IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

SOME PROPERTIES OF A KENMOTSU MANIFOLDADMITTING A NON-SYMMETRICNON-METRIC CONNECTION

Abhishek Singh
Assistant Professor
Department of Mathematics and Statistics
Dr. Ram Manohar Lohia Avadh University, Faizabad, U.P, India

Abstract. The objective of this paper is to investigate some properties of a Kenmotsu manifold admitting a non-symmetric non-metric connection. We study some properties of a Riemannian curvature tensor on a Kenmotsu manifold admitting a non-symmetric non-metric connection. Further, we also study Ricci solitonon a Kenmotsu manifold admitting a non-symmetric non-metric connection.

Key words and phrases. Kenmotsu manifold, non-symmetric non-metric connection, curvature tensor, Ricci tensor, scalar curvature, Ricci solitons.

2010 Mathematics Subject Classification.53C15, 53C25, 53C05, 53D10.

1. Introduction

The concept of Kenmotsu manifoldwas defined by K. Kenmotsu [10]. Kenmotsu proved that a locally Kenmotsu manifold is a warped product $I \times_f N$ of an interval I and a Kähler manifold N with warping function $f(t) = se^t$, where s is a non-zero constant. Kenmotsu [10] introduced a class of almost contact metric manifold and named as Kenmotsu manifold. Since then, the properties ofKenmotsu manifold have studied by several authors such as Sinha and Srivastava [14], Cihan [5] and many others. The main invariants of an affine connection are its torsion and curvature [6]. A connection ∇ is called torsion-free or symmetric if its torsion tensor $T(X,Y)$ vanishes and if it does not vanish then it is known as non-symmetric. A linear connection $\tilde{\nabla}$ on M^n is called non-symmetric if

$$\tilde{T}(X,Y) = 2g(\phi X, Y), (1.1)$$

holds $\forall\, X,\ Y$ on M^n. Further, A connection ∇ on a manifold M is called a metric connection if there is a Riemannian metric g in M such that $\nabla g = 0$, and if $\nabla g \neq 0$ then it is known as non-metric. Systematic study of semi-symmetric connection in a Riemannian manifold was introduced by Yano [16]. Golab [7], defined and studied quarter-symmetric connection in a differentiable manifold with affine connection. After that various properties of quarter-symmetric metric connection have been studied by many geometers like Rastogi [13], Mishra and Pandey [11], Yano and Imai [17], Pradeep Kumar, Venkatesha and Bagewadi [12] and many others. Later on some other authors [4] studied several connections. M. M. Tripathi [15] proved the existence of a new connection and showed that in particular cases.

On the other hand Ricci soliton is a natural generalization of Einstein metric and is also a self-similar solution to Hamilton's Ricci flow [8, 9]. The Ricci flow is an evolution equation for metrics on a Riemannian manifold defined as under:

$$\frac{\partial}{\partial t} g_{ij}(t) = -2R_{ij}.$$

A Ricci soliton (g, V, λ) on a Riemannian manifold (M, g) is defined as follows: $L_V g(X,Y) + 2S(X,Y) + 2\lambda g(X,Y) = 0,$ (1.2)

$\forall X, Y \in \chi(M)$, where $\chi(M)$ is a set of all smooth vector fields on M, S is a Ricci tensor, L_V is the Lie-derivative operator along the vector field V on M and λ is a real number. If the potential vector field V vanishes identically then the Ricci soliton becomes trivial and in this case the manifold is an Einstein one. The Ricci soliton is said to be shrinking, steady and expanding according to $\lambda < 0$, $\lambda = 0$ and $\lambda > 0$, respectively.

This present paper is organized as follows: Section 1 is introductory. Section 2 consolidates some basic informations about the Kenmotsu manifold.Section 3, concerned with the study of the curvature tensor as well as its properties in the Kenmotsu manifold admitting a non-symmetric non-metric connection. We study Ricci soliton on a Kenmotsu manifold admitting a non-symmetric non-metric connection in section 4.

2.Preliminaries

Let M be a $(2n+1)$-dimensional almost contact metric manifold with an almost contact metric structure (ϕ, ξ, η, g)consisting of a $(1,1)$tensor field ϕ, a vector field ξ, a 1-form η, and the Riemannian metric g on M satisfying [2]

$$\eta(\xi) = 1, \phi\xi = 0, \eta(\phi(X)) = 0, \quad g(X, \xi) = \eta(X), \quad (2.1)$$

$$\phi^2 X = -X + \eta(X)\xi, g(X, \phi Y) = -g(\phi X, Y), (2.2)$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), (2.3)$$

An almost contact metric structure (ϕ, ξ, η, g) is a Kenmotsu manifold [12] iff

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X. (2.4)$$

Hereafter we denote the Kenmotsu manifold of dimensional $(2n+1)$ by M.

From the above relation, it follows that

$$\nabla_X \xi = X - \eta(X)\xi, (2.5)$$

$$(\nabla_X \eta)Y = g(X,Y) - \eta(X)\eta(Y) = g(\phi X, \phi Y), (2.6)$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \quad (2.7)$$

$$R(\xi, X)Y = \eta(Y)X - g(X,Y)\xi, \quad (2.8)$$

$$R(\xi, X)\xi = X - \eta(X)\xi, \quad (2.9)$$

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), (2.10)$$

$$S(\phi X, \phi Y) = S(X,Y) + 2n\eta(X)\eta(Y), (2.11)$$

$$S(X, \xi) = -2n\eta(X), (2.12)$$

$$S(X,Y) = g(QX, Y), (2.13)$$

where R, S and Q denote the curvature tensor, Ricci tensor and Ricci operator of M, respectively, with respect to the Levi-Civita connection.

Definition 2.1. An almost contact metric manifold$M$ is said to be a generalizedη-Einstein manifold if the following condition

$$S(X,Y) = \lambda_1 g(X,Y) + \lambda_2 \eta(X)\eta(Y) + \lambda_3 g(\phi X, Y), (2.14)$$

holds on M, where λ_1, λ_2 and λ_3 are smooth functions on M. If $\lambda_3 = 0$, $\lambda_2 = \lambda_3 = 0$ and $\lambda_1 = \lambda_3 = 0$, then the manifold is called an η-Einstein, an Einstein and a special type of η-Einstein manifold, respectively.

3.A non-symmetric non-metric connection

Let us define, a linear connection $\tilde{\nabla}$[3] as

$$\tilde{\nabla}_X Y = \nabla_X Y + g(\phi X, Y)\xi, (3.1)$$

satisfying

$$\tilde{T}(X,Y) = 2g(\phi X, Y)\xi (3.2)$$

and

$$\left(\tilde{\nabla}_X g\right)(Y,Z) = -\eta(Z)g(\phi X, Y) - \eta(Y)g(\phi X, Z). (3.3)$$

for arbitrary vector fields X, Y and Z is said to be a non-symmetric non-metric connection. Also, we have

$$\left(\tilde{\nabla}_X \phi\right)(Y) = (\nabla_X \phi)(Y) + g(\phi X, \phi Y)\xi, (3.4)$$

$$\left(\tilde{\nabla}_X \eta\right)(Y) = (\nabla_X \eta)(Y) - g(\phi X, Y), (3.5)$$

$$\left(\tilde{\nabla}_X g\right)(\phi Y, Z) = -\eta(Z)g(\phi X, \phi Y). (3.6)$$

On replacing Y by ξ in the equation (3.1), we have

$$\left(\tilde{\nabla}_X \xi\right) = \nabla_X \xi. (3.7)$$

On replacing $X = \xi$ in the equation (3.3), we have

$$(\tilde{\nabla}_\xi g)(Y,Z) = 0. (3.8)$$

The curvature tensor $\tilde{R}$ of the connection $\tilde{\nabla}$ is defined as follows

$$\tilde{R}(X,Y)Z = \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y]} Z, (3.9)$$

where $X, Y, Z \in TM$, using equation (3.1), we have

$$\tilde{R}(X,Y)Z = R(X,Y)Z + g\big((\nabla_X \phi)Y, Z\big)\xi - g\big((\nabla_Y \phi)X, Z\big)\xi$$

$$+g(\phi Y, Z)\nabla_X \xi - g(\phi X, Z)\nabla_Y \xi, (3.10)$$

where

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z (3.11)$$

is the Riemannian curvature tensor [2] of Levi-Civita connection.

Using equations (2.1), (2.2), (2.4) and (2.5), we have

$$\tilde{R}(X,Y) = R(X,Y)Z + 2\eta(Z)g(\phi X, Y)\xi - g(\phi X, Z)Y + g(\phi Y, Z)X. (3.12)$$

Contracting equation (3.12) with respect to X, we have

$$\tilde{S}(Y,Z) = S(Y,Z) + 2ng(\phi Y, Z). (3.13)$$

Using equation (2.13) in equation (3.13), we have

$$\tilde{Q}(Y) = Q(Y) + 2n(\phi Y). (3.14)$$

Contracting equation (3.13), we have

$$\tilde{r} = r. (3.15)$$

where $\tilde{S}(Y,Z)$; $S(Y,Z)$,$\tilde{Q}$; Q and $\tilde{r}$; r are the Ricci tensors, Ricci operators and scalar curvatures of a non-symmetric non-metric connection $\tilde{\nabla}$and Levi-Civita connection ∇.

Hence, we have the following theorems:

Theorem 3.1. In a Kenmotsu manifold the scalar curvature is invariant with respect to a non-symmetric non-metric connection $\tilde{\nabla}$ and Levi-Civita connection ∇.

Theorem 3.2. In a Kenmotsu manifold admitting a non-symmetric non-metric connection$\tilde{\nabla}$, the curvature tensor, Ricci tensor, Riccioperator and scalar curvature are given by equations (3.12), (3.13), (3.14) and (3.15) respectively.

4.Ricci soliton on a Kenmotsu manifold admitting a non-symmetric non-metric connection

Let (g,ξ,λ) be a Ricci soliton on a Kenmotsu manifold with respect to a non-symmetric non-metric connection $\tilde{\nabla}$, then we have

$$(\tilde{L}_\xi g)(X,Y)+2\tilde{S}(X,Y)+2\lambda g(X,Y)=0. (4.1)$$

Now

$$(\tilde{L}_\xi g)(X,Y)=g(\tilde{\nabla}_X\xi,Y)+g(X,\tilde{\nabla}_Y\xi). (4.2)$$

Using (2.1), (2.3), (2.5) and (3.7) in (4.2), we have

$$(\tilde{L}_\xi g)(X,Y)=2[g(X,Y)-\eta(X)\eta(Y)]. (4.3)$$

Using (3.13) and (4.3) in (4.1), we have

$$S(X,Y)=-(1+\lambda)g(X,Y)+\eta(X)\eta(Y)-2ng(\phi X,Y). (4.4)$$

Hence, we have the following theorem:

Theorem 4.1. If a Ricci soliton (g,ξ,λ)in a Kenmotsu manifold admitting the connection$\tilde{\nabla}$ then the manifold is a generalized η-Einstein manifold.

Let (g,V,λ) be the Ricci soliton on a Kenmotsu manifold admitting a non-symmetric non-metric connection $\tilde{\nabla}$ such that V is pointwise collinear with ξ that is $V=b\xi$, where b is a function. Then (1.2) holds and follows that

$$bg(\tilde{\nabla}_X\xi,Y)+(Xb)\eta(Y)+bg(X,\tilde{\nabla}_Y\xi)+(Yb)\eta(X)+2\tilde{S}(X,Y)+2\lambda g(X,Y)=0. (4.5)$$

Using (2.1), (2.5), (3.7) and (3.13) in (4.5), we have

$$2(b+\lambda)g(X,Y)+(Xb)\eta(Y)+(Yb)\eta(X)$$

$$-2b\eta(X)\eta(Y)+2S(X,Y)+4ng(\phi X,Y)=0. (4.6)$$

Replacing Y by ξ in (4.6) and using (2.1), (2.12), we have

$$(Xb)=-\{2(\lambda-n)+\xi b\}\eta(X). (4.7)$$

Again replacing X by ξ in (4.7) and using (2.1), we have

$$(\xi b)=(n-\lambda). (4.8)$$

By virtue of (4.8), above equation (4.7) takes the form

$$(Xb)=(n-\lambda)\eta(X). (4.9)$$

By applying d on (4.9), we have

$$(n-\lambda)d\eta=0. (4.10)$$

Since $d\eta\neq 0$, (4.10) yields

$$\lambda = n. (4.11)$$

Using (4.11) in (4.9), we obtain b is a constant.

Hence, we have the following theorem:

Theorem 4.2. If a Ricci soliton (g, V, λ)in a Kenmotsu manifold admitting the connection$\widetilde{\nabla}$ such that $V = b\xi$, then V is a constant multiple of ξ and the Ricci soliton is always expanding.

REFERENCES

[1] A. Friedmann and A. Schouten, Über die Geometric der halbsymmetrischen Übertragung; Math., Zeitschr. 21, 211-223, 1924.
[2]B. B. Sinha & A. K. Srivastava, Curvatures on Kenmotsu manifold; Indian J. Pure Appl. Math., 22(1), 23-28, 1991.
[3] C. S. Bagewadi and G. Ingalahalli, Ricci solitons in Lorentzian α-Sasakian manifolds; Acta Math. Acad. P. N., Vol-28, 59-68, 2012.
[4] D. E. Blair, Contact manifolds in Riemannian geometry; Lecture Notes in Math. No. 509, springer Verlag, 1976.
[5] K. Kenmotsu, A class of almost contact Riemannian manifolds; Tohoku Math. J., 24, 93-103, 1971.
[6] K. T. Pradeep Kumar, C. S. Bagewadi and Venkatesha, On projective ϕ-symmetric K-contact manifold admitting quarter-symmetric metric connection; Differ. Geom. Dyn. Syst., 13, 128-137, 2011.
[7] K. Yano, On semi-symmetric metric connections; Rev. Roumaine Math. Press Apple. 15, 1579-1586, 1970.
[8] K. Yano and T. Imai, Quarter-symmetric metric connections and their curvature tensor; Tensor, N. S., 38, 13-18, 1982.
[9] M. M. Tripathi, A new connection in a Riemannian manifold; International Electronic Journal of Geometry Vol. 1 No. 1 pp. 15-24, 2008.
[10] O. Cihan & U. C. De, On the quasi-conformal curvature tensor of a Kenmotsu manifolds; Mathematica Pannonica, 17 (2), 221-228, 2006.
[11] R. S. Mishra and S. N. Pandey, On quarter-symmetric metric F-connections; Tensor, N. S., 34, 1-7, 1980.
[12] S. C. Rastogi, On quarter-symmetric metric connection; C. R. Acad. Sci. Bulgar, 31, 811-814, 1978.
[13] S. K. Chaubey, On semi-symmetric non-metric connection; Prog. of Math., Vol. 41-42, 11-20, 2007.
[14] S. K. Chaubey, Pankaj, and S. K. Yadav, Projective curvature tensor of Riemannian manifolds admitting a projective semi-symmetric connection; Universal Journal of Mathematics and Applications, 3(2), 78-85, 2020.
[15] S. Golab, On semi-symmetric and quarter-symmetric linear connections; Tensor, N. S., 29, 249-254, 1975.

[16] R. S. Hamilton, The formation of singularities in the Ricci flow, Survey in Diff. geometry, vol. II (Cambridge, M A, 1993), (1995).
[17] R. S. Hamilton, The Ricci flow on surfaces in Mathematics and general relativity, Contemp. Math. Amer.Math. Soc., Providence, RI, 17 (1988).