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BIANCHI TYPE-IX BOUNCING COSMOLOGICAL MODEL WITH VISCOUS FLUIDS

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Abstract: The bounce in viscous fluid cosmology with inhomogeneous viscous fluids in Bianchi Type-IX space-time has been investigated by considering different forms of scale factor. The general features of the fluids which realize them and the possibility to have an acceleration after the bounce have been discussed.

***Index Terms* - Bianchi Type-IX Space Time, Scale Factor, Viscous Fluid, Bounce, Energy Conditions.**

I. INTRODUCTION

Oscillating universes are alternatives to standard Big-bang cosmology to avoid the Big-bang singularity [1-3]. Several interesting models are obtained by replacing standard matter by difference source of contents. Among them, the bounce solutions (where a contraction is flowed by an expansion at a finite time) are quite interesting [4-5]. The idea that instead of initial singularity, the universe has emerged from a cosmological bounce furnishes an alternative scenario to the Big bang theory. In the matter bounce scenarios [6-8], the initial contraction of the universe is in matter-dominated stage, after that a universe without initial singularity appears leading to an expanding phase. In the context of bouncing cosmology, inclusion of viscosity broadens the applicability of the considered theory. In the early universe, (inflation period) viscosity has been proposed for a graceful exit.

Cai *et al.* [9] have obtained dark energy cosmological model of bouncing universe with quintom matter. Novell and Bergliaffa [4] have made detail discussions on bouncing cosmologies. Sadatian [10] has studied rip singularity scenario and bouncing universe in a Chaplygin gas dark energy model. Myrzakulov and Sebastiani [11] have investigated bouncing solutions by considering the source of energy as viscous fluid. Singh *et al.* [12] have discussed bounce conditions for FRW models in modified gravity theories. Brevik and Timoshkin [13] have obtained model with inhogeneous dark fluid and dark matter leading to a bounce cosmology. A critical review of classical bouncing cosmologies is stated by Battefeld and Peter [14]. Bouncing cosmology from $f(R)$ gravity and $f(R)$ bi-gravity obtained by Bamba *et al.* [15]. Recently, Singh *et al.* [16] have investigated bouncing cosmologies in Brans Dicke theory.

Motivated by these studies, in this chapter, an attempt is made to study LRS Bianchi Type-V bouncing cosmological model in general theory of gravitation. The universe is filled with a viscous fluid as a source of energy. The field equations are solved by taking the expansion scalar (θ) proportional to shear scalar (σ) which gives $A = B^m$, $(m \neq 1)$ where m is proportionality constant. This chapter is organized as follows : In section [2], the model and field equations have been presented. The field equations have been solved in section [3]. In section [4], the cosmological model has been obtained by choosing the average scale factor $a(t)$ of the form $a(t)=\sqrt{a_0^2+\beta^2 t^2}$, where a_0, β are non-zero positive constants. Some physical properties of the model are studied in section [5]. Section [6] contains conclusion.

II. METRIC AND FIELD EQUATIONS

Bianchi Type-IX space time is considered in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + (B^2 \sin^2 y + A^2 \cos^2 y) dz^2 - 2A^2 \cos y\, dx dz, \qquad (1)$$

where A and B are functions of cosmic time t.

The energy-momentum tensor for the viscous fluid is given by

$$T_{ij} = \rho u_i u_j + p(u_i u_j - g_{ij}) \,, \tag{2}$$

where u_i are the co-moving four velocity vectors, ρ is the energy density and g_{ij} is the metric tensor.

The fluid in the universe is inhomogeneous viscous fluid with equation of state [17-19].

$$p = \gamma(\rho)\rho - B(a, H, \dot{H}, \ldots\ldots) \,, \tag{3}$$

where the equation of state parameter γ may depend on ρ and bulk viscosity B is a general function of a, H and its derivatives. a is the average scale factor.

From the equations (2) and(3), the energy-momentum tensor for the viscous fluid is

$$T_{ij} = \rho u_i u_j + (\gamma(\rho)\rho + B(\rho, H, \dot{H}, \ldots\ldots))(u_i u_j - g_{ij}) \,. \tag{4}$$

The Einstein's field equations in general theory of gravitation are,

$$R^i_j - \frac{1}{2} R g^i_j = -\frac{8\pi G}{c^4} T^i_j \,.$$

Here, we have assumed that $8\pi G = c = 1$ in proper units.

The Ricci scalar for the Bianchi Type-IX metric is given by

$$R = 2\left(\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{1}{B^2}\right) .$$

The Einstein's field equations take the form

$$R^i_j - \frac{1}{2} R g^i_j = -T^i_j \,. \tag{5}$$

With the help of equation(4), the field equations (5) for the metric (1) are

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{A^2}{4B^4} = \rho \,, \tag{6}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{3A^2}{4B^4} = -p \,, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{A^2}{4B^4} = -p \,, \tag{8}$$

where an overhead dot $\left(\bullet\right)$ represents differentiation with respect to *t*.

The energy-conservation equation, which is the consequence of the field equations (5) is given by,

$$T^{ij}_{;j} = 0 \,,$$

where,

$$T^{ij}_{;j} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^j}\left(T^{ij}\sqrt{-g}\right) + T^{jk}\Gamma^i_{jk}$$

which simplifies to

$$\dot{\rho} + 3H\left(\rho + p\right) = 0 \,. \tag{9}$$

Using equation(3), equation (9) reduces to

$$\dot{\rho} + 3H\left(1 + \gamma(\rho)\right)\rho = 3HB(\rho, a, H, \dot{H} \ldots\ldots) \tag{10}$$

From the thermodynamic point of view, for positive entropy change in an irreversible process, the bulk viscosity must be a positive quantity [20]. The cosmological parameter Ω is defined as:

$$\Omega = 1 + \frac{1}{a^2 H^2} \,. \tag{11}$$

The quantity Ω in general may be different from (1).

By a bouncing universe, we mean a universe that undergoes a collapse, attains a minimum and then subsequently expands. For a successful bounce in Bianchi Type-IX model, during contraction phase $a(t)$ is decreasing i.e. $(\dot{a}(t) < 0)$ and then in the expanding phase, the scale factor is increasing i.e. $(\dot{a}(t) > 0)$. At the bounce point i.e. at $(t = t_b)$, the minimal necessary condition is

i) $\dot{a}(t_b) = 0$ and

ii) $\ddot{a}(t) > 0$ for $t \in (t_b - \epsilon, t_b) \cup (t_b, t_b + \epsilon)$, for small $\epsilon > 0$.

For non-singular bounce $a(t_b) \neq 0$. These conditions may not be sufficient for a non-singular bounce.

The bounce bevaviour of cosmological model is also relized using energy conditions as mentioned in [21]. In terms of (3) and(4), energy conditions can be stated as:

Null Energy Condition (NEC) is satisfied when $\rho + p \geq 0$.

Weak Energy Condition (WEC) is satisfied when $\rho \geq 0$ and $\rho + p \geq 0$.

Domenenet Energy Condition (DEC) is satisfied when $\rho \geq |p|$.

Strong Energy Condition (SEC) is satisfied when $\rho + p \geq 0$ and $\rho + 3p \geq 0$.

It is clear that, violation of NEC will lead to a violation of other energy conditions. (i.e. SEC) realizing the bounce.

III. SOLUTION OF FIELD EQUATION

The field equations (6) to (8) are a system of three highly nonlinear differential equations in four unknowns A, B, ρ and γ. The system is thus initially undetermined. We need one extra condition for solving the field equations completely. We assume that the scalar expansion (θ) is proportional to shear (σ). This condition leads to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \alpha_0 \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right),$$

which yields

$$\frac{\dot{A}}{A} = m\frac{\dot{B}}{B}$$

where α_0 and m are arbitrary constants.

Above equation after integration reduces to

$$A = \eta (B)^m, \tag{12}$$

where η is an integration constant.

Here, for simplicity and without loss generality, we assume that $\eta = 1$.

Hence, we have

$$A = (B)^m, (m \neq 1),$$

Collins *et al.* have pointed out that for spatially homogenous metric, the normal congruence to the homogenous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant [22].

IV. FLUID WITH MODEL $a(t) = \sqrt{a_0^2 + \beta^2 t^2}$:

The bouncing cosmological model has been obtained by choosing the average scale factor *a(t)* of the form (Molina-Parıs & Visser, 1999)

$$a(t) = \sqrt{a_0^2 + \beta^2 t^2}\ , \tag{13}$$

where a_0, β are non-zero positive constants.

The above scale factor is the temporal analogue of the toy model traversable wormhole [23]. One may get phenomenological quintom bouncing model with proper renormalization of a_0 ,β [9].

The Hubble parameter is given by

$$H(t) = \frac{\dot{a}}{a} = \frac{\beta^2 t}{a_0^2 + \beta^2 t^2}\ . \tag{14}$$

In terms of geometrical quantities, we have

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = \frac{(\beta a_0)^2}{(a_0^2 + \beta^2 t^2)^2}\ . \tag{15}$$

For the metric(1), the average scale factor is given by

$$a(t) = (AB^2)^{\frac{1}{3}}\ , \tag{16}$$

From the equations (13) and(16), we have

$$a(t) = (AB^2)^{\frac{1}{3}} = \sqrt{a_0^2 + \beta^2 t^2}$$

$$\Rightarrow \qquad AB^2 = \left(a_0{}^2 + \beta^2 t^2\right)^{\frac{3}{2}}. \tag{17}$$

Using equation(12), it reduces to

$$B^m B^2 = \left(a_0{}^2 + \beta^2 t^2\right)^{\frac{3}{2}}.$$

$$\Rightarrow \qquad B = \left(a_0{}^2 + \beta^2 t^2\right)^{\frac{3}{2(m+2)}}. \tag{18}$$

Using equation(18), equation (1) leads to

$$A = \left(a_0{}^2 + \beta^2 t^2\right)^{\frac{3m}{2(m+2)}}. \tag{19}$$

With the help of equations (18) and(19), the metric (1) becomes

$$ds^2 = -dt^2 + (a_0{}^2 + \beta^2 t^2)^{\frac{3m}{m+2}} dx^2 + (a_0{}^2 + \beta^2 t^2)^{\frac{3}{m+2}} dy^2$$

$$+\left[(a_0{}^2 + \beta^2 t^2)^{\frac{3}{m+2}} \sin^2 y + (a_0{}^2 + \beta^2 t^2)^{\frac{3m}{m+2}} \cos^2 y\right] dz^2. \tag{20}$$

$$-2(a_0{}^2 + \beta^2 t^2)^{\frac{3m}{m+2}} \cos y\, dx\, dz$$

Equation (20) represents Bianchi Type-IX bouncing cosmological model with the viscous fluid in general theory of relativity.

V. SOME PHYSICAL PROPERTIES OF THE MODEL

For the cosmological model (20), the physical quantities such as spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy A_m, shear scalar σ^2, energy density ρ are obtained as follows:

The spatial volume is in the form

$$V = a^3 = (a_0{}^2 + \beta^2 t^2)^{\frac{3}{2}}. \tag{21}$$

The Hubble parameter is

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{\beta^2 t}{a_0{}^2 + \beta^2 t^2}. \tag{22}$$

The expansion scalar is

$$\theta = 3H = \frac{3\beta^2 t}{a_0{}^2 + \beta^2 t^2}. \tag{23}$$

The mean anisotropy parameter A_m is

$$A_m = \frac{2(m-1)^2}{(m+2)^2} = const. \neq 0, \quad \text{for} \quad m \neq 1. \tag{24}$$

The shear scalar is

$$\sigma^2 = \frac{3(m-1)^2 \beta^4 t^2}{(m+2)^2 (a_0{}^2 + \beta^2 t^2)^2}. \tag{25}$$

It is observed that

$$\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = \frac{1}{3}\frac{(m-1)^2}{(m+2)^2} \neq 0, \quad \text{for} \quad m \neq 1. \tag{26}$$

The mean anisotropy parameter A_m and $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$ is constant. Hence the model is anisotropic throughout the evolution of the universe, except at $m = 1$ (i.e. the model does not approach isotropy).

The matter-energy density is given by

$$\rho = \frac{9(2m+1)\beta^4 t^2}{(m+2)^2 (a_0{}^2 + \beta^2 t^2)^2} + \frac{1}{(a_0{}^2 + \beta^2 t^2)^{\frac{3}{m+2}}} - \frac{1}{4}(a_0{}^2 + \beta^2 t^2)^{\frac{3(m-2)}{m+2}}, \tag{27}$$

$$\rho + p = \frac{6(5m+1)\beta^4 t^2}{(m+2)^2 (a_0{}^2 + \beta^2 t^2)^2} - \frac{6\beta^2}{(m+2)(a_0{}^2 + \beta^2 t^2)} + \frac{1}{2}(a_0{}^2 + \beta^2 t^2)^{\frac{3(m-2)}{m+2}}, \tag{28}$$

$$\rho - p = \frac{6\beta^4 t^2}{(m+2)\left(a_0{}^2 + \beta^2 t^2\right)^2} - \frac{6\beta^2}{(m+2)\left(a_0{}^2 + \beta^2 t^2\right)} + \frac{2}{(a_0{}^2 + \beta^2 t^2)^{\frac{3}{m+2}}} \tag{29}$$

$$-(a_0{}^2 + \beta^2 t^2)^{\frac{3(m-2)}{m+2}}$$

$$\rho + 3p = \frac{54 m\beta^4 t^2}{(m+2)^2 (a_0{}^2 + \beta^2 t^2)^2} - \frac{18\beta^2}{(m+2)(a_0{}^2 + \beta^2 t^2)} - \frac{2}{(a_0{}^2 + \beta^2 t^2)^{\frac{3}{m+2}}}$$

$$+2(a_0{}^2 + \beta^2 t^2)^{\frac{3(m-2)}{m+2}} \tag{30}$$

The cosmological parameter for the closed universe takes the form,

$$\Omega = 1 + \frac{1}{a^2 H^2},$$

$$\Omega = 1 + \frac{1}{\beta^2} + \frac{a_0{}^2}{\beta^4 t^2}. \tag{31}$$

The bulk viscosity is

$$B(a, H, \dot{H} ......) = 3H\zeta. \tag{32}$$

In this specific case, equation(3), takes the form

$$p = -\rho + 3H\zeta \tag{33}$$

From equation(28), we have

$$3H\zeta = \rho + p$$

$$= \frac{6(5m+1)H^2}{(m+2)^2} - \frac{6\beta^2}{(m+2)a^2} + \frac{1}{2}(a_0{}^2 + \beta^2 t^2)^{\frac{3(m-2)}{(m+2)}}.$$

Dividing by $3H$

$$\zeta(H, a) = \frac{2(5m+1)H}{(m+2)^2} - \frac{2\beta^2}{(m+2)Ha^2} + \frac{(a_0{}^2 + \beta^2 t^2)^{\frac{3(m-2)}{(m+2)}}}{6H}. \tag{34}$$

Fig. 1 represents the plots of time versus (a) Average scale factor (b) Hubble parameter (c) Energy density (ρ) (d) $\rho + p$ (e) $\rho - p$ (f) $\rho + 3p$.

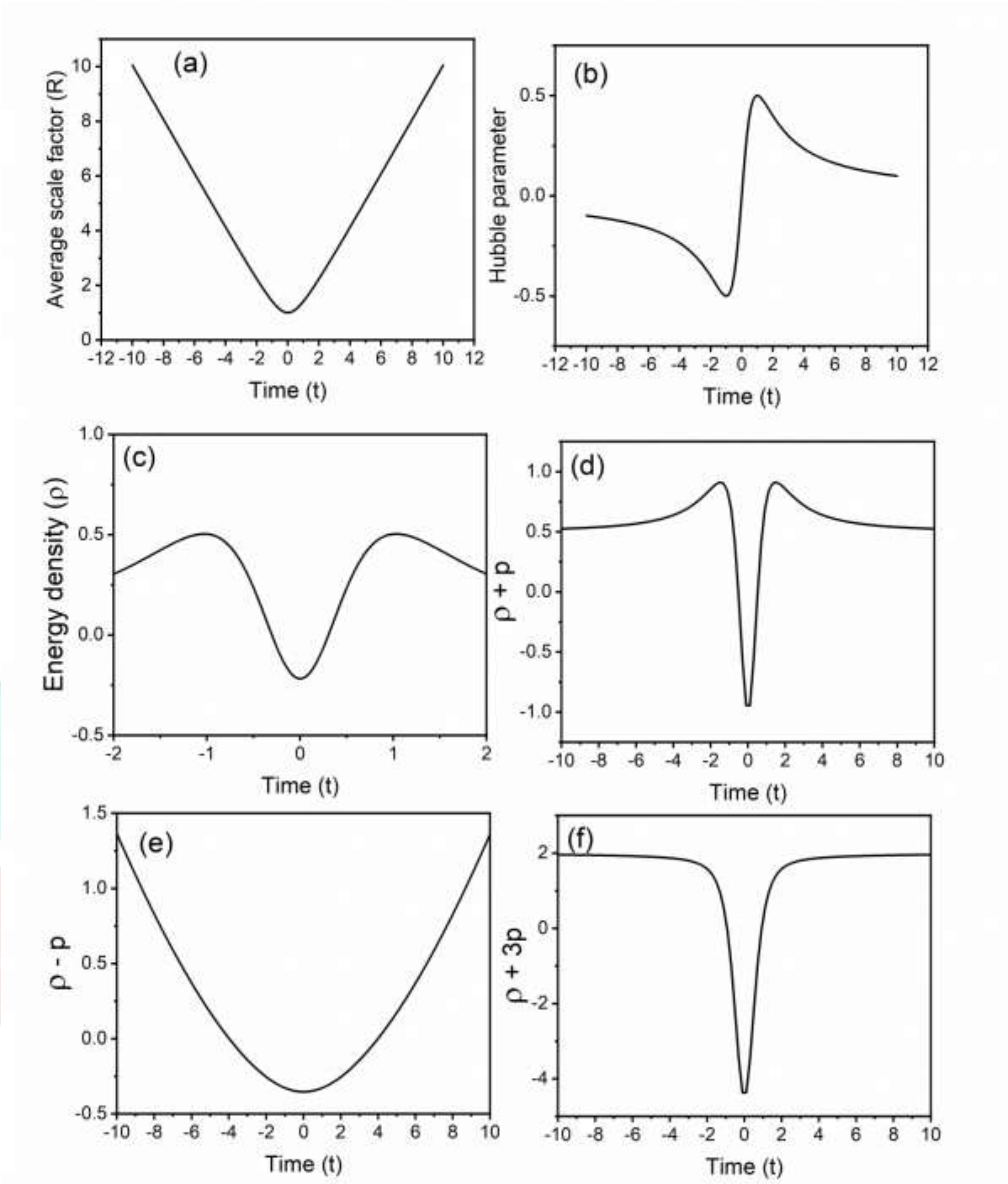

Fig. 1 Plots of time versus (a) Average scale factor (b) Hubble parameter (c) Energy density (ρ) (d) $\rho+p$ (e) $\rho-p$ (f) $\rho+3p$

Discussion : Fig. (1) (a) is the plot of time versus average scale factor for the values $a=1$, $b=1$. It is seen that, during contraction phase, the average scale factor $a(t)$ is decreasing (i.e. $\dot{a}(t)<0$) and then in the expanding phase $a(t)$ is increasing (i.e. $\dot{a}(t)>0$). Hence, the minimal necessary conditions (i) and (ii) for the bounce at time $t=0$ are satisfied [16].

(i) $\dot{a}(t)=0$ at $t=0$

and (ii) at $t=0$, $\ddot{a}(t)>0$ for $t\in(0-\in,0)\,\mathrm{U}(0,0+\in)$,

where $\in$ is very small.

Fig. (1) (b) is the plot of time versus Hubble parameter for the values $a=1$, $b=1$. At *t=0*, we get *H=0* and $a(0)=a_0$ with $\dot{H}(t)>0$ in small neighborhood of $t=0$, provided $a_0>\beta t$. Thus it satisfies the necessary condition of bounce [24]. After the bounce, the universe expands in an accelerated way.

To realize the bounce in our model, let we obtain the values of ρ, $\rho+p$, $\rho-p$ and $\rho+3p$ for $m=2$.

$$\rho=(2.8)\beta^4t^2(a_0{}^2+\beta^2t^2)^{-2}+(a_0{}^2+\beta^2t^2)^{-0.75}-0.25,$$

$$\rho + p = (4.1)\beta^4 t^2 (a_0{}^2 + \beta^2 t^2)^{-2} - (1.5)\beta^2 (a_0{}^2 + \beta^2 t^2)^{-1} + 0.5$$

$$\rho - p = (1.5)\beta^4 t^2 (a_0{}^2 + \beta^2 t^2)^{-2} + 2(a_0{}^2 + \beta^2 t^2)^{-0.75} - (1.5)\beta^2 (a_0{}^2 + \beta^2 t^2)^{-1} - 1$$

$$\rho + 3p = (6.75)\beta^4 t^2 (a_0{}^2 + \beta^2 t^2)^{-2} - (4.5)\beta^2 (a_0{}^2 + \beta^2 t^2)^{-1} - 2(a_0{}^2 + \beta^2 t^2)^{-(0.75)} + 2.$$

From Fig. (1) (c), (d), (e) and (f) are the plots of time versus energy density ρ, $\rho + p$, $\rho - p$ and $\rho + 3p$. It is observed that WEC, NEC and DEC energy conditions are satisfied at $t = 0$ but SEC is violated at $t = 0$. Hence, the bounce is realized in our model at $m = 2$.

In equation(33), When the scale factor becomes large, $\zeta(H) \approx \dfrac{2(5m+1)H}{(m+2)^2}$ and then we can treat it as a fluid with a bulk viscosity of $o(H)$. Therefore, with $\zeta(H, a)$, we can recognize bouncing universe with the scale factor(13).

In another example, we take $\gamma(\rho) = \gamma$ (constant) and $B(\rho, a, H, \dot{H}......) = 4H\zeta$ with $\zeta = \in \rho$, where $\in$ is a constant.
From equation(9), we have

$$\dot{\rho} = -3H(\rho + p).$$

Using equation(3), we get

$$\dot{\rho} = -3H(\rho + p)$$
$$= -3H(\gamma(\rho)\rho - B(a, H, \dot{H}, ...)).$$

Since, $\zeta = \in \rho$ *and* $B(a, H, \dot{H}, ...) = 3H\zeta$, we have

$$\dot{\rho} = 3H\rho(\in - (1+\gamma))$$

$$\Rightarrow \quad \frac{\dot{\rho}}{\rho} = -3H(1 + \gamma - \in).$$

Using equation(14), we get

$$\frac{\dot{\rho}}{\rho} = -3\left(\frac{\beta^2 t}{a_0{}^2 + \beta^2 t^2}\right)(1 + \gamma - \in).$$

Integrating with respect to t,

$$\rho = \frac{a_1}{(a_0{}^2 + \beta^2 t^2)^{\frac{3(1+\gamma-\in)}{2}}}, \qquad (35)$$

where, a_1 is a constant of integration.
Also, from equation (3) we get

$$p = \gamma(\rho)\rho - B(a, H, \dot{H}, ...)$$

Using $\gamma(\rho) = \gamma = const.$ *and* $B(a, H, \dot{H}, ...) = 3H\zeta$, we get

$$p = (\gamma - 3H \in)\rho. \qquad (36)$$

The scenario with $\gamma =$ constant, $\zeta = \in \rho$ the energy density of the bouncing universe will decrease with increasing time (provided, $(1 + \gamma > \in)$) and also the bulk viscosity. For the large time t, p will become negative.

VI. Conclusion

Bianchi Type-IX cosmological has been investigated with viscous fluid by considering specific forms of the scale factors proposed by Molina-Parris and Visser [21]. In the cosmological model, it is realized that there is existence of bounce at point $t = 0$.

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