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Analogical Study of Newton-Raphson Method & False Position Method

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Abstract: In this paper we do the analogical study of two different numerical methods in respect to iteration. Any numerical analysis model or method is faster if and only if a sequence of model solutions with increasingly refined solution domains approaches to a fixed value. Newton–Raphson and False Position methods are very effective numerical procedures used for solving nonlinear equations of the form f(x) = 0.Their analyses can be carried out from comparing Newton-Raphson and False Position. Given that a model is consistent, it is not feasible to apply any method to a problem without testing the size of the time and iterative steps which form the discrete approximation of the solution of the given problem. And we can say that Newton Raphson is best and faster than False Position method. The contents of this note should also be a useful exercise/example in the application of polynomial interpolation and divided differences in introductory courses in numerical analysis.

***Index Terms* - Comparison , Newton-Raphson , False Position**

I. INTRODUCTION

Given a number of total students in a school and you want to know if all the students fit in an assembly ground, you need to know how many lines need to be formed at a minimum. Given the dimension of a door, you need to know how big plywood can be passed through the door. As we know from school days and still we have studied about the solutions of equations like quadratic equations ,cubical equations and polynomial equations and having roots in the form of $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ where a,b,c are the coefficient of equation. But nowadays it is very difficult to remember the formulas for higher degree polynomial equations. You can't do these without one thing — square root. Whether it is finding the square root of a number or square root of a sum of squares, a function (or command) to find the square root of a number is needed. Hence to remove these difficulties there are few numerical methods are Newton-Raphson and False Position methods .

1.1 Newton-Raphson Method

Newton-Raphson Method (also called the **Newton's** method) is a recursive algorithm for approximating the root of a differentiable function. We know simple formulas for finding the roots of linear and quadratic equations, and there are also more complicated formulae for cubic and quartic equations. The Newton-Raphson method is a method for approximating the roots of polynomial equations of any order. In fact the method works for any equation, polynomial or not, as long as the function is differentiable in a desired interval. The Newton-Raphson method (also known as Newton's method) is a way to quickly find a good approximation for the real-valued function f(x)=0 . It uses the idea that a continuous and di�erentiable function can be approximated line tangent to it.Newton's method is always convergent if the initial approximation is sufficiently close to the root. Newton's method converges quadratically.

Let x_0 be a good estimate of r and let $r = x_0 + h$. Since the true root is r, and $h = r - x_0$, the number h measures how far the estimate x_0 is from the truth.
Since h is ' small ' we can use the linear (tangent line) approximation to conclude that

$$0 = f(r) = f(x_0 + h) \approx f(x_0) + hf'(x_0)$$

and therefore, unless $f'(x_0)$ is close to 0,

$$h \approx \frac{f(x_0)}{f'(x_0)}$$

It follows that

$$r = x_0 + h \approx x_0 - \frac{f(x_0)}{f'(x_0)}$$

Our new improved (?) estimate x_1 of r is therefore given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

The next estimate x_2 is obtained from x_1 in exactly the same way as x_1 was obtained from x_0:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Continue in this way. If x_n is the current estimate, then the next estimate x_{n+1} is given by

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

1.1.1 Example

Find a root of an equation $f(x) = x^3 - 3x - 5$ upto 3 decimal point using Newton Raphson method .

Solution:

Here $x^3 - 3x - 5 = 0$

Let $f(x) = x^3 - 3x - 5$

$$\frac{d}{dx}(x^3 - 3x - 5) = 3x^2 - 3$$

$\therefore f'(x) = 3x^2 - 3$

Here

x	0	1	2	3
***f*(*x*)**	-5	-7	-3	13

Here $f(2) = -3 < 0$ and $f(3) = 13 > 0$

$\therefore$ Root lies between 2 and 3

$$x_0 = \frac{2+3}{2} = 2.5$$

$x_0 = 2.5$

<u>1st iteration :</u>

$f(x_0) = f(2.5) = 2.5^3 - 3 \cdot 2.5 - 5 = 3.125$

$f'(x_0) = f'(2.5) = 3 \cdot 2.5^2 - 3 = 15.75$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2.5 - \frac{3.125}{15.75}$$

x_1=2.3016

According reccurance put the value of x_1 and we get from to 2nd iteration and $x_2 = 2.2793$ and goes then

<u>3rd iteration :</u>

$f(x2) = f(2.2793) = 2.2793^3 - 3 \cdot 2.2793 - 5 = 0.0034$

$f'(x2) = f'(2.2793) = 3 \cdot 2.2793^2 - 3 = 12.5855$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 2.2793 - \frac{0.0034}{12.5855}$$

$x_3 = 2.279$

So we stop finding value of x after 3rd iteration because we get same result upto 3 decimal point .

So approximate root of the equation $x^3 - 3x - 5 = 0$ using Newton-Raphson method is 2.279.

1.2 False Position Method

The reason behind Regula-Falsi method is referred also as False Position Method is that it is a trial and error method of solving problem by substituting value for the unknown variable and test the function based up on that decide the next interval to find the solution of the equation. False position method is a very old method for computing the real roots of an algebraic equation . In simple terms, the method is the trial and error technique of using test ("false") values for the variable and then adjusting the test value according to the outcome. This is sometimes also referred to as "guess and check". Versions of the method predate the advent of algebra and the use of equations.

To get the update formula for the false-position method, recall that the equation for the line between x_l and x_u is given by

$$y = f(x_u) + \frac{f(x_l) - f(x_u)}{x_l - x_u}(x - x_u)$$

Hence, to find the intersection of this line with the *x* axis, we must solve the equation

$$0 = f(x_u) + \frac{f(x_l) - f(x_u)}{x_l - x_u}(x_r - x_u)$$

This leads directly to the update formula

$$x_r = x_u - f(x_u)\frac{(x_l - x_u)}{f(x_l) - f(x_u)}$$

Example : 1.2.1

Find a root of an equation *f*(*x*)=*x*3-3*x*-5 using False Position method (regula falsi method)

Solution:

Here $x^3 - 3x - 5 = 0$

Let $f(x) = x^3 - 3x - 5$

Here

X	0	1	2	3
f(*x*)	-5	-7	-3	13

1st iteration :

Here $f(2) = -3 < 0$ and $f(3) = 13 > 0$

∴ Now, Root lies between $x_0 = 2$ and $x_1 = 3$

$$x_2 = x_0 - f(x_0) \cdot \frac{(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = 2 - (-3) \cdot \frac{3 - 2}{13 - (-3)}$$

$x_2 = 2.1875$

$f(x_2) = f(2.1875) = 2.1875^3 - 3 \cdot 2.1875 - 5 = -1.095 < 0$

Now goes 2nd iteration , 3rd iteration upto where we get same value upto 3 decimal. Then Final we get

8th iteration :

Here $f(2.2789) = -0.0009 < 0$ and $f(3) = 13 > 0$

∴Now Root lies between $x_0 = 2.2789$ and $x_1 = 3$

$$x_9 = x_0 - f(x_0) \cdot \frac{(x_1 - x_0)}{f(x_1) - f(x_0)}$$

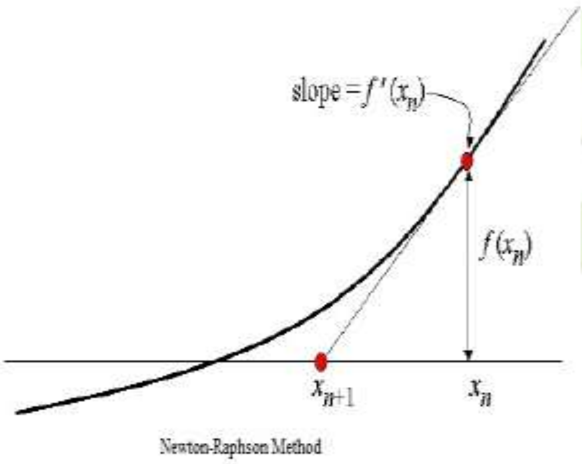
$$x_9 = 2.2789 - (-0.0009) \cdot 3 - \frac{2.27891}{3 - (-0.0009)}$$

$x_9 = 2.279$

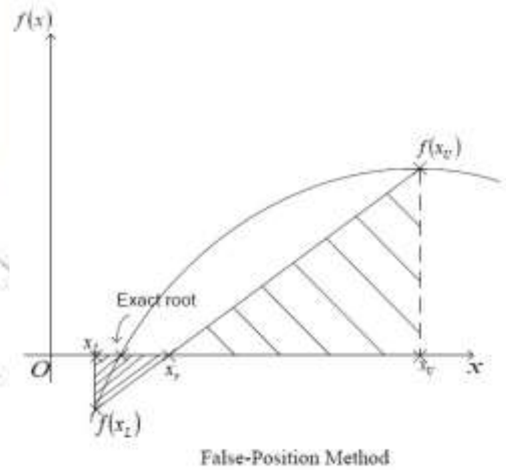
$f(x_9) = f(2.279) - 2.279^3 - 3 \cdot 2.279 - 5 = -0.0003 < 0$

Approximate root of the equation *x*3-3*x*-5=0 using False Position mehtod is 2.279

2. Graphical Representation



Newton-Raphson Method



False-Position Method

3. Analogical analysis :

N	x_0	$f(x_0)$	$f'(x_0)$	x_1
1	2.5	3.125	15.75	2.3016
2	2.3016	0.2874	12.8919	2.2793
3	2.2793	0.0034	12.5855	2.279

Table 1 . Newton-Raphson Method

V/s

n	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$
1	2	-3	3	13	2.1875	-1.095
2	2.1875	-1.095	3	13	2.2506	-0.3518
3	2.2506	-0.3518	3	13	2.2704	-0.1084
4	2.2704	-0.1084	3	13	2.2764	-0.0329
5	2.2764	-0.0329	3	13	2.2782	-0.01
6	2.2782	-0.01	3	13	2.2788	-0.003
7	2.2788	-0.003	3	13	2.2789	-0.0009
8	2.2789	-0.0009	3	13	2.279	-0.0003

Table 2 . False-Position Method

4.Conclusion

As discussed in above sections the two numerical methods have tables with step of iteration. Comparing the Newton-Raphson method and the False-Position method, we noticed that theoretically, Table shows that among Newton-Raphson method is faster than False Position method according to iteration. In False Position method, we calculate only one more value of each step i.e. $f(x_n)$ while in Newton's method we require two calculations $f(x_n)$ and $f'(x_n)$. So Newton's method generally requires less number of iterations . False Position method is surly convergent while Newton's method is conditionally convergent. But once, Newton's method converges, it converges faster. If f'(x) is large near the root, correction to be applied is smaller in case of Newton's method and this method is preferred. When f'(x) is small near the root, correction to be applied is large in case of Newton's method and hence in this case False-Position method should be applied.

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