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# KINETIC EFFECT OF RELATIVITY IN A LOW -$\beta$ PLASMA

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**Abstract:** Kinetic effect of relativity in a low-$\beta$ plasma for electrons and ions are studied analytically. Here $\beta = \frac{8\pi n_o T}{B_G{}^2}$ is the ratio of the kinetic to the magnetic pressure is taken into account. Also M = Mach number is the ratio of the wave velocity to the Alfven velocity. Q, the electron to ion mass ratio. Both compressive and rarefactive solitons are found to exist in a definite range of $K_Z$, the direction of propagation of the Kinetic Alfven solitary wave with respect to the direction of magnetic field for assigned values of $\beta$ , $v_A$, Alfven velocity and M.

**I.Introduction:** Many research workers have proved that propagation of wave can be studied with $\beta > \frac{m_e}{m_i}$ (the electron- to- ion mass ratio)..Hasegawa and Mima(1976) and Yu and Shukla(1978) investigated the wave solitons for $1 >> \beta >> \frac{m_e}{m_i}$. Dominating electron inertia over electron pressure in the magnetic field direction establishes the equilibrium. Shukla et.al.(1982) established the existence of new kind of super-Alfvenic soliton for the case where ion inertia and displacement current are neglected.

Both supersonic and subsonic rarefactive wave solitons were studied by Kalita and Kalita (1986) with low $\beta$ (<< $\frac{m_e}{m_i}$) plasma. Kumar and Srivastava(1991) studied the non linear behavior of waves using Schrodinger equation.Using modified Schrodinger equation Buti(1991) established the parallel propagation with large amplitude depending on $\beta$ . Properties of kinetic Alfven wave solitons were modified using magnetohydrodynamic solutions. Low frequency kinetic Alfven waves and electrostatic ion –cyclotron waves are supported by low $\beta$ plasma with two dimensional structure. To neglect resonance for hydromagnetic waves $\omega < \Omega_i$ is taken into consideration so that ion cyclotron frequency is greater than velocity component of Alfven solitons.

In this paper we have applied relativistic effect on low $\beta$ plasma solitons and investigate the nature of wave solitons. Supersonic compressive and subsonic rarefactive waves are found to exist in this case.

**II.Dynamics of motion:** We consider relativistic effect on a low- $\beta$ plasma with electron pressure gradient, finite electron inertia and current density. Also negligible inertia for highly magnetized electrons of are assumed to move only in the direction of the magnetic field $B_0 Z$ where Z is the unit vector along the z-axis. The low-$\beta$ (<<1) assumption for high magnetic pressure, the electric field can be written in terms of two potentials $\phi$ and $\psi$ as $E_x = -\frac{\partial \phi}{\partial x}$, $E_z = -\frac{\partial \psi}{\partial z}$

The field equation governing the dynamics of plasma in (x,z) plane, after normalizing the density by the equilibrium density $n_0$, distances by Debye length $\lambda_D$, the potentials by $\frac{T_e}{e}$, velocity by Alfven velocity $v_A$ and time by the ion gyroperiod $\Omega_i^{-1}$ is given by

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z}\left(n_e v_{ez}\right) = 0 \qquad (1)$$

$$\frac{\partial v_{ez}}{\partial t} + v_{ez}\frac{\partial v_{ez}}{\partial z} + \frac{v_A^{\ 2}}{2c^2}\frac{\partial v_{ez}^{\ 3}}{\partial t} + \frac{v_A^{\ 2}}{2c^2}\left(v_{ez}\frac{\partial v_{ez}^{\ 3}}{\partial z}\right) = \frac{\beta}{2Q}\frac{\partial \psi}{\partial z} - \frac{\beta}{2Qn_e}\frac{\partial n_e}{\partial z} \qquad (2)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i v_{ix}\right) + \frac{\partial}{\partial z}\left(n_i v_{iz}\right) = 0 \qquad (3)$$

$$v_{ix} = -\frac{\beta}{2(1+v_A^{\ 2} v_{ix}^{\ 2})}\frac{\partial^2 \phi}{\partial t \partial x} \qquad (4)$$

$$\frac{\partial v_{iz}}{\partial t} + v_{ix}\frac{\partial v_{iz}}{\partial x} + v_{iz}\frac{\partial v_{iz}}{\partial z} + \frac{v_A^{\ 2}}{2c^2}\left(v_{ix}\frac{\partial v_{iz}^{\ 3}}{\partial x}\right) + \frac{v_A^{\ 2}}{2c^2}\left(v_{iz}\frac{\partial v_{iz}^{\ 3}}{\partial z}\right) = -\frac{\beta}{2}\frac{\partial \psi}{\partial z} \qquad (5)$$

$$\frac{\partial^4(\phi-\psi)}{\partial x^2 \partial z^2} = \frac{2}{\beta}\left[\frac{\partial^2 n_e}{\partial t^2} + \frac{\partial^2}{\partial t \partial z}\left(n_i v_{iz}\right)\right] \qquad (6)$$

**III.Derivation of energy integral:** We assume that the wave is propagating obliquely to the external magnetic field and depends on the quantity $\eta = k_x x + k_z z - Mt$ (7)

Where $M = \frac{v}{v_A} = \frac{wave\ velocity}{Alfen\ velocity}$ and $k_x^{\ 2} + k_z^{\ 2} = 1$

Using the new coordinate $\eta$ moving with the wave in a quasi-neutral plasma, we get from (1)-(5), after integration and simplification,

Using the new coordinate $\eta$ moving with the wave in a quasi-neutral plasma, we get from (1)-(5), after integration and simplification,

$$v_{ez} = \frac{M}{k_z}\left(1 - \frac{1}{n}\right)$$

$$\frac{\partial \psi}{\partial \eta} = \left\{\frac{1}{n} - \frac{2QM^2}{\beta k_z^{\ 2} n^3} - \frac{3QM^4(n-1)^2}{\beta c^2 k_z^{\ 4} n^5}\right\}\frac{\partial n}{\partial \eta}$$

$$k_x v_{ix} + k_z v_{iz} = M\left(1 - \frac{1}{n}\right)$$

$$\frac{\partial^2 \phi}{\partial \eta^2} = \frac{2(1+v_A^{\ 2})}{\beta M k_x}\left(v_{ix} + v_{ix}^{\ 3} v_A^{\ 2}\right)$$

$$\frac{\partial v_{iz}}{\partial \eta} = \frac{\beta k_z n}{2M}\frac{\partial \psi}{\partial \eta}$$

Eliminating $v_{ix}$ from (6) in terms of new coordinate $\eta$ (after integrating twice) gives

$$k_x^2 k_z^2 \frac{\partial^2(\phi-\psi)}{\partial \eta^2} = \frac{2}{\beta}\left[M^2 n(1+Q) - M^2 Q + \frac{\beta k_z^2}{2} n(1-n) + \frac{QM^4}{2c^2 k_z^2}\left(n-3+\frac{3}{n}-\frac{1}{n^2}\right) - M^2\right] \qquad (8)$$

In deducing these equations we have applied the boundary conditions

$$v_{ix} = v_{iz} = v_{ez} = 0, \quad \phi = \psi = \frac{\partial n}{\partial \eta} = 0 \ at \ n = 1 \ as \ |n| \to \infty$$

And the charge-neutrality relation $n_e = n_i = n$

Putting the values of $\frac{\partial^2 \psi}{\partial \eta^2}$ and $\frac{\partial^2 \phi}{\partial \eta^2}$ into (8) and integrating twice, we get

$$\frac{d}{d\eta}\left[\left(\frac{1}{n} - \frac{2QM^2}{\beta k_z^2 n^3} - \frac{3QM^4 v_A^2}{\beta c^2 k_z^4}\frac{(n-1)^2}{n^5}\right)\frac{dn}{d\eta}\right] =$$

$$\frac{2(1+v_A^2)}{\beta M k_x^2}\left\{\frac{\beta k_z^2}{2M}(1-n) + M(1+Q)\left(1-\frac{1}{n}\right) + \frac{QM^3 v_A^2}{2c^2 k_z^2}\left(1-\frac{3}{n}+\frac{3}{n^2}-\frac{1}{n^3}\right)\right\} -$$

$$\frac{2}{\beta k_x^2 k_z^2}\left\{M^2 n(1+Q) - M^2(1+Q) + \frac{\beta k_z^2}{2} n(1-n) + \frac{QM^4 v_A^2}{2c^2 k_z^2}\left(n-3+\frac{3}{n}-\frac{1}{n^2}\right) - M^2\right\}$$

Multiplying both sides by $\left(\frac{1}{n} - \frac{2QM^2}{\beta k_z^2 n^3} - \frac{3QM^4 v_A^2}{\beta c^2 k_z^4}\frac{(n-1)^2}{n^5}\right)\frac{dn}{d\eta}$ and integrating once, we get the energy integral for classical particles in a potential well as

$$\frac{1}{2}\left(\frac{dn}{d\eta}\right)^2 + \psi(n, m, \beta, v_A, k_x, k_z) = 0$$

Where the Sagdeev Potential $\psi(n)$, a function of density n with variable parameters $M, k_x, k_z, v_A$ *and* $\beta$ , is given by

$$\psi(n, m, \beta, v_A, k_x, k_z) = \frac{-1}{\left(\frac{1}{n} - \frac{2QM^2}{\beta k_z^2 n^3} - \frac{3QM^4 v_A^2}{\beta c^2 k_z^4}\frac{(n-1)^2}{n^5}\right)^2}$$

$$\left[\left\{\frac{k_z^2(1+v_A^2)}{M^2 k_x^2}(\log n - n + 1) + \frac{2(1+Q)(1+v_A^2)}{\beta k_x^2}\left(\log n + \frac{1}{n} - 1\right) + \right.\right.$$

$$\frac{QM^2 v_A^2(1+v_A^2)}{\beta c^2 k_x^2 k_z^2}\left\{\begin{matrix}\log n + 3\left(1-\frac{1}{n}\right) + \frac{3}{2}\left(1-\frac{1}{n^2}\right) - \\ \frac{1}{3}\left(1-\frac{1}{n^3}\right)\end{matrix}\right\}$$

$$-\frac{2M^6(1+Q)}{\beta k_x^2 k_z^6}(n-1)+\frac{2M^2Q}{\beta k_x^2k_z^2}\log n-\frac{1}{k_x^2}\left(n-\frac{n^2}{2}-1+\frac{1}{2}\right)-\frac{M^4Qv_A^2}{\beta c^2k_x^2k_z^4}\left\{\begin{matrix}(n-1)+3\left(1-\frac{1}{n}\right)-\frac{1}{2}\left(1-\frac{1}{n^2}\right)\\ -3\log n\end{matrix}\right\}$$

$$+\frac{2M^2}{\beta k_x^2k_z^2}\log n\Bigg\}-\Bigg\{\frac{2Q\left(1+v_A^2\right)}{\beta k_x^2}\frac{1}{n}\left(1-\frac{1}{n}\right)+\frac{2QM^2\left(1+v_A^2\right)(1+Q)}{\beta^2k_x^2k_z^2}\left\{\frac12\left(1-\frac1{n^2}\right)-\frac13\left(1-\frac1{n^3}\right)+\right.$$

$$\frac{2Q^2M^4v_A^2\left(1+v_A^2\right)}{\beta^2c^2k_x^2k_z^4}\left\{\frac12\left(1-\frac1{n^2}\right)-\left(1-\frac1{n^3}\right)+\frac34\left(1-\frac1{n^4}\right)-\frac15\left(1-\frac1{n^5}\right)\right\}-\frac{4QM^4(1+Q)}{\beta^2k_x^2k_z^4}\left(1-\frac1n\right)+\frac{2Q^2M^4}{\beta^2k_x^2k_z^4}\left(1-\frac1{n^2}\right)$$

$$-\frac{2QM^2}{\beta k_x^2k_z^2}\left(1-\frac1n-\log n\right)-\frac{2Q^2M^6v_A^2}{\beta^2c^2k_x^2k_z^6}\left\{\left(1-\frac1n\right)-\frac32\left(1-\frac1{n^2}\right)+\left(1-\frac1{n^3}\right)-\frac14\left(1-\frac1{n^4}\right)\right\}+\frac{2QM^4}{\beta^2k_x^2k_z^4}\left(1-\frac1{n^2}\right)\Bigg\}$$

$$-\frac{3QM^2v_A^2\left(1+v_A^2\right)M^2}{\beta c^2k_x^2k_z^2}\left\{-\left(1-\frac1n\right)+\frac32\left(1-\frac1{n^2}\right)-\left(1-\frac1{n^3}\right)+\frac14\left(1-\frac1{n^4}\right)\right\}+$$

$$\frac{6Q(1+Q)M^4v_A^2\left(1+v_A^2\right)}{\beta^2c^2k_x^2k_z^4}\left\{\frac12\left(1-\frac1{n^2}\right)-\left(1-\frac1{n^3}\right)+\frac34\left(1-\frac1{n^4}\right)-\frac15\left(1-\frac1{n^5}\right)\right\}+$$

$$\frac{3Q^2M^6v_A^4\left(1+v_A^2\right)}{\beta^2c^4k_x^2k_z^6}\left\{\frac12\left(1-\frac1{n^2}\right)+\frac53\left(1-\frac1{n^3}\right)+\frac52\left(1-\frac1{n^4}\right)-2\left(1-\frac1{n^5}\right)+\frac56\left(1-\frac1{n^6}\right)-\frac17\left(1-\frac1{n^7}\right)\right\}-$$

$$\frac{6QM^6v_A^2(1+Q)}{\beta^2c^2k_x^2k_z^2}\left\{\left(\frac1{n^2}-\frac1n\right)+\frac13\left(1-\frac1{n^3}\right)\right\}+\frac{6Q^2M^6v_A^2}{\beta^2c^2k_x^2k_z^6}\left\{\frac12\left(1-\frac1{n^2}\right)-\frac23\left(1-\frac1{n^3}\right)+\frac14\left(1-\frac1{n^4}\right)\right\}-$$

$$\frac{3QM^4v_A^2}{\beta c^2k_x^2k_z^4}\left\{3\left(1-\frac1n\right)+\frac32\left(1-\frac1{n^2}\right)-\frac13\left(1-\frac1{n^3}\right)-\log n\right\}-$$

$$\frac{3Q^2M^8v_A^2}{\beta^2c^4k_x^2k_z^8}\left\{\left(1-\frac1n\right)-\frac52\left(1-\frac1{n^2}\right)+\frac{10}{3}\left(1-\frac1{n^3}\right)-\frac52\left(1-\frac1{n^4}\right)+\left(1-\frac1{n^5}\right)-\frac16\left(1-\frac1{n^6}\right)\right\}+$$

$$\frac{6QM^6v_A^2}{\beta^2c^2k_x^2k_z^6}\left\{\frac12\left(1-\frac1{n^2}\right)-\frac23\left(1-\frac1{n^3}\right)+\frac14\left(1-\frac1{n^4}\right)\right\}\Bigg]$$

**IV. Existence of solitary wave:** To find the existence of solitary waves it is necessary to study the behavior of the potential $\psi(n)$ near n=1 and n=N. For solitary wave solitons the required conditions are

$$\psi(1)=\psi(N)=\left(\frac{d\psi}{dn}\right)_{n=1}=0$$

$$\psi(n)<0$$

Expanding in Taylors series near n=1 and n =N we get for n=1

$$\psi(n)=-\frac{(n-1)^2}{2\left(k-\frac{2QM^2}{\beta K_Z^{\,2}}\right)^2}\left(\begin{array}{l}\frac{2QK\left(1+v_A^{\,2}\right)\left(k-\frac{6QM^2}{\beta K_Z^{\,2}}\right)}{\left(k-\frac{2QM^2}{\beta K_Z^{\,2}}\right)}-\frac{K^2K_Z^{\,2}\left(1+v_A^{\,2}\right)}{M^2K_x^{\,2}}-\frac{2M^2K}{\beta K_x^{\,2}K_Z^{\,2}}+\frac{K^2}{K_x^{\,2}}+\\ \frac{2K(1+Q)\left(1+v_A^{\,2}\right)}{\beta K_x^{\,2}}+\frac{4QM^4(1+Q)}{\beta^2K_x^{\,2}K_Z^{\,4}}+\frac{4QK(1+Q)}{\beta\ K_x^{\,2}}-\frac{2QM^2(1+Q)\left(1+v_A^{\,2}\right)}{\beta^2K_x^{\,2}K_Z^{\,2}}\end{array}\right)$$

and for n=N

$$\psi(n)=\frac{-(n-N)}{\left(\frac{1}{N}-\frac{2QM^2}{\beta k_z^{\,2}N^3}-\frac{3QM^4v_A^{\,2}}{\beta c^2k_z^{\,4}}\frac{(N-1)^2}{N^5}\right)^2}$$

$$\left[\left\{\frac{k_z^{\,2}\left(1+v_A^{\,2}\right)}{M^2k_x^{\,2}}\left(\frac{1}{N}-1\right)+\frac{2(1+Q)\left(1+v_A^{\,2}\right)}{\beta k_x^{\,2}}\left(\frac{1}{N}-\frac{1}{N^2}\right)+\frac{QM^2v_A^{\,2}\left(1+v_A^{\,2}\right)}{\beta c^2k_x^{\,2}k_z^{\,2}}\left\{\frac{1}{N}-\frac{3}{N^2}+\frac{3}{N^3}-\frac{1}{N^4}\right\}\right.\right.$$

$$-\frac{2M^6(1+Q)}{\beta k_x^{\,2}k_z^{\,6}}+\frac{2M^2Q}{\beta k_x^{\,2}k_z^{\,2}}\frac{1}{N}-\frac{1}{k_x^{\,2}}(1-N)-\frac{M^4Qv_A^{\,2}}{\beta c^2k_x^{\,2}k_z^{\,4}}\left\{1-\frac{3}{N}+\frac{3}{N^2}-\frac{1}{N^3}\right\}$$

$$\left.+\frac{2M^2}{\beta k_x^{\,2}k_z^{\,2}}\frac{1}{N}\right\}-\left\{\frac{2Q\left(1+v_A^{\,2}\right)}{\beta k_x^{\,2}}\left(\frac{2}{N^3}-\frac{1}{N^2}\right)+\frac{2QM^2\left(1+v_A^{\,2}\right)(1+Q)}{\beta^2k_x^{\,2}k_z^{\,2}}\left(\frac{1}{N^3}-\frac{1}{N^4}\right)\right.$$

$$+\frac{2Q^2M^4v_A^{\,2}\left(1+v_A^{\,2}\right)}{\beta^2k_x^{\,2}k_z^{\,4}C2}\left(\frac{1}{N^3}-\frac{3}{N^4}+\frac{3}{N^5}-\frac{1}{N^6}\right)-\frac{4QM^4(1+Q)}{\beta^2k_x^{\,2}k_z^{\,4}}\frac{1}{N^2}+\frac{4Q^2M^4}{\beta^2k_x^{\,2}k_z^{\,4}}\frac{1}{N^3}+$$

$$\left.\frac{2QM^2}{\beta\ k_x^{\,2}k_z^{\,2}}\left(\frac{1}{N^2}-\frac{1}{N}\right)-\frac{2Q^2M^6v_A^{\,2}}{\beta^2c^2k_x^{\,2}k_z^{\,6}}\left(\frac{1}{N^2}-\frac{3}{N^3}+\frac{3}{N^4}-\frac{1}{N^5}\right)+\frac{4QM^4}{\beta^2k_x^{\,2}k_z^{\,4}}\frac{1}{N^3}\right\}-$$

$$-\frac{3QM^2v_A^{\,2}\left(1+v_A^{\,2}\right)M^2}{\beta c^2k_x^{\,2}k_z^{\,2}}\left(\frac{1}{N^5}-\frac{3}{N^4}+\frac{3}{N^3}-\frac{1}{N^2}\right)+\frac{6Q(1+Q)M^4v_A^{\,2}\left(1+v_A^{\,2}\right)}{\beta^2c^2k_x^{\,2}k_z^{\,4}}\left(\frac{1}{N^3}-\frac{3}{N^4}+\frac{3}{N^5}-\frac{1}{N^6}\right)+$$

$$\frac{3Q^2M^6v_A^{\,4}\left(1+v_A^{\,2}\right)}{\beta^2c^4k_x^{\,2}k_z^{\,6}}\left(\frac{1}{N^3}-\frac{5}{N^4}+\frac{10}{N^5}-\frac{10}{N^6}+\frac{5}{N^7}-\frac{1}{N^8}\right)-+$$

$$\frac{6Q^2M^6v_A^{\,2}}{\beta^2c^2k_x^{\,2}k_z^{\,6}}\left(\frac{1}{N^3}-\frac{2}{N^4}+\frac{1}{N^5}\right)-\frac{3QM^4v_A^{\,2}}{\beta c^2k_x^{\,2}k_z^{\,4}}\left(\frac{1}{N^4}-\frac{3}{N^3}+\frac{3}{N^2}-\frac{1}{N}\right)-\frac{6QM^6v_A^{\,2}(1+Q)}{\beta^2c^2k_x^{\,2}k_z^{\,2}}\left(\frac{1}{N^2}-\frac{2}{N^3}+\frac{1}{N^4}\right)$$

$$\frac{3Q^2M^8v_A^{\,4}}{\beta^2c^4k_x^{\,2}k_z^{\,8}}\left(\frac{1}{N^2}-\frac{5}{N^3}+\frac{10}{N^4}-\frac{10}{N^5}+\frac{5}{N^6}-\frac{1}{N^7}\right)+\frac{6QM^6v_A^{\,2}}{\beta^2c^2k_x^{\,2}k_z^{\,6}}\left(\frac{1}{N^3}-\frac{2}{N^4}+\frac{1}{N^5}\right)$$

**V.Discussion:** In the low $K_z$ plasma under consideration supersonic compressive and subsonic rarefactive solitons found to exist depending on $\beta$, M and $K_z$. The amplitude of the supersonic compressive (N>1) solitons uniformly increases as M(> $K_z$) and a fixed value of $\beta$=.009 for Kz =.6,.7,.8,.6,.9 [fig-1].

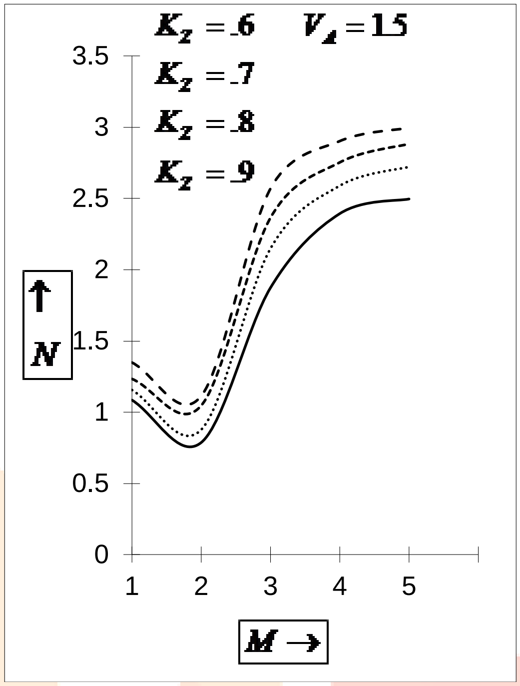
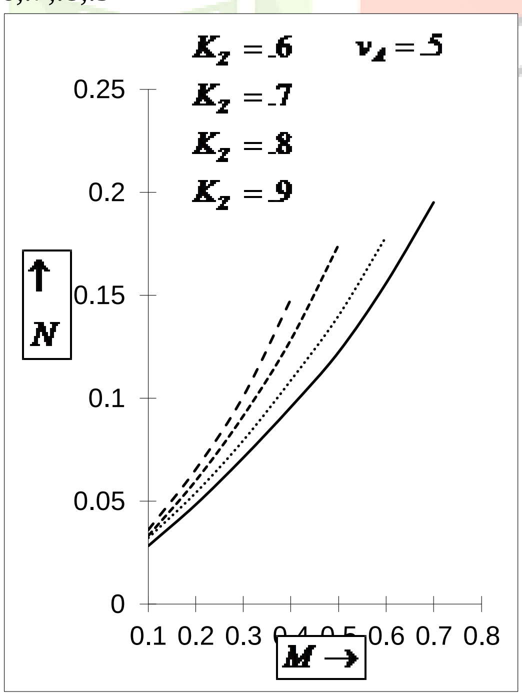



fig-1

Again the amplitude of the subsonic rarefactive (N<1) solitons uniformly increases as M(< $K_z$) and a fixed value of $\beta$=.0001 for $K_Z$ = .6,.7,.8,.9




[fig-2]

Again the amplitude of the supersonic compfressive (N>1) solitons uniformly decreases as M(> $K_z$) and a fixed value of $\beta$=.009 for M=2,3,4,5 [fig-3].

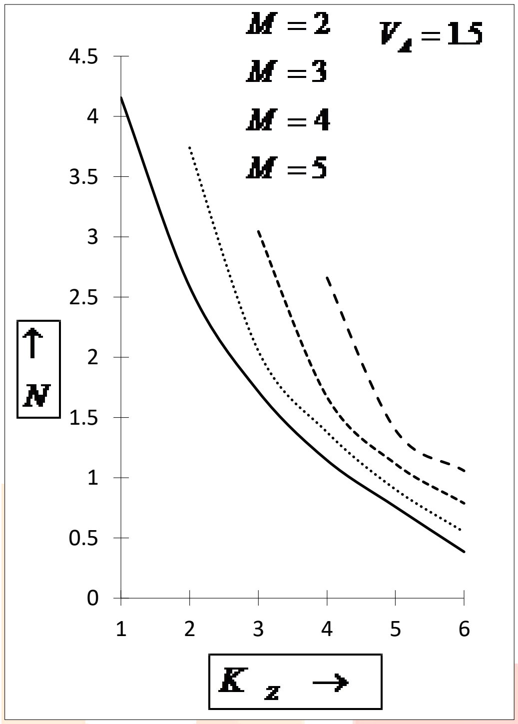



fig-3

Again the amplitude of the subsonic rarefactive (N<1) solitons uniformly decreases as M(< $K_z$) and a fixed value of $\beta$=.001 for M=.1,.2,.3,.4,.5 [fig-4].

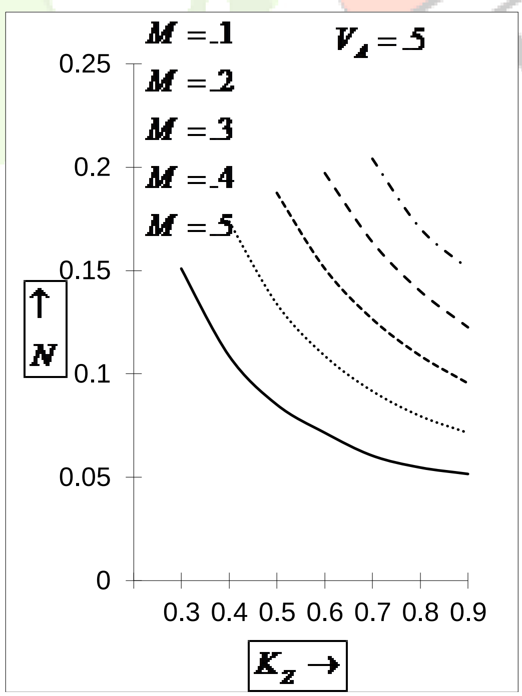



fig-4

Finally consideration of electron pressure gradient with relativistic effect are found to suppress the speed (M< $K_z$ ) of both types of Alfven solitons.

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