.2.25 :

In example (3.13) the fuzzy set $\tilde{I}$, $\tilde{J}$ are fuzzy separated sets in $\tilde{A}$ but not fuzzy δ-separated sets in $\tilde{A}$

Since: $\mu_{cl(\tilde{I})}(x) = \tilde{D}$, $\mu_{cl(\tilde{J})}(x) = \tilde{G}$

$\min\{ \mu_{cl(\tilde{I})}(x) , \mu_{\tilde{J}}(x)\}= \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{cl(\tilde{J})}(x), \mu_{\tilde{I}}(x)\}= \mu_{\tilde{\emptyset}}(x)$

Hence $\tilde{I}$ and $\tilde{J}$ are fuzzy separated in $\tilde{A}$.

But :

$\mu_{\delta cl(\tilde{I})}(x) = \tilde{B}^c$, $\mu_{\delta cl(\tilde{J})}(x) = \tilde{C}^c$

$\min\{ \mu_{\delta cl(\tilde{I})}(x) , \mu_{\tilde{J}}(x)\} \neq \mu_{\tilde{\emptyset}}(x)$

but , $\min\{ \mu_{\delta cl(\tilde{J})}(x), \mu_{\tilde{I}}(x)\} \neq \mu_{\tilde{\emptyset}}(x)$

Then $\tilde{C}$ and $\tilde{F}$ are not fuzzy δ-separated sets in $\tilde{A}$.

Theorem 3.26 :

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy a-separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$,

then $\min\{ \mu_{\delta cl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\}= \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{\delta cl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\}= \mu_{\tilde{\emptyset}}(x)$

since $\mu_{\delta cl(\tilde{B})}(x)$ and $\mu_{\delta cl(\tilde{C})}(x)$ are fuzzy δ-closed sets in $\tilde{A}$

Implies that by proposition (1.3.3)

$\min\{ \mu_{acl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\}= \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{acl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\}= \mu_{\tilde{\emptyset}}(x)$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy a-separated in $\tilde{A}$. ■

Remark 3.27 :

The converse of theorem (326) is not true in general as following example shows:-

Example 3.28 :

In example (3.16) the fuzzy set $\tilde{D}$, $\tilde{E}$ are fuzzy a-separated sets in $\tilde{A}$ but not fuzzy δ-separated sets in $\tilde{A}$

Since: $\mu_{acl(\tilde{D})}(x) = \tilde{B}$, $\mu_{acl(\tilde{E})}(x) = \tilde{E}$

$\min\{ \mu_{acl(\tilde{D})}(x) , \mu_{\tilde{E}}(x)\}= \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{acl(\tilde{E})}(x), \mu_{\tilde{D}}(x)\}= \mu_{\tilde{\emptyset}}(x)$

Hence $\widetilde{D}$ and $\tilde{E}$ are fuzzy a-separated in $\tilde{A}$.

But :

$\mu_{\delta cl(\widetilde{D})}(x) = \tilde{F}^c$, $\mu_{\delta cl(\tilde{G})}(x) = \tilde{E}$

min{ $\mu_{\delta cl(\widetilde{D})}(x)$, $\mu_{\tilde{E}}(x)$} $\neq \mu_{\tilde{\emptyset}}(x)$

but , min{ $\mu_{\delta cl(\tilde{E})}(x)$, $\mu_{\widetilde{D}}(x)$} $\neq \mu_{\tilde{\emptyset}}(x)$

Then $\widetilde{D}$ and $\tilde{E}$ are not fuzzy δ-separated sets in $\tilde{A}$.

Theorem 3.29 :

If ($\tilde{A}$,$\tilde{T}$) is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy $feebly$-separated sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy $feebly$-separated sets in $\tilde{A}$,

then min{ $\mu_{feebly cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{feebly cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\alpha cl(\tilde{B})}(x)$ and $\mu_{\alpha cl(\tilde{C})}(x)$ are fuzzy $feebly$-closed sets in $\tilde{A}$

Implies that by proposition (1.3.3)

min{ $\mu_{Sp cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{Sp cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated in $\tilde{A}$. ■

Remark 3.30 :

The converse of theorem (3.29) is not true in general as following example shows:-

Example 2.2.31 :

Let X = { a , b } and $\tilde{B}$, $\tilde{C}$, $\widetilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$, $\widetilde{K}$, $\tilde{L}$, $\widetilde{M}$, $\widetilde{H}$, $\widetilde{N}$ are fuzzy subset in $\tilde{A}$ where

$\tilde{A}$ = { (a , 0.7) , (b , 0.7) }

$\tilde{B}$ = { (a , 0.4) , (b , 0.7) } , $\tilde{C}$ = { (a , 0.7) , (b , 0.4) }

$\widetilde{D}$ = { (a , 0.4) , (b , 0.4) } , $\tilde{E}$ = { (a , 0.3) , (b , 0.0) }

$\tilde{F}$ = { (a , 0.0) , (b , 0.2) } , $\tilde{G}$ = { (a , 0.3) , (b , 0.3) }

$\widetilde{K}$ = { (a , 0.4) , (b , 0.0) } , $\tilde{L}$ = { (a , 0.0) , (b , 0.4) }

$\widetilde{M}$ = { (a , 0.0) , (b , 0.2) } , $\widetilde{H}$ = { (a , 0.3) , (b , 0.3) }

$\widetilde{N}$ = { (a , 0.4) , (b , 0.0) }

The fuzzy topology defined on $\tilde{A}$ is

$\tilde{T}$ = { $\emptyset$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\widetilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$, $\widetilde{K}$, $\tilde{L}$, $\widetilde{M}$, $\widetilde{H}$, $\widetilde{N}$ }

Then $\tilde{E}$ and $\tilde{G}$ are fuzzy Sp-separated sets in $\tilde{A}$ but not fuzzy $feebly$-separated sets in $\tilde{A}$.

Since: $\mu_{Spcl(\tilde{E})}(x) = \tilde{F}^{c}$, $\mu_{Spcl(\tilde{G})}(x) = \tilde{D}^{c}$

$\min\{ \mu_{Spcl(\tilde{E})}(x) , \mu_{\tilde{G}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{Spcl(\tilde{G})}(x), \mu_{\tilde{E}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Hence $\tilde{E}$ and $\tilde{G}$ are fuzzy Sp-separated sets in $\tilde{A}$.

But :

$\mu_{feeblycl(\tilde{E})}(x) = \tilde{N}^{c}$, $\mu_{feeblycl(\tilde{G})}(x) = \tilde{H}^{c}$

$\min\{ \mu_{feeblycl(\tilde{E})}(x) , \mu_{\tilde{G}}(x)\} \neq \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{feeblycl(G)}(x), \mu_{\tilde{E}}(x)\} \neq \mu_{\tilde{\emptyset}}(x)$,

then $\tilde{E}$ and $\tilde{G}$ are not fuzzy $feebly$-separated sets in $\tilde{A}$.

Theorem 3.32 :

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy ϐ-separated sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy ϐ-separated sets in $\tilde{A}$,

then $\min\{ \mu_{ϐcl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{ϐcl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Since $\mu_{ϐcl(\tilde{B})}(x)$ and $\mu_{ϐcl(\tilde{C})}(x)$ are fuzzy ϐ-closed sets in $\tilde{A}$

Implies that by proposition (1.3.3)

$\min\{ \mu_{Spcl(\tilde{B})}(x) , \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and

$\min\{ \mu_{Spcl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy Sp-separated in $\tilde{A}$. ■

Remark 3.33 :

The converse of theorem (3.32) is not true in general as following example shows:-

Example 3.34 :

Let X = { a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$ are fuzzy subset in $\tilde{A}$ where

$\tilde{A}$ = { (a , 0.5) , (b , 0.5) } , $\tilde{B}$ = { (a , 0.5) , (b , 0.0) } ,

$\tilde{C}$ = { (a , 0.0) , (b , 0.3) } , $\tilde{D}$ = { (a , 0.5) , (b , 0.3) } ,

$\tilde{E}$ = { (a , 0.0) , (b , 05) } , $\tilde{F}$ = { (a , 0.3) , (b , 0.0) } ,

$\tilde{G}$ = { (a , 0.3) , (b , 0.5) } , $\tilde{H}$ = { (a , 0.0) , (b , 0.4) } ,

$\tilde{I}$ = { (a , 0.4) , (b , 0.0) }

The fuzzy set $\tilde{H}$ and $\tilde{I}$ are fuzzy Sp-separated sets in $\tilde{A}$ but not fuzzy ꞵ-separated sets in $\tilde{A}$.

Since: $\mu_{Sp\mathrm{cl}(\tilde{H})}(x) = \tilde{E}$, $\mu_{Sp\mathrm{cl}(\tilde{I})}(x) = \tilde{B}$

min{ $\mu_{Sp\mathrm{cl}(\tilde{H})}(x)$, $\mu_{\tilde{I}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{Sp\mathrm{cl}(\tilde{I})}(x)$, $\mu_{\tilde{H}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Hence $\tilde{H}$ and $\tilde{I}$ are fuzzy Sp-separated sets in $\tilde{A}$.

But : $\mu_{ꞵ\mathrm{cl}(\tilde{H})}(x) = \tilde{F}^{c}$, $\mu_{ꞵ\mathrm{cl}(\tilde{I})}(x) = \tilde{C}^{c}$

min{ $\mu_{ꞵ\mathrm{cl}(\tilde{H})}(x)$, $\mu_{\tilde{I}}(x)$} $\neq \mu_{\tilde{\emptyset}}(x)$ and

min{ $\mu_{ꞵ\mathrm{cl}(\tilde{I})}(x)$, $\mu_{\tilde{H}}(x)$} $\neq \mu_{\tilde{\emptyset}}(x)$,

then $\tilde{D}$ and $\tilde{E}$ are not fuzzy ꞵ-separated sets in $\tilde{A}$.

Remark 3.32 :

Figure - 2 – illustrates the relation between fuzzy δ-separated set and some types of fuzzy separated sets.

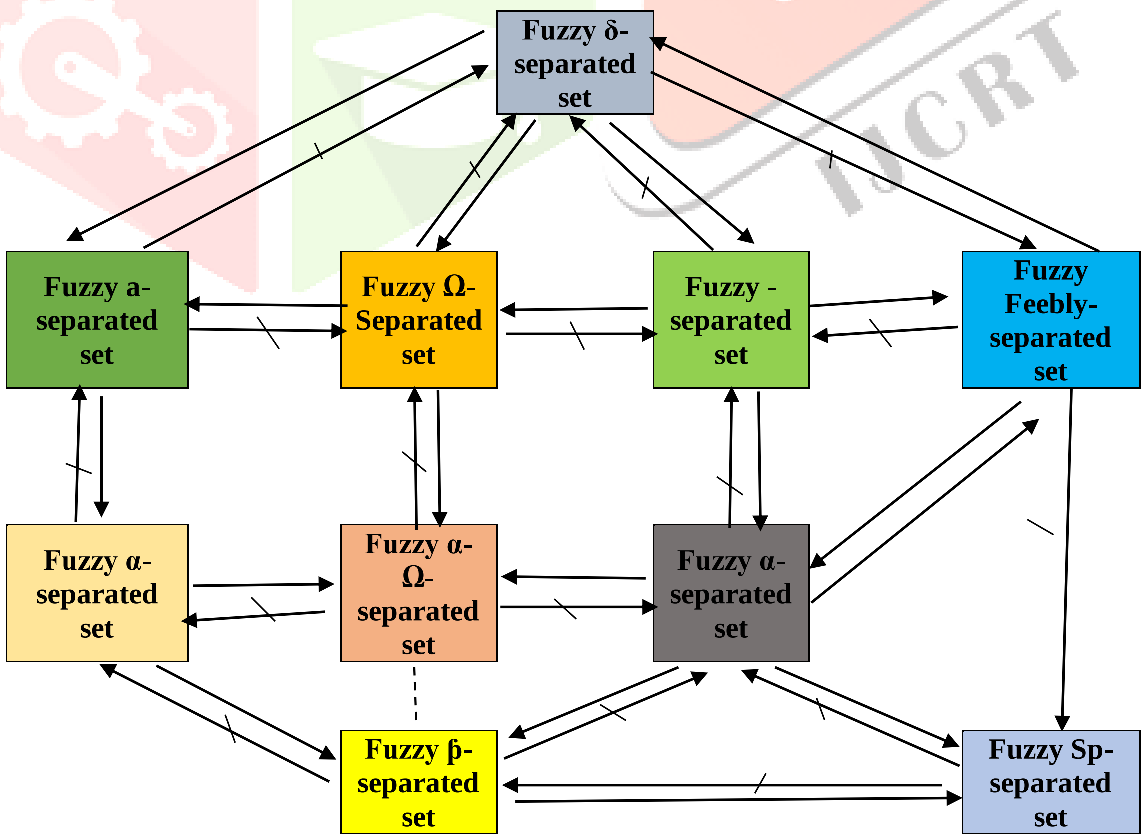

Figure -2-

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