



# An Efficient Fourth Order PDE-Based Nonlinear Filter Adapted to Rician Noise for Restoration and Enhancement of Magnetic Resonance Images

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## ABSTRACT

In this manuscript, for the removal of Rician noises from MR images a fourth order PDE (partial differential equation) based nonlinear filter is proposed. The filter implemented contains two terms *firstly*, the data fidelity term and *secondly*, prior function term. By minimizing the negative log likelihood of Rician probability density function (pdf) we get the fidelity term. A prior function is introduced because the solution of the first term has ill-posed nature. This prior function is a nonlinear anisotropic diffusion based filter. To maintain the trade off balance between these two terms a regularization parameter is introduced. In terms of PSNR at various noise levels the comparative study and performance analysis of the suggested model with other standard methods is presented for Brain Web dataset. From the results obtained, it is clear that the method implemented is performing better noise removal as compared to other methods.

**Keywords:** Nonlinear fourth order PDE based filter, 2D MR images, Rician's probability distribution function, Rician noise reduction.

## 1. INTRODUCTION

In the field of medical imaging magnetic resonance imaging (MRI) is considered as one of the most effective and powerful tool. Many issues come across MR imaging such as different oblique recreation time values in overlapping resonance, and low signal to noise ratio (SNR) [1]. Due to low SNR at the time of acquisition process, one of the disturbing conditions is thermal noise which oscillates the signal randomly and retrograde the quantitative measurements for further clinical analysis [2, 3, 4]. Therefore this process is not a common practice in clinic, because reducing the thermal noise and increasing the SNR can be used by averaging of MR signals, but due to increase in the acquisition time of MRI limits their utilization in many situations where long replications are not viable, such as for unstable biological compounds. Higher the resolution of the desired SNR more will be the acquisition time of the MRI because there is ingrained settlement between high SNR and resolution of MR images [2, 5].

Thus, for the removal of noise and increasing the accuracy there are several alternatives performed after the acquisition such as clinical diagnostic system, post-processing, de-noising and enhancement techniques. The type of image determines whether the noise is Gaussian or Rician distributed noise in MRI. Over magnitude MR images, usually noise estimation is done because this is the typical output of the scanning process [6], phase and magnitude data are directly related to the anatomical and physiological information of significance [7]. Additive white Gaussian noise having equal variances in imaginary and real parts, corrupted complex valued raw data of an MRI in the frequency domain  $k$ -space.

Henkelman [8] introduced for the first time the aftermath of noise on magnitude magnetic resonance images, scrutinized for the estimation of magnitude MRI, from a noisy data degraded by Rician noise. A lot of filtering methods that are based on the principle of signal averaging are used in natural spatial pattern redundancy of images. The edges of the image get blurred by the Gaussian filter also it affects the high-frequency regions of the images.

Therefore this filter is commonly used in some de-noising applications [9]. This problem gets quenched by using the edge preserving filters like anisotropic diffusion filters (ADFs) [10, 11, 12].

For MRI de-noising, several approaches were proposed in literature such as machine learning-based approaches [13, 14, 15, 16, 17, 18], discrete cosine transformation-based filter [9], maximum likelihood approach [4, 7], principal component analysis-based technique [20], conventional approach [21], and wavelet-based approaches [22, 23].

In case of Rician noise removal, the methods mentioned above only remove high-frequency signal components that results in blurring the boundaries and introducing some extra bias in the quantification process. Hence, there is a need of using more advanced image restoration methods to overcome this drawback. Therefore, to address these problems a PDE based [32] technique adapted to Rician noise is proposed for restoration and enhancement of MR images and also its comparative analysis with other standard methods is presented.

The paper comprises of five sections: Section 1, presents introduction; Section 2, presents the noise distribution in MR images; Section 3, presents methods and model; Section 4, presents an experimental setup, results and discussions, and finally Section 5, presents the conclusion of the work.

## 2. NOISE IN MR IMAGES

Rician noise amplitude is given as  $f(x, y) = f_R(x, y) + f_I(x, y)$  where  $f_R(x, y)$  is the zero mean and  $f_I(x, y)$  is independent Gaussian random variables for some variance. Field intensity of Rician noise is given as

$$n(x, y) = |f(x, y)|^2 = f_R^2 + f_I^2.$$

The image observation model for Rician noise reads:

$$I_o = I(x, y) \times n(x, y) + \eta(x, y) \quad (1)$$

where  $I_o(x, y)$  is the observed Rician noised image;  $I(x, y)$  is the original noise free image and  $n(x, y)$  is the multiplicative noise with zero mean and known variance  $n^2$  and  $\eta(x, y)$  is the detector noise which is additive in nature. Assuming the detector noise to be zero, the general observation model reads:

$$I_o(x, y) = I(x, y) \times n(x, y) \quad (2)$$

By transforming the complex value into magnitude image the noise free MR image can be calculated directly from its real and imaginary components. The anatomical and physiological quantities in the MRI are shown by the magnitude image [2, 7]. For automated computer analysis [5], the magnitude MR images are real valued and can be visualized easily. The conversion of the MRI distribution from Gaussian to Rician is performed by the non linear operation at the time of transformation.

$$P(I/M, \sigma) = \frac{M}{\sigma^2} \exp\left(-\frac{M^2 + I^2}{\sigma^2}\right) J_0\left(\frac{IM}{\sigma^2}\right) u(M) \quad (3)$$

where  $I$  is the amplitude of a noise-free image,  $M$  is the magnitude MR image,  $\sigma^2$  shows the Gaussian noise variance,  $J_0(\cdot)$  is the modified zero order Bessel function, and  $u(\cdot)$  is the unit step Heaviside function.

## 3. METHODS AND MODELS

By minimizing the following nonlinear energy function of the image  $I$  in a continuous domain  $\Omega$ , we get the Rician noise removal and regularization of MR image data:

$$E(I) = \arg \min_m \left\{ \int_{\Omega} [L(P(I/M)) + \lambda \cdot \phi(\|\nabla I\|)] \right\} \quad (4)$$

$L(P(I/M))$  - which denotes the negative likelihood term of Rician distributed noise in MRI. In the process of filtering the dissimilarities at a pixel between  $M$  and its estimated value  $I$  is measured by log likelihood term. In equation (3) the value of unit step Heaviside function  $u(\cdot)$  is equal to one for positive integer. To compute the log likelihood of above mentioned Rician probability distribution function, firstly we take the logarithm of both sides and to make the calculation simple and fast, we try to make simpler the third term in equation (3), i.e., zero order modified Bessel's function  $J_0\left(\frac{IM}{\sigma^2}\right)$ . The zero order Bessel functions solution i.e.  $J_0(\cdot)$  for positive integer is provided in the Abramowitz and Stegun handbook of mathematical functions [25], which shows that:

$$J_0(z) = \sum_{k_1=0}^{\infty} \frac{\left(-\frac{z^2}{4}\right)^{k_1}}{k_1!^2} \quad (5)$$

Now considering the logarithm of the above function and we obtain the simplified solution with logarithmic domain.

Then putting the value of  $z = \frac{IM}{\sigma^2}$  in the above equation then we differentiating equation (5) get the third term of equation (3) as follows:

$$\frac{\partial}{\partial I} \left\{ \log j_0\left(\frac{IM}{\sigma^2}\right) \right\} = \frac{2k_1}{I} \quad (6)$$

To procure maximum likelihood of  $I$ , the first order derivative of (3) Rician's pdf w.r.t.  $I$  reads:

$$L\{P(I/M)\} = -\frac{I}{\sigma^2} + \frac{2k_1}{I} \quad (7)$$

When we differentiate equation (7) w.r.t.  $I$ , three times more then we get fourth order Rician's pdf reads:

$$L'''\{P(I/M)\} = -\frac{12k_1}{I^4} \quad (8)$$

where  $L'''\{P(I/M)\}$  is expressed as the likelihood term in equation (4). The factor that leads to de-noising of image data are maximization of log likelihood or minimization of the negative log likelihood, but this problem is an ill-posed problem and hence regularization is needed. That is why the second term in equation (4) is needed which acts as a regularization or penalty function. In the equation (4), regularization parameter i.e.  $\lambda$ , is responsible for making a fair balance between the two terms which are fidelity measured by maximum likelihood function and regularization or penalty function in the proposed models. For deriving anisotropic diffusion based filter defined by Perona and Malik [12] is the suitable choice for the energy term  $\phi(\|\nabla I\|)$  based on energy function.  $\phi(\|\nabla I\|) = \|\nabla I\|^2$  which is the gradient norms of the image. Putting the value of  $\phi(\|\nabla I\|) = \|\nabla I\|^2$  in equation (4) we get:

$$E(I) = \arg \left\{ \int_{\Omega} \left[ L(P(I/M)) + \lambda \cdot \phi(\|\nabla I\|^2) \right] \right\} \quad (9)$$

Perona and Malik originally nominate the anisotropic diffusion based filter and the final model for the restoration of Rician noise corrupted MR image is given as:

$$\frac{\partial I}{\partial t} = L'''\{P(I/M)\} + \lambda \cdot \text{div}(c(\|\nabla I\|)\nabla I) \quad (10a)$$

with initial condition

$$I_{t=0} = I_0 \quad (10b)$$

The initial condition as the noisy image  $I_0$  of the PDE equation (10b) creates after certain iterations till its convergence the filtered de-noised Rician image. Where term  $c(\|\nabla I\|)$  in equation (10a) is known as conductivity coefficient and given as:

$$c(\|\nabla I\|) = \frac{1}{1 + \frac{\|\nabla I\|^2}{\gamma^2}} \quad (11)$$

where  $\gamma$  is the gradient threshold that differentiates the homogeneous regions and area of edges and contours, the value of  $\gamma$  always be greater than zero.

$$\frac{\partial I}{\partial t} = -\frac{12k_1}{I^4} + \lambda \Delta.(c(\|\nabla I\|)\nabla I) \quad (12a)$$

with initial condition

$$I_{t=0} = I_0 \quad (12b)$$

### 3.1 DISCRETISATION OF THE PROPOSED MODEL

The equation (8a) and (10a) can be discretized using finite difference schemes [26], is the proposed model. The proposed PDE based model is represented in the discretized form given as:

$$I^{n+1}(x, y) = I^n(x, y) + \Delta t \left[ L \{P(I/M)\} + \lambda \cdot \text{div}(c(\|\nabla I^n(x, y)\|) \nabla I^n(x, y)) \right]$$

$$I^{n+1}(x, y) = I^n(x, y) + \Delta t \left[ \left( -\frac{12k_1}{I^n(x, y)} \right) + \lambda \cdot \text{div}(c(\|\nabla I^n(x, y)\|) \nabla I^n(x, y)) \right]$$

The Von Neumann analysis [26], indicates that condition require  $\Delta t / \Delta x \leq 1/4$  for the numerical scheme, given by above equation to become stable.

## 4. RESULTS AND DISCUSSION

All clinical investigations must be conducted For simulated (synthetic) and real (clinical) data sets of normal brain MR images, Brain Web database [27] is used to compare the potency of the suggested method. Brain Web data bases [27] consist of three modalities (pulse sequences) which are T1, T2 and PD weighted. MATLAB R2014 used for the implementation of proposed method as well as for the other standard method used for the comparison purposes. The performance of restoration results are analyzed for images artificially degraded by Rician noise. Non-local means (NLM) filter [3], Adaptive Wiener Filter (AWF) [28], Fuzzy-based hybrid filter [29], MF [30], and BM3D de-noising filter [31] are well known existing techniques used for comparing the proposed method. The best setups as proposed by the authors and the free parameters of these methods are used during experimentation. For best results the relevant values of the parameters are given below:

- BM3D: for calculating level of the noise, variance of actual noise is taken and other parameter adjusted according to the authors in the article.
- MF: size 3×3 of the window.
- AWF: using a 5×5 neighbourhood window. To achieve the best performance, variance of noise is manually located to the real value.
- Fuzzy based hybrid filter: the parameters are adjusted according to the authors in the article.
- NLM filter: actual noise variance is used for the calculation of noise level, radius of the searching area=5, radius of the local area=3, the correction constant=0.15.

For de-noising MR images adjustment of parameters are done empirically and the Table 2 show all the parameter setup using the proposed scheme. Quantitative comparison is done with the help of peak signal-to-noise ratio (PSNR). Performance of the proposed approach is compared with few standard de-noising methods on simulated and real MR data sets. The MR data taken are artificially corrupted with a variance of the range 5–30% for the evaluation of quantitative metrics. All these de-noising methods are computed for over 5-50 iterations or till convergence which are totally based on PSNR average restoration results.

Figure 1 illustrate detailed results, obtained with the close up view of the restored images for better inspection, in order to compare the visual performance, existing and proposed approaches, incorporates real image, noisy image and the restored image. The visual results for simulated MR slice is corrupted with 10 % level of Rician noise, in Figure1. The BM3D and AWF are unable to smooth out noise completely and provide a blurred output as it can be observed from the

Figure 1 Median filter is good at preserving some details at the cost of some noisy spots, NLM filter removes the noise completely, but taking much more time and most of the image structural information has been lost. Similarly, fuzzy based hybrid filter provides better results with noisy spots than NLM filter. To overcome these limitations, the proposed approach produces better results. On the basis of quantitative and visual results it is apparent that the proposed approach has produced more accurate results such as more noise removing ability, and preservation of edges and structural information, at all levels of Rician noise.

In Figure 2 the performance analysis and comparative study of the suggested method along with other standard methods are represented on the basis of quantitative results (PSNR) for different levels of Rician noise. Also it is observed that PSNR at low as well as at high rates of Rician noise has given much better restoration results of the proposed PDE based technique as compared to other existing methods.

At low-noise corrupted detailed regions the non-local filter [3] gives good results while the local filter [30] gives better performance at smooth regions degraded with noise, whereas, the proposed technique performs better in low as well as high noise corrupted image data. The hybrid filter [29] is based on noise contamination and its region characteristic, which adaptively assign appropriate fuzzy weights to local and non-local filters and the restoration results are better than NLM [3] based method.

Figure 2 shows that with the increase in noise rate proposed filter's efficiency also increases. The proposed technique easily differentiates the low and high noise regions at higher noise rates hence the better result are obtained.

**Figure 1- simulated t1 weighted MR image with Rician noise (a) original image (b) 10% noisy image (c) BM3D (d) MF (e) AWF (f) NLM (g) Fuzzy filter (h) proposed method**

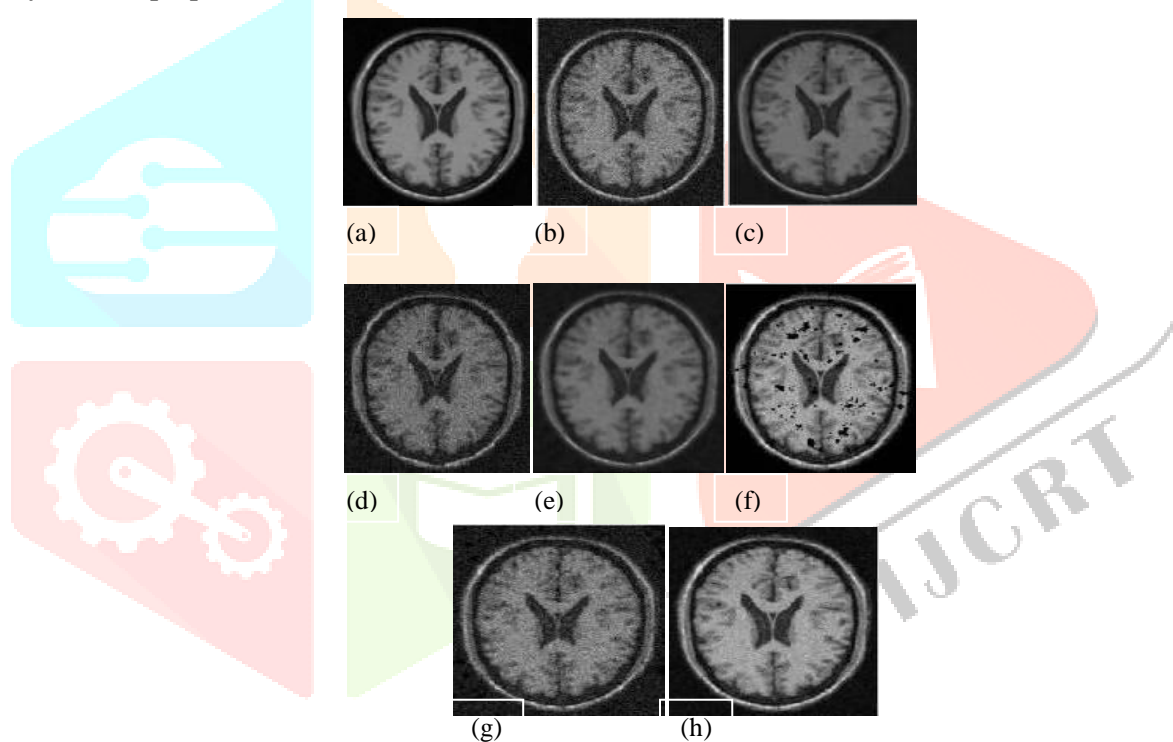
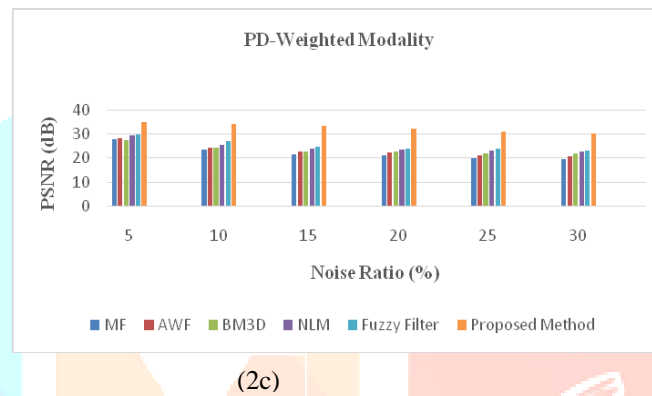
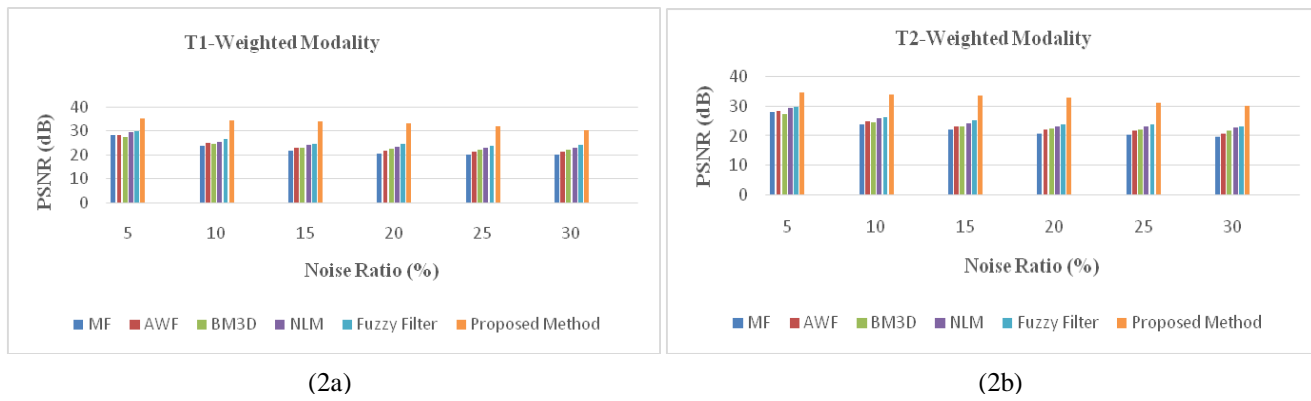


Figure 2- (2a) PSNR based comparison of T1-weighted modality; (2b) PSNR based comparison of T2-weighted modality; (2c) PSNR based comparison of PD-weighted modality



## CONCLUSION

For restoration and enhancement of magnetic resonance images an efficient fourth order PDE based nonlinear filter adapted to Rician noise has been proposed in this article. In this scheme likelihood term, regularization parameter and regularization function have been observed in an innovative way. The mathematical calculation of log likelihood of Rician noise pdf has been simplified. On the basis of obtained description results in terms PSNR the proposed method gives better performance (de-noising) than existing techniques over simulated MR brain images. Further, visual results clearly point that the proposed technique has the capability of better noise removal.

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