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SERVICE SURRENDER QUEUES WITH FINITE RANGE OF SERVICE HOLDING TIME AND TRUNCATION OF PROBABILITY AT BOTH THE ENDS

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Abstract:

The present literature of queueing theory is restricted to the "only" queue to a service counter.It does not focus on the queues formed when the service counter has no service to offer to the customers after some time. Moreover the services are such that they can be surrendered to the service counter by the customers after using it for some finite period of time. Here queueing systems have two different queues namely a 'primary queue' and a 'secondary queue'. The existence of secondary queue is possible only when service counter has limited service and the system has a characteristic of 'Service surrender facility'. This paper contains analysis of secondary queues where a model with finite range of service holding time and incomplete range (truncation at both the ends) of probability of service surrender is considered. The expected waiting time of a customer in the entire queueing system and service wastage rate are derived in this paper.

Index Terms - Secondary queues, Service holding time, Service surrender, Service wastage.

1. INTRODUCTION

A queueing system is composed of customers arriving for service, waiting for service if it is not immediate, and if having waited for service, leaving the system after being served or sometimes without being served. Up till now in the literature of queueing analysis, right from A.K. Erlang [2] to Feller,W [3], D.G. Kendall [5],D.V.Lindley [7], N.U. Prabhu and U.N. Bhat [8] and till the recent years the entire study is focused on the one and only one queue which is formed in front of a service counter.

S.P.Kane and P.B.Lakhani [4] pointed out that there are some queueing systems where a customer not only has to wait in single queue but also has to wait in another if did not served in the first queue due to limited service with the service counter. The existence of second queue is possible only when system has a characteristic of 'service surrender facility' which means returning back the utilized service by the customer.

It should be noted that in every queueing system it is not possible for a customer to surrender the service. Some examples where service surrender facility is not available are

1) A patient in a queue for taking service from a doctor and
2) A customer in a queue for food from a restaurant.
3) A customer being served by a barber.

In contrary, some examples where service surrender facility is available are

1) A queue of customers for acquiring a railway reservation ticket.

2) A queue of customers for getting a locker in the bank.
3) A queue of persons to borrow book(s) from the library.

1.1 Primary and secondary queues

Let there be a service counter where the service is available only to a finite number of customers (say N). Initially the system works and after offering the service to N customers, service is not available to any of the next waiting customers in queue. From this point of time waiting customers have an option either to register themselves into a 'waiting list' or to quit the system. This waiting list now becomes a secondary queue. Thus the customer initially joins the 'primary queue' and after reaching the service counter and knowing that all the services are exhausted has a choice whether to join the 'secondary queue' or to quit the system. The duration for maximum waiting time in the secondary queue is uncertain. Following the terminology of Bartholomew D.J. and Forbes A.F.[1], we define Service Holding Time (SHT) of the queueing system as the average duration of time for which a customer holds the service, before the service is surrendered.

2. Analysis of the secondary queue

Let p be the probability that a customer enjoys the service for a unit time. The probability that he surrenders the service some time during a unit time is (1-p) = q (say).Therefore the probability that a customer completes 't' units of time before he surrenders the service is a Geometric variable with p.m.f.

$P(T = t) = p^{t-1}(1-p) \qquad t = 1,2, \ldots, \infty$

As the probability 'p' differs from person to person randomly, taking any value in the range [0,1], it is appropriate to consider it as a random variable.

The appropriate distribution of p may be considered as Beta distribution with parameters 'a' and 'b' which is given by

$$f(p) = \frac{1}{B(a,b)}\, p^{a-1}(1-p)^{b-1} \qquad a, b > 0 ; 0 < p < 1 .$$

Hence the situation can be well studied by considering Compound Geometric Beta distribution. Thus SHT(T) can be assumed to follow a compound Geometric Beta distribution, which can be utilized to further analyze the secondary queues.

P.B.Lakhani and S.P.Kane [9] have given analysis of secondary queues where a model with infinite range of service holding time and truncation at both the ends ($\alpha_1 \le p \le \alpha_2$) is considered.

Here the distribution of p truncated at both the ends is given by

$$g(p) = \frac{f(p)}{\int_{\alpha_1}^{\alpha_2} f(p)dp}$$

$$= \frac{p^{a-1}(1-p)^{b-1}}{\int_{\alpha_1}^{\alpha_2} p^{a-1}(1-p)^{b-1}}$$

$$= \frac{p^{a-1}(1-p)^{b-1}}{[B_{\alpha_2}(a,b) - B_{\alpha_1}(a,b)]}$$

Where $B_{\alpha_1}(a,b)$ and $B_{\alpha_2}(a,b)$ are incomplete Beta functions.

The following probability function of compound Geometric doubly truncated Beta distribution is derived in this paper of P.B.Lakhani and S.P.Kane [9].

$$P(T=t)=f_t=\frac{[B_{\alpha_2}(a+t-1,b+1)-B_{\alpha_1}(a+t-1,b+1)]}{[B_{\alpha_2}(a,b)-B_{\alpha_1}(a,b)]} \quad ...(1)$$

Where $a,b>0 : 0<\alpha_1<\alpha_2<1 : t=1, 2....\infty$

To make the expression (1) more convenient to handle the values of a and b both are substituted equal to 1, in the above paper [9] and the following probability function is obtained.

$$P(T=t)=f_t=\frac{[B_{\alpha_2}(t,2)-B_{\alpha_1}(t,2)]}{[B_{\alpha_2}(1,1)-B_{\alpha_1}(1,1)]}$$

$$=\frac{\left\{\alpha_2^{\,t}[1+t(1-\alpha_2)]-\alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}{t(t+1)(\alpha_2-\alpha_1)} \quad ...(2)$$

$$0<\alpha_1<\alpha_2<1 \; ; \; t=1,2,...,\infty$$

2.1 Model with finite range of SHT and $\alpha_1 \le p \le \alpha_2$:

Let us consider a queueing system that works for some finite time and after that the entire system vanishes. This means that the customers cannot utilize the service for indefinite period of time. The service utilization stops after a fixed time period even after the customer does not surrender his service. Further this implies that if a customer wants, he can surrender his service before reaching that fixed time point and as such the next one in the queue gets the service. The customers who are on 'waiting list' are also in the system for a fixed time irrespective of the fact that they get the service or not.

So here SHT ranges from zero to some finite number say 'A'. Hence for the analysis of this situation we need to obtain the p.m.f. of the doubly truncated compound Geometric Beta distribution. Before that we make a few assumptions as follows :

Assumption 1 :

The event of surrendering the service and getting a service to a customer who is registered on the 'waiting list' are allowed to occur only at discrete time points.

Assumption 2 :

These time points are equipped spaced.

Let T be the SHT of a customer in the system . Let 'A' be the maximum length of service utilization. So here SHT ranges from zero to some finite number say 'A'. In other words we truncate T to the right at A and the range of 'p' is truncated at both the ends i.e. $\alpha_1 \le p \le \alpha_2$. We consider this model for particular values of a and b (i.e. with a =1 and b =1). $\therefore$ From equation (2) we write p.m.f. of doubly truncated compound Geometric Beta distribution as

From equation (2) we get

$$P(T=t)=f_t=\begin{cases}\dfrac{K.\left\{\alpha_2^{\,t}[1+t(1-\alpha_2)]-\alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}{t(t+1)(\alpha_2-\alpha_1)} & 0<t\le A\\[2ex] 0 & \text{o.w}\end{cases}$$

$$\sum_{t=1}^{\infty} f_t = 1$$

$$\Rightarrow \sum_{t=1}^{A} f_t + \sum_{t=A+1}^{\infty} f_t = 1$$

$$\text{i.e.} \sum_{t=1}^{A} f_t = \frac{\sum_{t=1}^{A} K.\left\{\alpha_2^{\,t}[1+ t(1-\alpha_2)] - \alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}{t(t+1)(\alpha_2-\alpha_1)} = 1$$

$$\Rightarrow K = \frac{t(t+1)(\alpha_2-\alpha_1)}{\sum_{t=1}^{A}\left\{\alpha_2^{\,t}[1+t(1-\alpha_2)] - \alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}$$

$$\therefore f_t = \frac{\left\{\alpha_2^{\,t}[1+t(1-\alpha_2)] - \alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}{\sum_{t=1}^{A}\left\{\alpha_2^{\,t}[1+t(1-\alpha_2)] - \alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}$$

$$\therefore P(T=t) = f_t = \frac{(1-\alpha_2)(1-\alpha_1)\left\{\alpha_2^{\,t}[1+t(1-\alpha_2)] - \alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}{\left\{\begin{array}{l} 2(\alpha_2-\alpha_1) - (A+2)[(1-\alpha_1)\alpha_2^{\,A+1} - (1-\alpha_2)\,\alpha_1^{\,A+1}] \\ + A[(1-\alpha_1)\,\alpha_2^{\,A+2} - (1-\alpha_2)\alpha_1^{\,A+2}] \end{array}\right\}} \quad \ldots(3)$$

The above probability function can be of vital importance which can be utilized to further analyze the secondary queues. Pratibha B.Lakhani and S.P.Kane [6] have given analysis of secondary queues where a model with infinite range of service holding time and truncation of probability of service surrender to the right is considered. Pratibha B.Lakhani and S.P.Kane [9] have given analysis of secondary queues where a model with infinite range of service holding time and truncation of probability of service surrender at both the ends is considered. Pratibha B.Lakhani and N.S.Kane [10] have given analysis of secondary queues where a model with finite range of service holding time and complete range of probability of service surrender (i.e.from 0 to 1)is considered. Pratibha B.Lakhani and S.P.Kane [11] have given analysis of secondary queues where a model with finite range of service holding time and truncation of probability of service surrender to the right is considered.Pratibha B.Lakhani [12] has given analysis of secondary queues where a model with infinite range of service holding time and complete range of probability of service surrender (i.e.from 0 to 1)is considered.

2.1.1: Average waiting time of a customer in the secondary queue

The average duration of utilizing the service by a customer is

$$E(t) = \sum_{t=1}^{A} t.f_t$$

Using (3) we get

$$E(t) = \frac{(1-\alpha_2)(1-\alpha_1)\sum_{t=1}^{A} t\left\{\alpha_2^{\,t}[1+t(1-\alpha_2)]-\alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}{\begin{Bmatrix}2(\alpha_2-\alpha_1)-(A+2)[(1-\alpha_1)\alpha_2^{\,A+1}-(1-\alpha_2)\,\alpha_1^{\,A+1}]\\ +A[(1-\alpha_1)\,\alpha_2^{\,A+2}-(1-\alpha_2)\alpha_1^{\,A+2}]\end{Bmatrix}}$$

$$= \frac{1}{\begin{Bmatrix}[2(\alpha_2-\alpha_1)-(A+2)[(1-\alpha_1)\alpha_2^{\,A+1}-(1-\alpha_2)\alpha_1^{\,A+1}]\\ +A[(1-\alpha_1)\alpha_2^{\,A+2}-(1-\alpha_2)\alpha_1^{\,A+2}]\end{Bmatrix}}$$

$$\cdot\left\{\frac{\begin{array}{l}(1-\alpha_1)^2\{\alpha_2(2+\alpha_2)-[A^2+3A+2]\,\alpha_2^{\,A+1}+[2A^2+3A-1]\alpha_2^{\,A+2}-A^2\alpha_2^{\,A+3}\}\\ -(1-\alpha_2)^2\{\alpha_1(2+\alpha_1)-[A^2+3A+2]\alpha_1^{\,A+1}+[2A^2+3A-1]\alpha_1^{\,A+2}-A^2\alpha_1^{\,A+3}\}\end{array}}{(1-\alpha_1)(1-\alpha_2)}\right\} \quad \ldots(4)$$

2.1.2: The distribution function of T

Using (3), the distribution function is obtained as follows.

$$F_T = \sum_{t=1}^{T} f_t = \frac{\sum_{t=1}^{T}(1-\alpha_2)(1-\alpha_1)\left\{\alpha_2^{\,t}[1+t(1-\alpha_2)]-\alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}{\begin{Bmatrix}2(\alpha_2-\alpha_1)-(A+2)[(1-\alpha_1)\alpha_2^{\,A+1}-(1-\alpha_2)\,\alpha_1^{\,A+1}]\\ +A[(1-\alpha_1)\,\alpha_2^{\,A+2}-(1-\alpha_2)\alpha_1^{\,A+2}]\end{Bmatrix}}$$

$$= \frac{(1-\alpha_2)(1-\alpha_1)\sum_{t=1}^{T}\left\{\alpha_2^{\,t}[1+t(1-\alpha_2)]-\alpha_1^{\,t}[1+t(1-\alpha_1)]\right\}}{\left\{\begin{array}{l} 2(\alpha_2-\alpha_1)-(A+2)[(1-\alpha_1)\alpha_2^{A+1}-(1-\alpha_2)\,\alpha_1^{A+1}] \\ +A[(1-\alpha_1)\,\alpha_2^{A+2}-(1-\alpha_2)\alpha_1^{A+2}] \end{array}\right\}}$$

$$F_T = \frac{\left\{\begin{array}{l} 2(\alpha_2-\alpha_1)-(T+2)(1-\alpha_1)\alpha_2^{T+1}+T(1-\alpha_1)\alpha_2^{T+2} \\ +(T+2)(1-\alpha_2)\alpha_1^{T+1}-T(1-\alpha_2)\alpha_1^{T+2} \end{array}\right\}}{\left\{\begin{array}{l} 2(\alpha_2-\alpha_1)-(A+2)\left[(1-\alpha_1)\alpha_2^{A+1}-(1-\alpha_2)\alpha_1^{A+1}\right] \\ +A\left[(1-\alpha_1)\alpha_2^{A+2}-(1-\alpha_2)\alpha_1^{A+2}\right] \end{array}\right\}} \qquad \ldots(5)$$

2.1.3: Survival function of the customer

The survival function of the customer in the system is

$$G_T = 1 - F_{(T-1)}$$

$$= 1 - \frac{\left\{\begin{array}{l} 2(\alpha_2-\alpha_1)-(T+1)(1-\alpha_1)\alpha_2^{T}+(T-1)(1-\alpha_1)\alpha_2^{T+1} \\ +(T+1)(1-\alpha_2)\alpha_1^{T}-(T-1)(1-\alpha_2)\alpha_1^{T+1} \end{array}\right\}}{\left\{\begin{array}{l} 2(\alpha_2-\alpha_1)-(A+2)\left[(1-\alpha_1)\alpha_2^{A+1}-(1-\alpha_2)\alpha_1^{A+1}\right] \\ +A\left[(1-\alpha_1)\alpha_2^{A+2}-(1-\alpha_2)\alpha_1^{A+2}\right] \end{array}\right\}}$$

$$= \frac{\left\{\begin{array}{l} -(A+2)\left[(1-\alpha_1)\alpha_2^{A+1}-(1-\alpha_2)\alpha_1^{A+1}\right] \\ +A\left[(1-\alpha_1)\alpha_2^{A+2}-(1-\alpha_2)\alpha_1^{A+2}\right]+(T+1)(1-\alpha_1)\alpha_2^{T} \\ -(T-1)(1-\alpha_1)\alpha_2^{T+1}-(T+1)(1-\alpha_2)\alpha_1^{T}+(T-1)(1-\alpha_2)\alpha_1^{T+1} \end{array}\right\}}{\left\{\begin{array}{l} 2(\alpha_2-\alpha_1)-(A+2)[(1-\alpha_1)\alpha_2^{A+1}-(1-\alpha_2)\alpha_1^{A+1}] \\ +A[(1-\alpha_1)\alpha_2^{A+2}-(1-\alpha_2)\alpha_1^{A+2}] \end{array}\right\}}$$

…(6)

2.1.4 : Service wastage of the model

The service wastage of the model S_T is given by

$$S_T = \frac{f_T}{G_T}$$

$$= \frac{(1-\alpha_2)(1-\alpha_1)\left\{\alpha_2^{T}[1+T(1-\alpha_2)]-\alpha_1^{T}[1+T(1-\alpha_1)]\right\}}{\left\{\begin{array}{l} A[(1-\alpha_1)\alpha_2^{A+2}-(1-\alpha_2)\alpha_1^{A+2}] \\ -(A+2)[(1-\alpha_1)\alpha_2^{A+1}-(1-\alpha_2)\alpha_1^{A+1}] \\ +(T+1)[(1-\alpha_1)\alpha_2^{T}-(1-\alpha_2)\alpha_1^{T}] \\ -(T-1)[(1-\alpha_1)\alpha_2^{T+1}-(1-\alpha_2)\alpha_1^{T+1}] \end{array}\right\}}$$

… (7)

Thus service wastage indicates the rate of surrender of the service. If service wastage S_T is more then there is more rate of service surrender. Hence S_T is one of the most important characteristics of the model.

In table 1 below we have presented a few values of S_T obtained using above formula for various values of T taking different values of other parameters .

Table 1

Value of S_T with finite renge of SHT and $\alpha_1 \angle p \angle \alpha_2$

T	S_T				
	A=50		α_1 = 0.25		
	α_2 = 0.35	α_2= 0.50	α_2 = 0.65	α_2 = 0.75	α_2 = 0.85
5	0.576770	0.430147	0.280594	0.186299	0.100652
10	0.617303	0.461304	0.305802	0.205882	0.112346
15	0.628838	0.472215	0.317029	0.215927	0.119701
20	0.634024	0.478261	0.323701	0.222286	0.125303
25	0.637051	0.482143	0.328133	0.226795	0.130468
30	0.639085	0.484849	0.331338	0.230589	0.136657
35	0.640559	0.486853	0.334109	0.235401	0.146608
40	0.641690	0.488672	0.339161	0.246912	0.167725
45	0.643900	0.498343	0.368032	0.293925	0.231709
50	1.000000	1.000000	1.000000	1.000000	1.000000

T	S_T				
	A=50		α_2 = 0.85		
	α_1 = 0.25	α_1 = 0.35	α_1= 0.50	α_1 = 0.65	α_1 = 0.75
5	0.100652	0.098741	0.089847	0.069856	0.050245
10	0.112346	0.112308	0.111332	0.103184	0.086192
15	0.119701	0.119700	0.119619	0.117105	0.106086
20	0.125303	0.125303	0.125297	0.124582	0.118095
25	0.130468	0.130468	0.130467	0.130270	0.126619
30	0.136657	0.136657	0.136657	0.136603	0.134599
35	0.146608	0.146608	0.146608	0.146593	0.145515
40	0.167725	0.167725	0.167725	0.167721	0.167155
45	0.231709	0.231709	0.231709	0.231707	0.231430
50	1.000000	1.000000	1.000000	1.000000	1.000000

T	S_T				
		$\alpha_1 = 0.25$		$\alpha_2 = 0.85$	
	A=5	A=7	A=10	A=15	A=20
1	0.143842	0.105342	0.079858	0.063063	0.056435
2	0.243612	0.170731	0.125844	0.097596	0.086726
3	0.345395	0.220789	0.154385	0.115982	0.101838
4	0.514999	0.276562	0.178198	0.128056	0.110668
5	**1.000000**	0.360021	0.204208	0.138309	0.117191
6		0.521297	0.237790	0.148737	0.123013
7		**1.000000**	0.286485	0.160449	0.128807
8			0.366405	0.174403	0.134924
9			0.525130	0.191824	0.141629
10			**1.000000**	0.214637	0.149205
11				0.246243	0.158012
12				0.293404	0.168545
13				0.371858	0.181535
14				0.528785	0.198116
15				**1.000000**	0.220165
16					0.251049
17					0.297491
18					0.375158
19					0.531034
20					**1.000000**

We also plot a graph of S_T versus T in figure 1.

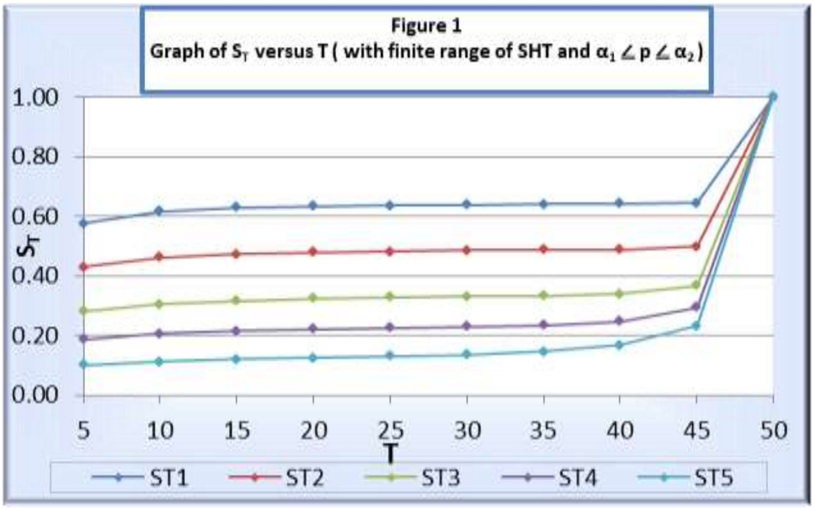
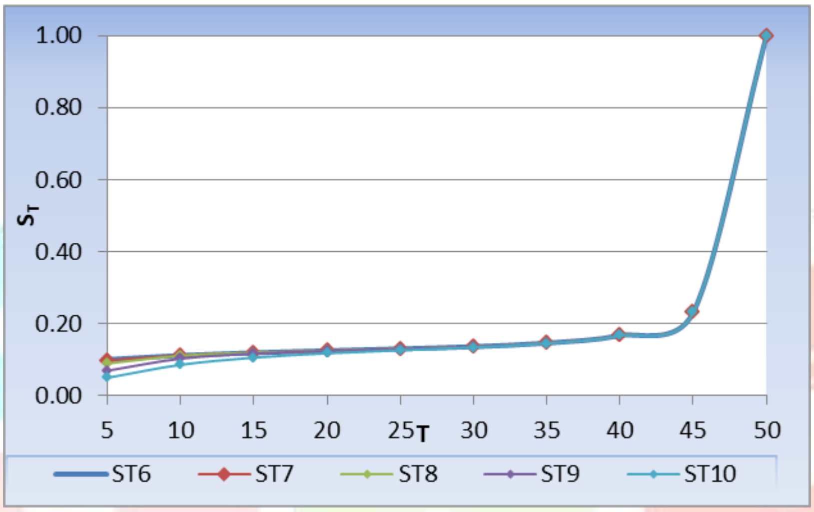
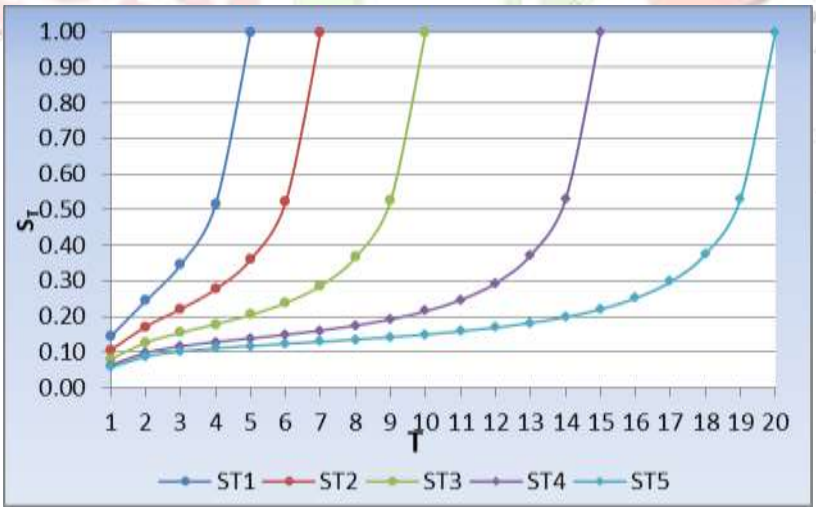


Figure 1
Graph of S_T versus T (with finite range of SHT and $\alpha_1 \angle p \angle \alpha_2$)
1.00
0.80
0.60
0.40
0.20
0.00
S_T
5 10 15 20 25 30 35 40 45 50
T
ST1 ST2 ST3 ST4 ST5


1.00
0.80
0.60
0.40
0.20
0.00
S_T
5 10 15 20 25 30 35 40 45 50
T
ST6 ST7 ST8 ST9 ST10


1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
S_T
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
T
ST1 ST2 ST3 ST4 ST5

3.GENERAL INTERPRETATIONS

The only aim of developing this new model of secondary queueing systems is to estimate the average waiting time of the customer in the secondary queue as well as to understand the degree of instability of the secondary queue using service wastage S_T.

Analysis of service surrender queues is dominated mainly by a secondary queue. The main focus is on finding the expected waiting time of a customer in the entire system.

Note that the customer's waiting time in the system can be divided into two parts. The one is waiting time in the primary queue and the second is waiting time in the secondary queue. Let the average waiting time in primary queue be W, depending upon the model which the primary queue follows.

The average waiting time in the secondary queue is given by equation (4). Therefore the overall average waiting time in the system could be expressed as

$$W + \left(\frac{1}{\left\{ \begin{array}{l} [2(\alpha_2-\alpha_1)-(A+2)[(1-\alpha_1)\alpha_2^{A+1}-(1-\alpha_2)\alpha_1^{A+1}] \\ +A[(1-\alpha_1)\alpha_2^{A+2}-(1-\alpha_2)\alpha_1^{A+2}] \end{array} \right\}} \cdot \left\{ \frac{\begin{array}{l} (1-\alpha_1)^2\{\alpha_2(2+\alpha_2)-[A^2+3A+2]\alpha_2^{A+1}+[2A^2+3A-1]\alpha_2^{A+2}-A^2\alpha_2^{A+3}\} \\ -(1-\alpha_2)^2\{\alpha_1(2+\alpha_1)-[A^2+3A+2]\alpha_1^{A+1}+[2A^2+3A-1]\alpha_1^{A+2}-A^2\alpha_1^{A+3}\} \end{array}}{(1-\alpha_1)(1-\alpha_2)} \right\} \right)$$

The value of service wastage S_T is one of the most important characteristic of the model. It is a measure of disturbances in the secondary queue indicating the rate of service surrender. More the value of S_T more are the chances of getting the surrendered service to the new customers.

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