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Study on Games over the product of two Hausdorff topological spaces

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Abstract: We play several games- outdoor and indoor. But it is very interesting to play a game over a topological space. In this paper, we have tried to play a game over the product $X \times Y$ of two Hausdorff topological spaces X & Y.

Keywords: cozero set, compactness, rectangular, subparacompact closure covering etc.

1. Introduction:

We try to play a game over the product $X \times Y$ of two Hausdorff topological spaces X & Y. Firstly, an important result has been obtained by playing the game G(DC_m. X) where DC_m is the class of all spaces which have a discrete closed cover consisting of m-compact space, by defining rectangles in such product space. Then lastly, with the help of a lemma over a space X which has a closure preserving closed cover by m-compact sets it is proved that dim $(X \times Y) \leq$ dim X + dim Y where X be a collectionwise normal space, Y be a subparacompact space & $X \times Y$ is normal.

2. Games over the product space

2.1 Definitions:

(a) A subset $A \times B$ of a topological product $X \times Y$ is said to be a rectangle. For a rectangle E in $X \times Y$, E' and E'' denote the projection of E into X and Y repectively. So we have $E = E' \times E''$. A rectangle E is said to be a cozero, zero, open and closed rectangle if E' and E'' are cozero, zero, open and closed in $X \times Y$ respectively.

(b) A topological product $X \times Y$ is said to be strongly rectangular of each locally finite open cover of $X \times Y$ has locally finite refinement by cozero rectangles.

(c) A space is said to be m-compact if each of its open cover of power ≤ m has a finite subcover.

2.2 Theorem:

Let X be a collection-wise normal space and Y a subparacompact space with $\chi(Y) \leq m$. If player P has a winning strategy in the game G (DC_m, X) where DC_m is the class of all spaces which have a discrete closed cover consisting of m-compact space, then every open cover of $X \times Y$ with power ≤ m has a σ-discrete refinement by closed rectangles in $X \times Y$.

Proof:

Let s be a winning strategy of player P in G (Dc_m, X). Let C be an arbitrary open cover of $X \times Y$ with $|C| \leq m$. we construct:

(i). a sequence $\{J_n: n \leq 0\}$ collections of closed rectangles in $X \times Y$;

(ii). Sequence $\{< R_n, \psi_n >: n \geq 0\}$ of the pairs of collections R_n by closed rectangles in $X \times Y$.

(iii). The function $\psi_n : R_n \rightarrow R_{n-1}$ satisfyong the following five conditions:

(a). J_n is σ-discrete in $X \times Y$.

(b). R_n is σ-discrete in $X \times Y$.

(c). Each $F \in J_n$ is contained in some $G \in C$.

(d). If $(x, y) \in R_{n-1}$
and $(x, y) \in J_n$
Then there is R_n such that
$(x, y) \in R_n$,
and $\psi_n(R_n) = R_{n-1}$

(e). for an $R \in R_n$,
Let $U_k = X - R$,
and $U_k = X - (\psi_{k-1}, 0, \ldots, 0, \psi_k(R))$, for $1 \leq k \leq n-1$.

We put

$E_1 = S(\phi)$:

and $E_{k+1} = S(U_1, \ldots, U_k)$ for $1 \leq k \leq n-1$.

Then the finite series $< E_1, U_1, \ldots, E_n, U_n >$ is admissible for G(DC_m, X)

Let $J_n = \{\phi\}$

and $R_n = \{X \times Y\}$

We suppose that he above $\{J_n, 1 \leq n\}$ and $\{< R_n, \psi_n > : 1 \leq n\}$ are already constructed.

We pick an $R \in R_n$.

Let $< E_1, U_1, \ldots, E_n, U_n>$ be the admissible sequence in G(DC_m, X).

Hence there is a discrete collection $\{C_\alpha: \alpha \in \Omega(R)\}$ by m-compact closed sets in R' such that

$$S(U_1, \ldots, U_n) \cap R' = \cup \{C_\alpha : \alpha \in \Omega(R)\}$$

We can choose a discrete collection $\{W_\alpha : \alpha \in \Omega(R)\}$ of open sets in R' such that

$C_\alpha \subset W_\alpha$, for all $\alpha \in \Omega(R)$.

Since C_α is m-compact $|C| \leq m$, $\chi(y) \leq m$ and R^n is subparacompact.

There is a collection

$J_{n+1}^{\alpha} = \{\text{Cl}\, U_\lambda^{\alpha,i} \times H_\lambda : i = 1, \ldots, k_\lambda \text{ and } \lambda \in \wedge(k)\}$

By closed rectangle in R, satisfying the following four conditions:

(1). Each $U_\lambda^{\alpha,1}$ is open in R'.

(2). $C_\alpha \subset \cup\{U_\lambda^{\alpha,i}: i = 1, \ldots, k_\lambda\} \subset W_\alpha$.

(3). Each Cl $U_\lambda^{\alpha,i} \times H_\lambda$ is contained in some $G \in C$.

(4). $\{H_\lambda : \lambda \in \wedge(\alpha)\}$ is a σ- discrete closed cover of R^n. Then
$J_{n+1}(R) = \cup\{J_{n+1}^{\alpha} : \alpha \in \Omega(R)\}$ is a $\sigma - discrete$ in $X \times Y$.

Put $R_\lambda^{\alpha} = \{Cl\, W_\alpha - \cup\{U_\lambda^{\alpha,i} : 1 \leq i \leq k\} \times H_\lambda\}, for\ all\ \lambda \in \wedge(k)\}$.

Again put $R = (R' - \cup\{W_\alpha : \alpha \in \Omega(R)\} \times R^n$.

Moreover, we put

$R_{n+1}(R) = \{R \cup \{R_\lambda^{\alpha} : \lambda \in \wedge(\alpha)\} and\ \lambda \in \Omega(R)\}$

Then R_{n+1} (R) is also a σ-discrete collection by closed rectangles in RType equation here..

We set

$J_{n+1} = \cup\{J_{n+1}(R) : R \in R_n\}$;

and $R_{n+1} = \cup\{R_{n+1}(R) : R \in R_n\}$.

The function $\psi_{n+1}(R_{n+1}(R)) = \{R\}$,

for all $R \in R$.

From (a), J_{n+1} and R_{n+1} are σ-discrete refinement of C by closed rectangles in $X \times Y$.

2.3 Lemma:

Let X be a space which has a closure preserving closed cover J by m=compact sets. Then to each closed set E of X one can assign a discrete collection A(E) by m-compact closed subsets of E, satisfying the following two conditions:

(a). Each D ∈ A(E) is contained in some F ∈ J.

(b). If <E_1, E_2, …> is a decreasing sequence of closed sets of X such that

$E_1 \cap \left(\cup A(x)\right) = \phi.$

and $E_{n+1} \cap \left(\cup A(E_n)\right) = \phi, for\ all\ n \in N,$

then $\cap \{E_n : n \in N\} = \phi.$

Then following results obvious:

(a). If a space X has a σ-closure preserving closed cover by m-compact sets, then player P has a winning strategy in G(DC_m, X).

(b). Let X be a normal space if player P has winning strategy G(Dim_n, X), then dim X ≤ n.

3. Conclusion:

Let $X \times Y$ be a normal space with dim X ≤ m and dim Y ≤ n.

Let A× B be a product space such that A is m-compact and χ (B) ≤ m. Since the projection of A× B onto B is a closed map. A× B is rectangular. It follows from the product theorem of B. A. Paskynov that

Dim(A× B) ≤ dim A + dim B holds.

Thus, for all closed rectangle R in $X \times Y$ with R ∈ PC_m where PC_m denotes to the class of all product spaces with the first factor being m-compact.Type equation here.

We get

dim R ≤ dim R' – dim R^n ≤ m + n.

Therefore, each closed sets P of $X \times Y$ with P ∈ D (PC_m).

We get dim P ≤ m + n.

From the above result (1) of previous lemma it follows

Since $X \times Y$ is normal, it also follows that

Dim ($X \times Y$) ≤ m + n

4. References:

[1] Paskynov, B. A. 1975, On the dimension of rectangular products, Soviet Math. Dokl. 16, pp. 344-347.

[2] Mortia, K. & Hoshina, T. 1980On the rectangular products of topological spaces, Topology and its application II, pp. 47-57.

[3] Kumar, B.P. 1982 Lattice and topological approach to game theory Ph. D. thesis submitted to Bhagalpur University, Bhagalpur(Bihar).

[4] Von Neumann, J. & Morgaustern, O. 1947 Theory of games and economic behavior, Princeton University Press, Princeton 1944. 2nd ed. 1947.

[5] Dresher, M. A. W., Tucker, and P. Wolfe (eds.) 1957 Contributon to the theory of Games, III, Princeton, N.J. Princeton University Press.

[6] Kumar Sanjay & Kumar B.P. 2004 : covering Properties of spaces defined by Games and its fuzzification; Proc. Of the conf. on Adv. in Maths. & its Aplications.