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"Renewal Theorems for a Sequence of Discrete Fuzzy Random Renewal Variables in Fuzzy Random Renewal Processes"

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Abstract- Fuzzy random variable is a measure function from a probability space to a collection of fuzzy variables. Based on fuzzy random theory, this paper addresses Renewal theorems for a sequence of discrete fuzzy random variables .The relationship between the expected value of the fuzzy random renewal variable and the distribution functions is λ-pessimistic and λ-optimistic values of the interarrival times is discussed. Furthermore, the fuzzy random style of central limit theorem is provided. Finally, some limit theorems obtained in this paper can degenerate to the corresponding classical result in stochastic renewal process.

Index terms: Probability measure, Possibility measure, fuzzy variable, fuzzy random variable, fuzzy renewal process.

1. Introduction:

In a classical renewal process, the various variables such as interarrival times and other variables were assumed to be inexactly and characterized to be fuzzy variables. In this context, randomness and fuzziness are merged with each other. Consider the revised Fuzzy renewal process to deal with a type of uncertain process. Recently, a new definition of fuzzy random variable was described in [6] and defined a measurable function from the probability space to a collection of fuzzy variables and its expected value was defined as a scalar expected value operator. Based on a renewal process in which the interarrival was considered as iid fuzzy random variable. Some limit theorems obtained in this paper can degenerate to the corresponding classical result in stochastic renewal process.

1.1. Fuzzy variables:

Let ξ be a fuzzy variable on possibility space $(\varphi,\wp(\varphi),\mathrm{PoS})$,where φ is a universe, ,$\wp(\varphi)$ is a power set of φ , PoS is a possibility measure on $\wp(\varphi)$.Based on possibility measure (PoS),the necessity (Ne) and credibility (Cr) of the fuzzy event ($\xi > t$) can be expressed by

$$\mathrm{Ne}[\xi > r] = 1-\mathrm{PoS}(\xi < r) \ \ \&\ \mathrm{Cr}[\xi \geq r] = \tfrac{1}{2}[\mathrm{PoS}(\xi \geq r) + \mathrm{Ne}(\xi \geq r)] \qquad \text{---------(1.1.1)}$$

Definition:1.1.1. Let ξ be a fuzzy variable on possibility space $(\varphi,\wp(\varphi),\mathrm{PoS})$,and $\lambda\epsilon(0,1]$. Then

$$\xi'_{\lambda=\inf\{t|PoS(\xi\leq r)\geq\lambda\}} \ \text{and}\ \xi''_{\lambda=\sup\{t|PoS(\xi\geq r)\geq\lambda\}} \qquad \text{-------- (1.1.2)}$$

are called the λ-pessimistic value and the λ-optimistic value of ξ, respectively.

Definition: 1.1.2[6] Let ξ be a fuzzy variable on possibility space $(\varphi,\wp(\varphi),\mathrm{PoS})$.the expected value $E[\xi]$ is defined as

$$E[\xi] = \int_0^{\infty} \mathrm{Cr}[\xi \geq r]dr - \int_{-\infty}^{0} \mathrm{Cr}[\xi \leq r]dr \qquad \text{--------(1.1.3)}$$

1.2. Fuzzy random variables:

Definition: 1.2.1. A fuzzy random variable is a function ξ: $\Omega \rightarrow \mathcal{F}$(where $\mathcal{F}$ is a collection of fuzzy variable defined on possibility space) such that for any Borel set B of $\mathfrak{R}$,$\mathrm{PoS}(\xi(\omega) \in B)$ is a measurable function of ω .

Definition: 1.2.2. A fuzzy random variable ξ is said to be positive if and only if for any $\omega\epsilon\Omega$,

$\mathrm{PoS}(\xi(\omega) \leq 0)) = 0$.

Definition:1.2.3. [4] Let ξ be a fuzzy variable on the probability space $(\Omega,\mathcal{A}),\mathrm{Pr})$.Then its expected value $E[\xi]$ is defined by

$$E[\xi] = \int_{\Omega}[\int_0^{\infty} \mathrm{Cr}[\xi \geq r]dr - \int_{-\infty}^{0} \mathrm{Cr}[\xi \leq r]dr]\mathrm{Pr}(d\omega)$$

provided that at least one of two integrals is finite. Especially, if ξ is positive fuzzy random variable, then $E[\xi] = \int_{\Omega}[\int_0^{\infty} \mathrm{Cr}[\xi \geq r]dr]\mathrm{Pr}(d\omega)$.

Definition:1.2.4. [4] A fuzzy random variable $\xi_1, \xi_2, \ldots \xi_n$ is said to be iid if and only if

$\{\mathrm{PoS}(\xi_i(\omega) \in B_1), \mathrm{PoS}(\xi_i(\omega) \in B_2),\ldots, \mathrm{PoS}(\xi_i(\omega) \in B_m)\}$, i=1,2,…,n are iid random vectors for any Borel sets B_1, $B_2,\ldots, B_m$ of $\mathfrak{R}$ and any positive integer m.

2.1. Fuzzy random renewal process:

Let ξ_n denote the interarrival time between the (n-1) th and n th event,n=1.2,…..respectively.

Define $S_0=0$ and $S_n = \xi_1+ \xi_2+\ldots+ \xi_n$, $\forall n \geq 1$ ----------(2.1.1)

If the interarrival times are iid fuzzy random variables on probability space $(\Omega,\mathcal{A}),\mathrm{Pr})$.then the process $\{S_n, n \geq 1\}$ is called fuzzy random renewal process.

Let N(t) denote the total number of events that occurred by time t. Then, we have

$$N(t) = \max_{n\geq 0}\{n|0 < Sn \leq t\} \qquad \text{-------------(2.1.2)}$$

Furthermore, for each $\omega\epsilon\Omega$,N(t)(ω) is a fuzzy variable and its membership function is

$$\mu_{N(t)(\omega)}(n) = \mathrm{PoS}\{S_n(\omega) \leq t \leq S_{n+1}(\omega)\},\ n=0,1,2,\ldots \qquad \text{--------- -(2.1.3)}$$

We call N(t) the fuzzy random renewal variable.

For each $\omega\epsilon\Omega$, $\xi_i(\omega)$, $S_n(\omega)$, and $N(t)(\omega)$ are fuzzy variables and their λ-pessimistic and the λ-optimistic values can be expressed by

$\xi'_{i,\lambda}(\omega) = \inf\{r|PoS[(\xi i(\omega)] \leq r \geq \lambda\}$ ------------(2.1.4)

$\xi''_{i,\lambda}(\omega) = sup\{r|PoS[(\xi i(\omega)] \geq r \geq \lambda\}$ ------------(2.1.5)

$S'_{n,\lambda}(\omega) = \inf\{r|PoS[(Sn(\omega)] \leq r \geq \lambda\}$ ------------(2.1.6)

$S''_{n,\lambda}(\omega) = sup\{r|PoS[(Sn(\omega)] \geq r \geq \lambda\}$ ------------(2.1.7)

$N(t)'_{\lambda}(\omega)= \inf\{r|PoS[(N(t)(\omega)] \leq n \geq \lambda\}$ ------------(2.1.8)

$N(t)''_{\lambda}(\omega)= sup\{r|PoS[(N(t)(\omega)] \geq n \geq \lambda\}$, where $\lambda \epsilon (0,1]$ ------------(2.1.9).

It follows [14]from the proposition1 that

$S'_{n,\lambda}(\omega) = \sum_{i=1}^{n} \xi'_{i,\lambda}(\omega)$ -----------(2.1.10)

$S''_{n,\lambda}(\omega) = \sum_{i=1}^{n} \xi''_{i,\lambda}(\omega)$ ----------- (2.1.11)

Moreover,by [14],we have

$N(t)'_{\lambda}(\omega)=$ sup{ n | $S''_{n,\lambda}(\omega) \leq t\}$ ----------- (2.1.12)

$N(t)''_{\lambda}(\omega)=$ sup{ n | $S'_{n,\lambda}(\omega) \leq t\}$ ----------- (2.1.13)

Note : If $F'_{\lambda}(t)$, $F''_{\lambda}(t)$ are the common distribution function of the λ-pessimistic value of $\xi'_{i,\lambda}(\omega)$,i=1,2,…and λ-optimistic value of $\xi''_{i,\lambda}(\omega)$,i=1,2,…..,respectively.

Then , the following given theorems [12]can be expressed as a degenerates to a sequence of iid non-negative fuzzy random interarrival times and fuzzy random renewal variable.

2.2.Some limit theorems:

A1. The operations of fuzzy variables are determined by the generalized extension principle (4), and denotes any continuous Archimedean *t*-norm with an additive generator *f* .

A2. *Π* is a nonnegative real-valued function with *Π(0)* = 1, and *Π* is non-increasing on R+, non-decreasing on R−.

In condition A1, the generalized extension principle provides any continuous Archimedean *t*-norm operator for fuzzy variables. As to condition A2, the function *Π* is called a possibility function which is used to represent the possibility distribution of a fuzzy variable. Through possibility function *Π* , we can construct convex possibility distributions such as triangular and norm distributions of fuzzy variables, where such convexity is critical to our desired results.

Suppose that { ξ_n} is a sequence of fuzzy variables with the same possibility distribution *Π* , from [14, Lemma 5] we know the possibility distribution $\mu_{\xi_1+\cdots+\xi_n}$ is also non-increasing on R+, and non-decreasing on R−.

Denote *Ξ* as the support of possibility function Π, i.e., the closure of subset $\{t \in R \mid \Pi(t) > 0\}$ of R. For possibility distribution *Π* and Archimedean *t*-norm T with additive generater*f* , since $f : [0, 1] \to [0, \infty]$ is continuous and strictly decreasing, we know $f \circ \Pi : R \to [0, \infty]$ is nonincreasing on R−, nondecreasing on R+ with $f \circ \Pi(0) = 0$, and $f \circ \Pi(x) = f(0)$ for any $x \notin \Xi$

Now, we consider the convex hull of the composition function $f \circ \Pi$ on *Ξ* , denoted co$(f \circ \Pi)$, which is defined as

$$co(f \circ \Pi)(z) = \inf \sum_{k=1}^{n} \lambda_k (f \circ \Pi)(x_k) \qquad (z \in \Xi) \ldots\ldots \qquad (2.2.1)$$

where the infimum is taken over all representations of *z* as a (finite) convex combination $\sum_{k=1}^{n} \lambda_k x_k$ of points of

Ξ . From the knowledge of convex analysis (see Tiel [8]), we know co($f \circ \Pi$) is the largest convex function $h(x)$ such that $h(x) \le f \circ \Pi(x)$, for $x \in \Xi$.

Lemma2.2.1. Let ξ_k, k = 1, 2, . . .be a sequence of fuzzy variables with identical possibility distribution Π, and $S_n = \xi_1 + \cdots + \xi_n$. If co($f \circ \Pi$)(x) >0 for any nonzero x∈ Ξ, then

$$\lim_{n\to\infty} \mu_{\frac{1}{n}S_n} = \begin{cases} 1, & \text{if } z=0 \\ 0, & \text{otherwise} \end{cases}$$

Proof. The proof can be divided into the following three cases:

ase 1. $z = 0$. We have

$$\mu_{\frac{1}{n}S_n}(0) = \sup_{x_1+\cdots+x_n=0} T(\mu_{\xi_1}(x_1), \ldots, \mu_{\xi_n}(x_n)) \ge T(\Pi(0), \ldots, \Pi(0)) = 1.$$

Case 2. $z \ne 0$, $z \notin \Xi$. For any $\{x_1, x_2, \ldots, x_n\}$ with $x_1 + \cdots + x_n = nz$, there must be two points x_i and x_j, $1 \le i, j \le n$ such that $x_i \le z$ and $x_j \ge z$, which implies $T(\Pi(x_i), \Pi(x_j)) \le \Pi(z)$.

Therefore $T(\Pi(x_1), \ldots, \Pi(x_n)) \le \Pi(z)$. It follows

$$\mu_{\frac{1}{n}S_n}(z) = \sup_{x_1+\cdots+x_n=nz} T(\mu_{\xi_1}(x_1), \ldots, \mu_{\xi_n}(x_n)) \le T(\Pi(x_1), \ldots, \Pi(x_n)) \le \Pi(z)=0.$$

Case 3. $z \ne 0$, $z \in \Xi$. In this case, we note that if $x_k \notin \Xi$ for some k, then $f^{[-1]}\left(\sum_{k=1}^{n} f \circ \Pi(x_k)\right) = 0$. Therefore, by (2.1.4) and (2.1.10), we have

$$\mu_{\frac{1}{n}S_n}(z) = f^{[-1]}\left(\inf_{x_1+\cdots+x_n=nz} \sum_{k=1}^{n} f(\mu_{\xi_n}(x_k))\right)$$

$$\inf_{\substack{x_1+\cdots+x_n=nz \\ x_k\in\Xi, 1\le k\le n}} \frac{1}{n}\sum_{k=1}^{n} f\circ\Pi(x_k) \ge \mathrm{co}(f\circ\Pi)(z),$$

or, equivalently,

$$\inf_{\substack{x_1+\cdots+x_n=nz \\ x_k\in\Xi, 1\le k\le n}} \sum_{k=1}^{n} f\circ\Pi(x_k) \ge n \,.\, \mathrm{co}(f\circ\Pi)(z),$$

Since $f^{[-1]}$ is nonincreasing, we can deduce

$$\mu_{\frac{1}{n}S_n}(z) \le f^{[-1]}(n \,.\, \mathrm{co}\,(f\circ\Pi)(z))$$

Nothing that co($f \circ \Pi$)(r) >0 for any nonzero $r \in \Xi$, we have

$$\mu_{\frac{1}{n}S_n} \le f^{[-1]}(n \,.\, \mathrm{co}\,(f\circ\Pi)(z)) \to 0 \ (n\to\infty).$$

THEOREM 2.2.1:

Assume ξ_k is a sequence of i.i.d. fuzzy random variables with $\mu_{\xi_k(\omega)}(x) = \Pi(x - U_k(\omega))$ for almost every $\omega \in \Omega$, where U_k, k=1, 2, . . ., are random variables with finite expected values. If co($f \circ \Pi$) (x) >0 for any nonzero $x \in \Xi$, then we have

$$\sum_{k=1}^{n} \xi_k \xrightarrow{Ch} E[U_1].$$

THEOREM 2.2.2:

Assume $\{\xi_k\}$ is a sequence of i.i.d. fuzzy random variables with $\mu_{\xi_k(\omega)}(x) = \Pi(x - U_k(\omega))$ for almost every $\omega \in \Omega$, where U_k, k= 1, 2, . . ., are random variables with finite expected values. If co($f \circ \Pi$)(x) >0 for any nonzero

n

$x \in \Xi$, and $E[\xi 1] < \infty$, then we have

$$\lim_{n\to\infty} E\left[\frac{1}{n}\sum_{k=1}\xi_k\right] = E[U_1]$$

Lemma 2.2.2: Suppose ξ_k, k =1, 2, . . .are fuzzy random interarrival times with $\mu\ \xi_k(\omega)\ \Pi(x - U_k(\omega))$ for almost every $\omega\ \Omega$, where U_k are random variables, and N(t) is the fuzzy random renewal variable. If $U_k \geq a$ almost surely, then for any real number t, r>0, we have

$$Ch\left\{\frac{N(t)}{t} \geq r\right\} \leq \int_{\Omega} pos\left\{\frac{1}{\xi 1(\omega) - (U1(\omega) - a)} \geq r\right\} \Pr d\omega.$$

The main result which we provide a fuzzy analogue of the **central limit theorem** can stated below:

THEOREM:2.2.3:

Let $\{\xi k\}$ be a sequence of i.i.d.sequence of non-degenerate fuzzy random variables with on **a** probability space (Ω,$\mathcal{A}$),Pr).Suppose that $\xi_1(x)$ has a moment generating function, and let $Q\xi = Q\xi_1$

For each n,let $\overline{\xi n} = \frac{1}{n}\sum_{i=1}^{n}\xi i$, and let $\Phi_{co[Q\xi\backslash n]}$ be the fuzzy Gaussian random variable determined by

$co[Q\xi\backslash n]$.Then for each $\alpha \in (0, 1]$ there exists a constant A_α such that for all n,

$$d_H(\mathcal{L}_\alpha(\xi_n), \mathcal{L}_\alpha(\Phi_{co[Q\xi\backslash n]})) \leq A_\alpha n^{-1/2}.$$

Proof:

Now, consider the sequence of fuzzy random variables $\{\overline{\xi_n}\}$. Independence of the fuzzy random variables implies the independence of fuzzy random variables $\exp(t\xi_i)$. By n-fold convolution,EX.EY=EX*EY

$Ee^{t\overline{\xi_n}} = E\exp\{(\frac{1}{n}t)\sum_{i=1}^{n}\xi_i\} = \prod_{i=1}^{n} E\exp\{(1/n)t\xi_i\} = [E\exp\{(1/n)t\,\xi_1\}]^n$, -----------(2.3.1.)

where the latter expression denotes an n-fold product of fuzzy numbers. Thus for all $\alpha\in$ (0, 1], and all n,

$$\mathcal{L}_\alpha(Ee^{t\overline{\xi_n}}) = \prod_{i=1}^{n}\mathcal{L}_\alpha(E\exp\{(1/n)t\xi_i\})$$
$$= \left\{ z = \prod_{i=1}^{n} z_i : z_i \in E\exp((1/n)t\xi_i), i = 1,2,\ldots,n\right\}$$
$$= \left\{ z = \prod_{i=1}^{n} E\exp((1/n)tU_i) : U_i \in \mathcal{L}_\alpha(\xi_i), i = 1,2,\ldots,n\right\}$$

Suppose that $\{U_1, U_2,\ldots\}$ is one element of the Cartesian product $\{\mathcal{L}_\alpha(\xi_1), \mathcal{L}_\alpha(\xi_2), \ldots\}$

Letting $\mu_i = EU_i$, $\sigma_i = VarU_i$, $\overline{\mu_n} = (1/n)\sum_{i=1}^{n}\mu_i$, and $\overline{\sigma_i^2} = (1/n)\sum_{i=1}^{n}\sigma_i^2$.

$$\prod_{i=1}^{n} Ee^{\frac{t}{n}U_i} = e^{-\overline{\mu_n}t}\prod_{i=1}^{n} E\exp((t/n)(U_i - \mu_i)$$

Also, expanding the m.g.f. in a second-order Taylor series with infinitesimal remainder term,

$$\prod_{i=1}^{n} E\exp((t/\sqrt{n})(U_i - \mu_i) = \prod_{i=1}^{n}[1 + \frac{1}{2}\left(\frac{t^2}{n}\right)\sigma_i^2 + o(n)]$$

Let $\theta_{n_i} = \frac{1}{2}t^2\left(\frac{\sigma_i^2}{n}\right) + o(n)$.Since $\mathcal{L}_\alpha$ (Qξ) is bounded, there is some number M such that $\sigma_i^2 \leq M$ for all i. Also, since $\lim Q_n \in co[\mathcal{L}_\alpha(Q\xi)]$, then $\mu_* = \lim \overline{\mu_n}$ and $\sigma_*^2 = \lim \overline{\sigma_i^2}$ both exist, and $(\mu_*, \sigma_*^2) \in$ co co[$\mathcal{L}_\alpha$ (Qξ)] . Thus by a common lemma of calculus ([2], section 7.1),

$$\lim_{n\to\infty}\prod_{i=1}^{n} E\exp((t/\sqrt{n})(U_i - \mu_i) = e^{-\sigma^2 t^2/2}.$$

Slutsky's Theorem [10], along with the continuity theorem for moment generating functions, then implies that, as $n \to \infty$,$(1/\sqrt{n})\sum_{i=1}^{n}(U_i - \mu_i)/\sigma_*$, converges in distribution to a standard normal variable. Thus for any $\{U_1, U_2, \ldots\} \in \mathcal{L}_\alpha(\xi_1), \mathcal{L}_\alpha(\xi_2), \ldots\}$ the limiting distribution of

$\overline{U_n} = (1/n)\sum_{i=1}^{n}U_i$, suitably normalized, is Gaussian.

Let G_n be the distribution function of $(\overline{U_n} - \overline{\mu_n})/(\overline{\sigma_n}/\sqrt{n})$.

Also let $\Phi(x; \mu, \sigma^2)$ represent the distribution function of a Gaussian random variable with mean μz and variance σ^2. Then the Berry-Esseen Theorem [4] states that for all n,

$\mathrm{Sup}|G_n(x) - \Phi(x; 0, 1)| < 6r_n/\sigma_n^3$.

Thus, with H_n representing the distribution function of $(1/n)\sum_{i=1}^{n}U_i$

$$\sup_{x\in R}|H_n(x) - \Phi(x; \mu_n, \sigma_n^2/n| < 6(r_n/n^2)/(\sigma_n^2/n)^{3/2}.$$

Because the ξ_i are nondegenerate, and $\mathcal{L}_\alpha$ (QX) is compact, $\alpha \in (0, 1]$, there exists a positive value σ_α^2such that $0 < \sigma_\alpha^2 \leq \inf\{Var(U): U\in\mathcal{L}_\alpha(\xi)\}$.

Therefore for any $(\mu,\sigma^2) \in co[\mathcal{L}_\alpha (Q\xi)]$, $\sigma_\alpha^2 \leq \sigma^2$.

Also, as previously stated, $E \mid U|^3 \leq \Gamma_\alpha$ for all $U \in \mathcal{L}_\alpha(\xi)$. Thus, taking $A_\alpha = \sigma\, \Gamma_\alpha / \sigma_\alpha^3$,

$$\sup_{U\in\mathcal{L}_\alpha(\overline{\xi_n})} \inf_{V\in\mathcal{L}_\alpha(\Phi_{co(\mathcal{L}_\alpha (Q\xi\backslash n))})} d_k(U, V) < A_\alpha n^{-1/2} \qquad \text{……….. (2.3.2)}$$

The left-hand side of (2.3.2) is one of the component semi-metrics of $d_H(\mathcal{L}_\alpha(\overline{\xi_n}), \mathcal{L}_\alpha(\Phi_{co(\mathcal{L}_\alpha (Q\xi\backslash n))})$, . Essentially (2.3.2) states that the limiting form of $\mathcal{L}_\alpha(\overline{\xi_n})$ is contained in the set of Gaussian random variables with means and variances belonging to $co[\mathcal{L}_\alpha (Q\xi\backslash n)]$.

Therefore, for each i ,$\mu_i = EU_i$ and $\sigma^2 = Var\, U_i$. In addition, for every n, the Berry-Esseen Theorem holds for the sequence $\{\overline{U_n}\}$. That is, if H_n is the distribution function of $\overline{U_n}$,

then for all n, $\sup_{x\in R} |H_n(x) - \Phi (x;\mu, \sigma^2/n| < A_\alpha n^{-1/2}$.

It follows automatically that $\inf_{F\in\mathcal{L}_\alpha(\overline{\xi_n})} \sup |F(x) - \Phi (x;\mu, \sigma^2/n| < A_\alpha n^{-1/2}$, and

since (μ,σ^2) were arbitrary choices from $co[\mathcal{L}_\alpha(Q\xi)]$, also

This completes the proof of Theorem $\sup_{V\in\mathcal{L}_\alpha(\Phi_{co(\mathcal{L}_\alpha (Q\xi\backslash n))})} \inf_{U\in\mathcal{L}_\alpha(\overline{\xi_n})} d_k(U, V) < A_\alpha n^{-1/2}$.

Remark 1:

The public opinion poll discussed by Kwakernaak [41 is one such application. In these cases it is straightforward to verify, or in principle to disprove, the statements,

$$\sup_{\alpha>0} \max[E\, |U_\alpha^*|^3, E|\, U_\alpha^{**} \mid ^3] < \infty, \qquad \text{…………….. (2.3.3)}$$

and

$$\sup_{\alpha>0} (\text{ess sup } U_\alpha^* - \text{essinf } U_\alpha^{**} > 0 \qquad \text{………………(2.3.4)}$$

Corollary 2.2.1.: Suppose that, in addition to the assumptions of Theorem 3.1, also (2.3.3) and (2.3.4) hold. Suppose that $(\mu,\sigma^2) \in co[Q\xi]$, and let $T_n = (\overline{\xi_n} - \mu)/(\sigma/\sqrt{n})$.. Also let $Z = (\overline{\xi_n} - \mu)/\sigma$. Then, T_n converges in distribution to the fuzzy Gaussian random variable $\Phi_{co\,[QZ]}$, in the sense that $\delta_H(T_n, \Phi_{co\,[QZ]}) \to 0$ as $n\to\infty$

Moreover, there is a constant a such that for all n, $\delta_H(T_n, \Phi_{co\,[QZ]}) < An^{-1/2}$.

Remark 2: Corollary 3.1.1 is a statement for fuzzy random variables which closely resembles the conventional central limit theorem for real random variables.

3.CONCLUSION:

In this paper , the randomness and fuzziness are merged with each other. Consider the revised Fuzzy renewal process to deal with a type of uncertain process. Recently, a new definition of fuzzy random variable was described and defined a measurable function from the possibility space to a collection of fuzzy variables and its expected value was defined as a scalar expected value operator. Based on a renewal process in which the interarrival was considered as iid fuzzy random variable. From Lemma 2.2.1, Theorems2.2 1 and 2.2.2, we see that a critical convexity condition is that co*(f oΠ)(x)* >0 for any nonzero *x. Ξ* , which is also indispensable to the main results of the next section. This convexity condition is determined completely by the composition of the possibility function *Π* and the additive function of the chosen *t*-norm. The main result which we provide a fuzzy analogue of the **central limit theorem** can stated and proved using **distribution to the fuzzy Gaussian random variable.**

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