IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Bounce Behavior of LRS Bianchi Type- I Cosmological Model in General Relativity

Yogendra D. Patil

Assistant Professor,
Department of Mathematics,
Vidnyan Mahavidyalaya Malkapur 443101, Dist-Buldana, Maharshstra India

Abstract: *Bianchi Type-I cosmological model has obtained in the general theory of relativity. The source for energy momentum tensor is assumed a perfect fluid. The field equations have solved by using special form of the average scale factor* $R(t)=\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}}$ *proposed by Cai et al.[10]. The physical properties and the bouncing behavior of the model are also discussed.*

Index Terms - Bianchi Type- I space-time, bouncing universe, relativity, scale factor.

I. INTRODUCTION

Astronomical observational data obtained from high red shift surveys of Supernovae (S_nIa) by Riess et al. [1], Perlmutter et al. [2] and Bennett et al. [3] indicated that our universe is expanding with acceleration. Also, observations such as Cosmic Microwave Background Radiations [4] and Large-scale structure [5] provide indirect evidence for the late time accelerated expansion of the universe. The accelerating expansion of the universe is driven by a mysterious component with high negative pressure known as dark energy (DE). In spite of all these attempts, DE is still the open question to the theoretical physicists because its nature is unknown. According to the astronomical observations, the DE currently accounts for about 73% of the total mass/energy of the universe and only 27% of a combination of dark matter and baryonic matter [6]. The DE universe may have very interesting implications for the future [7,8]. A different way of accounting for the DE without any extra components is the modification of gravity [9,10].

The idea that instead of originating from a Big Bang singularity, the universe has emerged from a cosmological bounce has a long history [11]. Novello et al. [12,13] realized that a bouncing cosmology with a matter-dominated phase of contraction during which scales which are probed today, a cosmological observation exit a Hubble radius can provide an alternative to the current inflationary universe paradigm of cosmological structure formation. According to Cai et al. [10], the solution of the singularity problem of the standard Big Bang cosmology is known as bouncing universe. A bouncing universe with an initial contraction to a non-vanishing minimal radius and then subsequent an expanding phase provides a possible solution to the singularity problem of the standard Big Bang cosmology. Moreover, for the universe entering into the hot Big Bang era after the bouncing, the equation of state (EoS) of the matter content ω in the universe must transit from ω <- 1 to ω >-1 . In the contracting phase, the scale factor R(t) is decreasing, this means $\dot{R}(t) < 0$ and in the expanding universe, scale factor$\dot{R}(t) > 0$. Finally at the bouncing point, $\dot{R}(t) = 0$ and near this point $\ddot{R}(t) > 0$, for a period of time. It is also discussed with other view that in the bouncing cosmology, the Hubble parameter H passes across zero (H =0) from H < 0 to H > 0 . Cai et al. have investigated bouncing universe with quintom matter. He showed that a bouncing universe has an initial narrow state by a minimal radius and then develops to an expanding phase. This means for the universe arriving to the BigBang era after the bouncing, the EoS parameter should crossing from ω <-1 to ω >-1 . Sadatian [14] have studied rip singularity scenario and bouncing universe in a Chaplygin gas dark energy model. Recently, Bamba et al. [15] have investigated bounce cosmology from $f(R)$gravity and $f(R)$ bi-gravity. Astashenok [16] has studied effective energy models and dark energy models with bounce in frames of $f(T)$ gravity. Solomans et al. [17] have investigated bounce behavior in Kantowski-Sach and Bianchi cosmology. Silva et al. [18] have studied bouncing solutions in Rastall's theory with a barotropic fluid. Brevik and Timoshkin [19] have obtained inhomogeneous dark fluid matter leading to a bounce cosmology. Singh et al. [20] have studied k-essence cosmologies in Kantowski-Sach cosmological Sachs and Bianchi space-times.

In this paper, Bouncing behavior of Bianchi Type-I cosmological model has been obtained in the general theory of relativity. This work is organized as follows: In section 2, the metric and field equations have presented. The field equations have solved in section 3 by using the physical condition that the expansion scalar θ is proportional to shear scalar σ and the special form of average scale factor

$R(t)=\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}}$ proposed by Cai et al. [10]. The physical and geometrical behavior of the model have been discussed in section 4. In the last section 5, concluding remarks have been expressed.

II. METRIC AND FIELD EQUATIONS

LRS Bianchi Type-I metric is considered in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2+dz^2) \tag{1}$$

where $A(t)$ and $B(t)$ are scale factors and are functions of cosmic time t.
The energy-momentum tensor for a perfect fluid is

$$T_i^j = (\rho+p)u_iu^j - pg_i^j, \tag{2}$$

where p is the pressure, ρ is the energy density and g_j^i is a metric tensor. In co-moving coordinate system, u^i are the four co-moving velocity vectors which satisfy the condition

$u^iu_i = 0$, for $i=1,2,3$

and $u^iu_i = 1$, for $i=0$.

From equation(2), the components of energy-momentum tensor are

$$T_0^0=\rho\ , \qquad T_1^1=T_2^2=T_3^3=-p\ . \tag{3}$$

With the help of equation(3), the matter tensor is given by

$$T_j^i = diag.(\rho,-p,-p,-p). \tag{4}$$

For the perfect fluid, p and ρ are related by and equation of state

$$p=\omega\rho,\quad 0\le\omega\le 1. \tag{5}$$

The Einstein's field equations are given by

$$R_i^j-\frac{1}{2}g_i^jR=-T_i^j, \tag{6}$$

where R_i^j is a Ricci tensor, R is the Ricci scalar.
The Ricci scalar for the Bianchi Type-I metric is given by

$$R=2\left(\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+2\frac{\dot{A}\dot{B}}{AB}\right).$$

With the help of equations (4) and(5), the field equations(6), for the metric (1) are

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}=\rho, \tag{7}$$

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}=-\omega\rho\ , \tag{8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-\omega\rho\ . \tag{9}$$

Here the over dot $\left(\bullet\right)$ represents the differentiation with respect to t.

III. SOLUTION AND FIELD EQUATION

The field equations (7) to (9) are a system of three highly non-linear differential equations in four unknowns A, B, ρ and ω. The system is thus initially undetermined. We need one extra physical condition to solve the field equations completely.
We assume that the expansion scalar (θ) is proportional to the shear scalar (σ). This condition leads to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=\alpha_0\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right),$$

which yields

$$\frac{\dot{A}}{A}=m\frac{\dot{B}}{B},$$

where α_0 and m are constants.

Above equation, after integration reduces to

$$A = \eta (B)^m ,$$

where η is an integration constant.

Here for simplicity and without loss of generality, we assume that $\eta = 1$.

Hence, we have

$$A = (B)^m \ , \ (m \neq 1) . \qquad (10)$$

Collins *et al.* have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant [21].

In cosmology, the constant deceleration parameter is commonly used by several researchers [22-26], as it duly gives a power law for metric function or corresponding quantity.

The motivation to choose time-dependent deceleration parameter (DP) is the fact that the expansion of the universe was decelerating in the past and accelerating at present as observed by recent observations of Type Ia Supernova [1,2,27-29] and CMB anisotropies [3,31]. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a Universe which was decelerating in the past and accelerating at present, the DP must show signature flipping [31,33]. So, in general, the DP is not a constant but time variable. The motivation to choose the following scale factor is that it provides a time-dependent DP.

Under above motivations, we use a special form of deceleration parameter as

$$q = -\frac{R\ddot{R}}{R^2} = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{1}{2}\left[(1-\beta) - \frac{t_0}{(t-t_0)^2}\right], \beta < 1 \qquad (11)$$

where R is average scale factor of the Universe.

This form is proposed by Cai *et al.* [10] and then modified by Sadatian [14].

Integrating twice equation(11), the average scale factor which is time dependent is given by

$$R(t) = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}} , \qquad (12)$$

where t_0 is initial time and $\beta < 1$ is constant.

For the metric(1), the scale factor R is given by,

$$R(t) = (AB^2)^{\frac{1}{3}} . \qquad (13)$$

From the equations (12) and(13), we have

$$R(t) = (AB^2)^{\frac{1}{3}} = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}} .$$

$$\Rightarrow \qquad AB^2 = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{3}{1-\beta}} .$$

Using equation(10), it reduces to

$$B^m B^2 = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{3}{1-\beta}} .$$

$$\Rightarrow \qquad B = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{3}{(1-\beta)(m+2)}} . \qquad (14)$$

Using equation (14) , equation (10) leads to

$$A = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{3m}{(1-\beta)(m+2)}} . \qquad (15)$$

With the help of equations (14) and(15), the metric (1) becomes

$$ds^2 = dt^2 - \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{6m}{(1-\beta)(m+2)}} dx^2 - \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{6}{(1-\beta)(m+2)}} \left(dy^2 + dz^2\right) \qquad (16)$$

The equation (16) represents the Bianchi Type-I cosmological model in general relativity.

IV. PHYSICAL PROPERTIES OF THE MODEL

For the cosmological model (16), the physical quantities such as spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy A_m, shear scalar σ^2, energy density ρ, the equation of state parameter ω are obtained as follows:

The spatial volume is in the form

$$V = R^3 = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{3}{1-\beta}} . \qquad (17)$$

The Hubble parameter is given by

$$H = \frac{1}{3}\left[H_x + 2H_y\right] = \frac{1}{3}\left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right] = \frac{2(t-t_0)}{(1-\beta)}\left[(t-t_0)^2 + \frac{t_0}{(1-\beta)}\right]^{-1} . \qquad (18)$$

From fig. 1(b), the Hubble parameter $H<0$, for $t<1$ and $H>0$, for $t>1$ indicating that H passes across zero $(H=0)$ at $t=1$, which represents that the universe is bouncing at $t=1$.

The expansion scalar is

$$\theta = 3H = \frac{6(t-t_0)}{(1-\beta)}\left((t-t_0)^2 + \frac{t_0}{1-\beta}\right)^{-1} . \qquad (19)$$

The mean anisotropy parameter is

$$Am = 2\frac{(m-1)^2}{(m+2)^2} = const. \neq 0 \ , \quad \text{for } m \neq 1 . \qquad (20)$$

The shear scalar is

$$\sigma^2 = 12\frac{(m-1)^2 (t-t_0)^2}{(m+2)^2 (1-\beta)^2}\left[(t-t_0)^2 + \frac{t_0}{(1-\beta)}\right]^{-2} . \qquad (21)$$

It is observed that

$$\lim_{t\to\infty}\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} \neq 0 \ , \quad \text{for } m \neq 1 . \qquad (22)$$

The mean anisotropy parameter A_m is constant and $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2} \neq 0$ is also constant. Hence the model is anisotropic throughout the evolution of the universe except at $m=1$. i.e., the model does not approach isotropy.

The matter-energy density is given by

$$\rho = \frac{36(2m+1)(t-t_0)^2}{(1-\beta)^2 (m+2)^2}\left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{-2} \qquad (23)$$

From Fig. 1(d) , the energy density decreases at the early stage of evolution when $t<1$ and goes into the hot Big Bang era. The model bounces at $t=1$ and after bouncing the energy density rapidly increases for $t>1$.

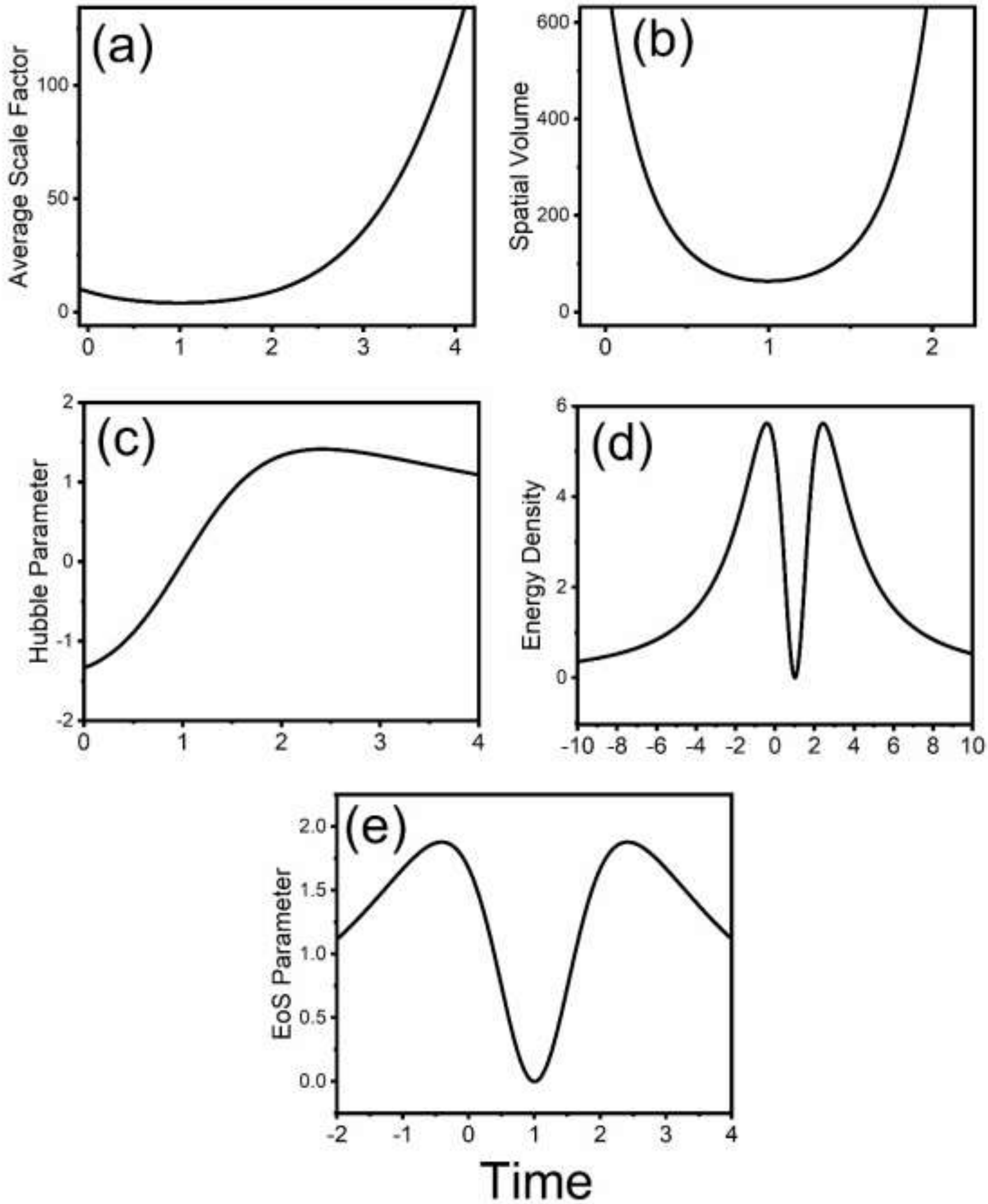
The equation of state parameter (EoS) is given by

$$\omega = -\left(\frac{\frac{108(t-t_0)^2}{(1-\beta)^2 (m+2)^2}\left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{-2} - \frac{24(t-t_0)^2}{(1-\beta)(m+2)}\left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{-2} + \frac{12}{(1-\beta)(m+2)}\left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{-1}}{\frac{36(2m+1)(t-t_0)^2}{(1-\beta)^2 (m+2)^2}\left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{-2}}\right) . \qquad (24)$$

It is seen that from fig. 1(e), before bouncing (at point $t=1$), the EoS parameter $\omega<-1$ and after the bounce, $\omega>-1$ for $t>1$. The equation of state parameter of the universe crosses from $\omega<-1$ to $\omega>-1$ .Hence, our model is bouncing at $t=1$. Thus, it is observed that, a bouncing universe model has an initial narrow state by a non-zero minimal radius and then develops to an expanding phase. After the bounce, the universe enters into the hot Big-Bang era.

To study the physical properties of Bianchi Type-I cosmological model (16), plots of time versus (a) average scale factor (b) spatial volume (c) Hubble parameter (d) energy density (e) EoS parameter for the values $\beta=0.5$, $t_0=1$, $m=2$ are shown in Fig. 1.

From Fig. 1(a), in the earlier stage, the average scale factor (R) is strictly decreasing $(\dot{R}(t)<0)$ and in the expanding phase, it



increases rapidly $(\dot{R}(t)>0)$. Hence our model is bouncing at some finite time $t=1$ $(\dot{R}(t)=0)$.

Fig-1 Plots of time versus - (a) Average scale factor (b) Spatial volume (c) Hubble Parameter (d) Energy density (e) EoS Parameter for the values $\beta=0.5$, $t_0=1$, $m=2$.

V. CONCLUSIONS

Bianchi Type-I cosmological model has been investigated in the general theory of relativity. The source for energy-momentum tensor is a perfect fluid. The field equations have been solved by using time dependent deceleration parameter. The mean anisotropy parameter A_m is constant and $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}(\neq 0)$ is constant, hence the model is anisotropic throughout the evolution of the universe except at $m = 1$. It is interesting to note that a bouncing universe model has an initial narrow state by non-zero minimal radius and then develop to expanding phase. After the bounce, the universe enters into the Hot Big-Bang era. The model has a bounce at some finite time $t = t_0$. In particular, for the values $\beta = 0.5$, $t_0 = 1$, $m = 2$, the model is bouncing at finite time $t_0 = 1$.

REFERENCES

[1] Riess, A.G.; Filippenko, A. V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliland, R.L.; Hogan, C.J.; Jha, S.; Kirshner, R.P.; et al. "Observational evidence from supernovae for an accelerating universe and a cosmological constant", Astronom. J., 116, 1009–1038, 1998.

[2] Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R.A.; Nugent, P.; Castro, P.G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D.E. et al. "Measurement of Ω and Λ from 42 high-redshift supernovae", Astrophys. J., 517, No.2, 565–586, 1999.

[3] Bennett, C. L., Halpern, M., Hinshaw, G., et al., "First year Wilkinson microwave anisotropy probe (WMAP) Observations: Preliminary maps and basic results", Astrophys. J. Suppl., 148, 1, 2003.

[4] Spergel, D.M. et al., Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology, Astro. Phys. J. Suppl. Ser. 170, 377, 2007.

[5] Tegmark, M. et al.; Cosmological Parameters from SDSS and NMAP, Phys. Rev. D 69, 103501, 2004.

[6] Kowalski, M.; Rubin, D.; Aldering, G.; Agostinho, R.J.; Amadon, A.; Amanullah, R.; Balland, C.; Barbary, K.; Blanc, G.; Challis, P.J.; et al., "Improved cosmological constants from new, old, and combined supernova data sets", Astrophys. J., 686, 749–778, 2008.

[7] Nojiri, S.; Odintsov, S.D.; Tsujikawa, S., "Properties of singularities in the (phantom) dark energy universe", Phys. Rev. D, 71, 063004, 2005. Universe 2015.

[8] Bamba, K.; Capozziello, S.; Nojiri, S.; Odintsov, S. D. "Dark energy cosmology: The equivalent description via different theoretical models and cosmography tests", Astrophys. Space Sci., 342, 155–228, 2012.

[9] Nojiri, S.; Odintsov, S.D., "Unified cosmic history in modified gravity: From f(R) theory to Lorentz noninvariant models.", Phys. Rep., 505, 59–144, 2011.

[10] Cai, Y. F., Qiu, T., Piao, Y. S., Li, M., Zhang, X. "Bouncing Universe with Quintom Matter", J. High Energy Phys. 0710, 071, 2007.

[11] Novello, M., Bergliaffa, S. E. P., "Bouncing cosmologies", Phys. Rept. 463, 127, 2008.

[12] Wands, D., "Duality invariance of cosmological perturbations spectra", Phys. Rev. D 60, 023507, 1999.

[13] Finelli, F., Brandenberger, R., "On the generations of a scale-invariant spectrum of adiabetic fluctuations in cosmological models with a contracting phase", Phys. Rev. D 65, 103522 (2002).

[14] Sadatian S. D., "Rip singularity scenario and bouncing universe in a Chaplygin gas dark energy model", Int. J. Theo. Phys., 53, 675-684, 2014. DOI 10.1007/s 10773-013-1855-1.

[15] Bamba K., Makarenko A. N., " Bounce cosmology from f(R) gravity and f(R) bi-gravity", arXiv 1309.3748v2 [hep-th] 2013.

[16] Astashenok A. V., "Effective dark energy models and dark energy models with bounce in frames of F(T) gravity", Astrophys. Spa. Sci. 351, 377-383, 2014. DOI: 10.1007/s 10509-014-1846-6.

[17] Solomans D., Dunsby P. K. S., Ellis G. F. R., "Bounce behavior in Kantowski-Sachs and Bianchi cosmologies", Classical and Quan. Grav., 23, (23); arXiv: [gr-qc] 0103087v2, 2006:DOI 10.1088/0264- 9381/23/23/001.

[18] Silva G. F., Piattella O. F., Fabris J. C., Casarini L., Barbosa T. O., "Bouncing solutions in Rastall's theory with a barotropic fluid", Grav. Cosm., 19, 156- 162, 2013. arXiv. 1212.6954v3 [gr-qc] 2014. 10.1142/S02773231400783.

[19] Brevik I., Obukhov V. V., Timoshkin A. V., "Bounce universe induced by an inhomogeneous dark fluid coupled with dark matter", Mod. Phys. Lett. A 29, Issue 15, 1450078, 2014. DOI: (2015) Universe 1, 24-37, 2014; arXiv 1404.11887v1 [gr-qc].

[20] Singh T., Chaubuy R., Singh A., "k-essence cosmologies in Kantowski-Sachs and Bianchi space-times", Canadian J. of Phys., 93, (11) 1319- 1323, 2015. DOI:10.1139/cjp-2015-0001.

[21] Collins C. B., Glass E. N., Wilkinson D. A., "Exact spatially homogeneous cosmologies", Gen. Relat. Grav., Vol. 12, No. 10, pp. 805-823, 1980.

[22] Akarsu, O., Kilinc, C. B., " LRS Bianchi type-I models with anisotropic dark energy and constant deceleration parameter", Gen. Relat. Gravit., 42, 119, 2010a.

[23] Akarsu, O., Kilinc, C. B., "Bianchi type-III models with anisotropic dark energy", General Relativity and Gravitation, Vol. 42, No.4, pp. 763-775, 2010b. doi:10.1007/s10714-009-0878-7.

[24] Amirhashchi, H., Pradhan, A., Saha, B., "Variable equation of state for Bianchi type-VI0 dark energy models", Astrophys. Space Sci., 333, 295-303, 2011.

[25] Saha, B., Amirhashchi, H., Pradhan, A., "Two-fluid scenario for dark energy models in an FRW universerevisited", Astrophys. Space Sci., Vol. 342, pp. 257- 267, 2012.

[26] Kumar, S., Singh, C.P., " Anisotropic dark energy models with constant deceleration parameter", Gen. Rel. Grav., 43, 1427, 2011.

[27] Riess, A. G., Sirolger, L. G., Tonry, J., et al., "Type Ia Supernova discoveries at z >1 from the Hubble Space Telescope: Evidence for the past deceleration and constraints on dark energy evolution," The Astrophysical Journal, Vol. 607, No. 2, 2004, pp. 665-678. doi:10.1086/383612.

[28] Tonry, J. L., Schmidt, B. P., Barris, B., et al. (Supernova Search Team Collaboration), "Cosmological results from high-z supernovae", Astrophys. J., 594, No. 1, 1-24, 2003.

[29] Clocchiatti, A., Schmidt, B. P., Filippenko, A. V., et al., " Hubble space telescope and ground-based observations of type Ia Supernovae at redshift 0.5: cosmological implications(High Z SN Search Collaboration)", Astrophys. J., 642, 1-21, 2006.

[30] Hanany, S., et al., "A measurement of the cosmic microwave background anisotropy on angular scales of 10 arcminutes to 5 degrees", Astrophys. J. 545, L5, 2000.

[31] Amendola, L., "Acceleration at z>1 ?,", Monthly Notices of the Royal Astronomical Society, 342, 221- 226, 2003.

[32] Padmanabhan, T., & Roychowdhury, T., "A theoretician's analysis of the supernova data and the limitations in determining the nature of dark energy", Monthly Notices of the Royal Astronomical Society, 344, 823-834, 2003.

[33] Riess, A. G., Nugent, P. E., Gilliland, R. L. et al. "The Farthest known supernova: Support for an accelerating universe and a glimpse of the epoch of deceleration", The Astrophysical Journal, Vol. 560, No. 1, pp. 49, 2001.