IJCRT.ORG   **ISSN : 2320-2882**

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Mathematical Modeling of Bhagavat Gita: Algebraic Field of Human Resources

Dr. J. Suresh Kumar

Assistant Professor and Research Supervisor,

Post Graduate and Research Department of Mathematics,

N.S.S. Hindu College, Changanacherry, Kottayam Dist., Kerala, India-686102

Abstract:

In this paper, we present a Mathematical model of the Bhagavat Gita as an Algebraic Field of the Human Resources for Professional Human Resource Management. An individual is considered as an algebraic field of Human Resources (Soft skills, Outer and Inner human Resources) together with addition (*Karma-Yoga*-Professional Aptitude) and multiplication (*Bhakti-Yoga*-Professional Attitude) operations. In this paper, we investigate the Human Resources Management principles mentioned in the Bhagavat Gita and their Mathematical formulation in the abstract algebraic terms.

Key Words: Algebra, Field, Human Resource Management, Bhagavat Gita, Aptitude, Attitude.

1. INTRODUCTION:

Bhagavat Gita, which is a part of the great Indian epic, Mahabharata, had studied by scholars from various fields like Management, Psychology, Education, Humanities, Upanishads, Politics, Social work, and Spiritual Sciences. Mathematical formulation of the Human Resource Management principles mentioned in the Gita is an active research area. Suresh Kumar [5, 6] initiated such a study and obtained two algebraic Group theoretic models of the science part of Bhagavat Gita and discussed the Bloom's Taxonomy levels of the cognizant domain of the learners and the continuous quality improvement principle of Ishikava. While most of the other studies concern with discussion of their own faculty with comparison or illustration from some selected parts of Bhagavat Gita, we propose a mathematical model of the entire Bhagavat Gita, an abstract algebraic field of Human resources.

The first chapter is only a preface and the slokas from 2.40 to 15.20 explains and concludes the science part of Bhagavat Gita. In this paper, we present and investigate the Mathematical model of the Science of Bhagavat Gita as an Algebraic Field Theory of Human Resources for Professional Quality Improvement and Human Resources Management. The only Mathematics used in this paper is the notion of an "algebraic field", which is the abstraction of the algebraic properties of our familiar number system under the addition and multiplication operations. In abstract algebra (*Bija-Ganitam*), objects in the universe with similar algebraic properties are investigated.

A Field is an Algebraic structure, denoted by a triplet $(F, +, \times)$, consisting of a non-empty set, F, an addition operation defined on F (denoted by +) and a multiplication operation defined on F (denoted by $\times$) and satisfying the following eleven properties:

1) Binary Addition: $a + b \in G$ for all $a, b \in F$

   This simply means that the sum of any two elements of F is again an element of F

2) Additive Associativity: $a + (b + c) = (a + b) + c$ for all $a, b, c \in F$

   This simply means that the sum of three given elements of F can be computed in any order.

3) Additive Identity: There is an element, say 0, in F such that $a + 0 = a$ for all $a \in F$

   This simply means that there is an element in F which doesn't alter any element when added to it.

4) Additive Inverse: For any a in F, there is an element, called as $-a$, in G so that $a + (-a) = 0$

   This simply means that every element of F has an additive inverse in F.

5) Additive Commutativity: $a + b = b + a$ for all $a, b \in F$

   This simply means that sum of any two elements in F can be done in any order.

6) Binary Multiplication: $a \times b \in F$ for all $a, b \in F$

This simply means that the product of any two elements of F is again an element of F

7) Multiplicative Associativity: $a \times (b \times c) = (a \times b) \times c$ for all $a, b, c \in F$

This simply means that the product of three given elements of F can computer in any order.

8) Multiplicative Identity: There is an element, say 1, in F such that $a \times 1 = a$ for all $a \in F$

This simply means there is an element in F which doesn't alter any element when multiplied by it.

9) Multiplicative Inverse: For any $a \in F, a \neq 0$, there is an element, say $a' \in F$ with $a \times a' = 1$

This simply means that every non-zero element of F has a multiplicative inverse in F.

10) Multiplicative Commutativity: $a \times b = b \times a$ for all $a, b \in F$

This simply means that product of any two elements in F can be done in any order.

11) Distributivity: $a \times (b + c) = (a \times b) + (a \times c)$ for every $a, b, c \in F$

This simply means addition and multiplication operations on F are not independence, but related.

In the next section, we will show that the set, F of all Human resources forms a field under the addition (Professional Aptitude) and multiplication (Professional Attitude) operations. Reader may refer any good book on Bhagavat Gita like [1] for slokas from Bhagavat Gita with translation. For Mathematical terms and definitions not explicitly defined here, one may refer Fraleigh [2].

2. MAIN RESULTS:

Let F be the set of all Human Resources of an individual (Soft skills, Outer and Inner Human Resources) and the Addition operation, denoted by +, (Professional Aptitude) and the Multiplication operation, denoted by x, (Professional Attitude) be two operations defined on F. While the Aptitude adjoins two Human resources to an enhanced blended Human resource/skill, the attitude orients two Human resources towards a standard virtuous Human resource/skill. In this section, we see that the Science part of Bhagavat Gita (2.40-15.20) explain the elven axioms of an algebraic field, (F, +, x).

The Mathematical model of the Bhagavat Gita is an Algebraic Field of Human Resources and it identifies a human being as an algebraic field of resources (Soft skills, Inner and Outer human resources) under the addition (Professional Aptitude) and the multiplication (Professional Attitude) operations. In this section, we will investigate Human Resources Management (HRM) principles mentioned in the Bhagavat Gita and show that their Mathematical formulations are the properties of an algebraic field.

2.1. Binary Addition (Karma-Yoga):

The second chapter, namely *Sankhya Yoga*, explains the absolute knowledge about the Cosmic Intellect, which is the supreme personality of godhead (as personified as Lord Krishna) and the need of an individual to control the senses, the mind and the intellect (such persons are called *SthitaPrajna*) to realize this knowledge in one's actions/decisions (2.40-2.72). This will lead to an enhanced Professional aptitude for one's actions/decisions, called as Karma-Yoga, whose nature is further explained in the third chapter, namely *Karma Yoga*, as the Right Professional Aptitude for one's actions/decisions.

Karma-Yoga is an Aptitude and is a way of action, thoughts, and willingness for individuals to orient them towards the realization of the right Professional Aptitude by which an individual can execute the actions/decisions without selfish motives but aiming only at the excellence, which is the supreme personality of Godhead, as Lord Krishna himself told in the tenth chapter (10.41):

Yadyat-Vibhutimad sattvam SreemadUrjithameva va

Tattadevavagacah tvam mama tejoAmsasambhavam

"Anything, prosperous and excellent in this universe, is a part of the cosmic intellect". Since inaction is the death, everyone has to act according to one's nature to live in this world. Not only the acts of thinking or talking, but all kinds of physical works are also included in Karma and only right action can help us to continue our life cycle. Karma as divine service is only a beginning and through such actions we can also reach the highest goal of human life [3].

The second chapter, namely *Sankhya-Yoga*, explains the nature of Consistent Cosmic Intellect (*Vyavasayatmika Biddhih*) (2.40-2.48) of an individual as the pre-requisite for the right Karma-Yoga. With this human resource quality an individual should have a Consistent mind (*Sthitha Prajnan*) (2.49-1.56), Consistent Senses-orientation (*Sthitha-Prajnan*) (2.57-2.64) and Consistent Intelligence (*Brahma-Nishta*) (2.65-2.72). When an individual can adjoin this cosmic knowledge with every actions/decisions, then that is defined as a *Karma Yoga* (Professional Aptitude) and this is our first Mathematical operation (addition) defined on the set F. The *Karma Yoga* (Professional Aptitude) of two human resources is another enhanced human resource and thus it is also an element of the set F. Hence the first axiom of an algebraic Field is satisfied.

That the above binary operation is mathematically well defined is explained in the third chapter, namely *Karma-Yoga*. The modus operandi of the *Karma Yoga* (Professional Aptitude) such as the nature of Professional actions-*Karma* (3.01-3.08), the need of Professional action to be with selfless motives resulting them as a service to Humanity (3.09-3.16), the inevitability of actions for individuals (3.17-3.24), the nature of the right Aptitude of actions (3.25-3.32) and the need of eliminating the greed and anger (3.33-3.43) for *Karma Yoga* (Professional Aptitude) are well explained in the third chapter.

2.2. Additive Associativity (*Jnana-Karma-Sanyasa yoga*):

The associativity property of the operation, *Karma-Yoga* is defined in the 4th chapter, namely *Jnana-Karma-Sanyasa yoga*, as the action of dealing multiple human resources with a sense of Knowledge of Renunciation (*Tyaga*). When an action requires more than two human resources, an individual of Karma-Yoga can create the right professional aptitude by adjoining them in any order. This defines the Mathematical property of associativity of human resources.

This continuous human resource quality improvement strategy is the pioneer notion of the modern continuous Quality Improvement Technology as popularized by the Japanese scientist, Ishikava [4], who proposed quality control techniques such as Quality circles, Pareto charts, Ishikawa diagrams etc. Thus, associativity of human resources means a continuous process of quality improvement of human resources for better professional action as explained below:

Yadayadahi dharmasya glanir bhavati bharathah

abhyuthanamAdharmasya Tadatmanam srujamyaham

paritranaya Sadhunam Vinasaya ca dushkritam

dharma samsthapanarthaya sambhavami yuge yuge (4.07-4.08)

"Whenever there is a decline in the quality (dharma) and wherever bad practices arise, the cosmic intellect of individuals will be stimulated (*Tatatmanam srjamaham*) and to retain the good practices and to eliminate the bad practices, our cosmic intellect works continuously from time to time (*sambhavami yuge yuge*)".

2.3. Additive Identity (Sanyasa-yoga):

The Sanyasa is a human resource, whose nature is explained in the 5th chapter, namely *Sanyasa-yoga*, and it is formally defined as the "Non-fruititive Aptitude towards actions/decisions". If we adjoin this with any other human resource in any action, the later human resource will stand as it is, since Sanyasa is non-fruititive nature. Thus this human resource, Sanyasa, is the Mathematical identity element of the algebraic field, F.

The nature and development of this human resource, Sanyasa is explained in the 5th chapter, namely *Sanyasa-yoga*. It is clearly stated that the *Karma-yoga* is the same as the *Jnana-yoga* (5.5). The nature of a true *Karma-yogi* (5.1-5.8) and the means of attaining it by *Jnana-yoga*, which is the absolute knowledge of the cosmic intellect (5.9-5.16) are explained. Further, it is explained that when a *Karma-yogi* becomes a *Jnana-yogi*, the individual can feel the goodness within himself thus becoming the real human being (5.17-5.24) and ultimately he discovers the knowledge of *Atma* and *Brahma*, thereby acquiring the rare bliss (*Brahmananda*) and peace (*Santih*) (5.25-5.30).

2.4. Additive Inverse (Nishkama Karma-Yoga):

Renunciation in any action (*Nishkama Karma-Yoga*) means discarding the benefits of the actions thereby concentrating on the quality of the way of doing the action. It can be developed in individuals by stimulating the Inner Human Resources, whose nature is explained in the 6th chapter, namely *Atma-Samyama Yoga*.

The nature of *Nishkama Karma-Yogi* is explained in the 6th chapter, namely *Atma-Samyama yoga*. It is explained that such an individual is with constantly controlled senses and mind in actions (6.01-6.08), with constantly controlled intellect in actions (6.09-6.15) and with constantly controlled thoughts and resources in actions (6.16-6.24). Further, it is described how *Nishkama Karma-Yoga* can be attained by an individual with non-constant mind (6.25-6.32), but through another human resource, namely, concentration (*sraddha*) (6.33-6.40) and the chapter concludes with a description of the nature of the individuals, who do not attain *Nishkama Karma-Yoga* and the fruitive results of the excellence of being a *Nishkama Karma-Yogi* (6.41-6.47). Thus, the nature of *Nishkama Karma-Yoga* is that it transforms the actions into non-fruititive actions, so that when it adjoins with any human resource, it will result in the Sanyasa human resource. This is the Mathematical definition of the additive inverse of human resources.

2.5. Additive Commutativity (*Jnana-Vigjnna Yoga*):

The 7th chapter, namely *Jnan Vigjnna Yoga*, begins with a description of the material structure (*Prakriti*) of the cosmic intellect of an individual (7.01-7.08) and then explains the material human resource qualities like the vision of equality, the three qualitative divisions of human resources, namely Virtuous (*Satva*), Passionate (*Rajas*) and Ignorant (*Tamas*), which help individuals to control their actions to attain the cosmic intellect (7.09-7.16).

It is further made clear that the absolute cosmic intellect (the supreme personality of Godhead) is inhuman nature and can be attained by any model of individual choice (7.17-7.24), which is in fact negates the modern models of religions, which are highly orthodox in their ideas and rituals and concludes with explaining the effects of realizing the cosmic intellect (7.25-7.30). Thus in the light of absolute cosmic intellect, the human resources can be adjoined to create a professional aptitude for one's actions/decisions and the order of integrating them into a *Karma-Yogi* does not matter, which is the Mathematical definition of Additive Commutativity.

2.6. Binary Multiplication (*Bhakti-Yoga*):

The 10th chapter, namely *Vibhuti-Yoga*, begins with description of the supreme personality of Godhead, as the absolute Cosmic Intellect of an individual that it is omnipresent and prior to all divine or human endeavors and is unknown to all. Arjuna requests Lord Krishna to explain it (10.17-10.18).

Katham vidyamaham yogim stvam sada parichinthayan

Keshukeshu ca bhaveshu chintyoAsi bhagavan maya?

Vistareatmano yogam vibhutim ca Janardhana

Bhuyah Kathaya triptirhi srunato nasti meAmritam

"O Lord Krishna (*Janardhana*), How should I meditate (*parichinthayan*) to know you? What are the forms for me to understand (*chintyo*) you? Tell me in details about your synthesis (*yoga*) and analysis (*vibhuti*). I am eager to hear the words, which are like nectar (*amrit*) for me". Here, we see that Arjuna's enquiry is similar to the modern educational psychological notion of Bloom's taxonomy levels of the cognizant domain of any learner. Arjuna asks four questions about the absolute Cosmic Intellect: (1) How should I know you? (Knowledge level), (2) How should I understand you? (Understanding level), (3) How can I know your yoga? (Analysis level) and (4) How can I know your prosperity? (Synthesis level)

The Mathematical answer to all these four questions is a single abstract one (10.41) that the supreme personality of Godhead, the absolute Cosmic Intellect of an individual, is the excellence or prosperity in any actions and this information is the pre-requisite for the *Bhakti-Yoga* (Professional Attitude). This quality human resource inculcates in the individuals an attitude which is suitable for the best utilization of the human resources of individuals.

In the 11th chapter, it is explained that mere human resources cannot attain the absolute cosmic intellect and need some special vision (11.01-11.18), and that the cosmic intellect is the Time, which annihilates all Human resources (10.31-41), and explains the state of mind of devotion (11.42-11.53) and concludes that the excellence (supreme cosmic intellect) can be realized in an individual's action only with devotional service mentality (11.54-11.55). This is defined as the *Bhakti-Yoga* (Professional Attitude) for Human resources. This is our second Mathematical multiplicative operation on Human resources.

The nature of the Bhakti-Yoga (Professional Attitude) is explained in the 12th chapter, namely *Bhakti-Yoga*, where the right Attitude towards one's actions/decisions is explained and that in turn shows that this Mathematical operation is well defined. *Bhakti-yoga* (Attitude) is that while using human resources towards an action, the individuals should condition their attitude by orienting their mind towards the cosmic intellect (excellence in that action) and be a constant minded (12.01-12.08). Thus, Bhakti-Yoga is a technique to condition oneself to fix the mind in the cosmic intellect for a Karma-Yogi, so that it has a multiplicative effect on the Karma-Yoga.

It is further explained that in case an individual is not capable of having such an attitude, then that person should condition his mind, intellect and body through yoga exercises (12.09). If even that is not possible, that person should dedicating all his actions to the cosmic intellect (excellence in that action) (12.10).If even that is also not possible, then hat person should simply surrender all his actions to the cosmic intellect (excellence in that action) (12.11). For such an attitude, individuals may adopt a concrete model of the cosmic intellect such as human form (Lord Krishna) because the abstract model is difficult for human beings to attain with this attitude (12.05):

kleshoAdhikatharaS thesham Avyakta sakta chetasam

avyakta hi gatiR dukham dehavadbhirApapyate

"It will be miserable for those who try to keep their concentration fixed in abstract form of the cosmic intellect, as it is almost impossible for the human individuals to fix their concentration (*Nishta*) in the abstract form of the cosmic intellect (*Brahma*)". Prabhupada [1] named this aspect of the human personality as "Krishna Consciousness" and he devoted his entire life for the practice and propagation of it. A Karma yogi doing actions with this attitude can succeed in it very quickly, as the former is an attempt to connect all actions to the cosmic intellect (12.12-12.20). Hence the *Bhakti yoga* has the multiplying effect on the human resources of individuals and mathematically it says that *Bhakti yoga* is the multiplication operation on human resources.

2.7. Multiplicative Associativity (*Kshtra-Kshetranja Yoga*)

The 13th chapter, namely *Kshtra-Kshetranja Yoga*, explains the relation between the knowledge and the structure of knowledge. *Kshetra* means the body of individuals whereas *Kshetranja* means that knowledge of the body. The relation between the two is explained (13.01-13.12) and it is further modeled as the relation between the senses (*Indriya*) and sensory knowledge (13.13-13.19), the mind (*Chittam*) and the mental knowledge (13.20-13.24) as well as the intellect (*Buddhi*) and intellectual knowledge (13.25-13.25) about the cosmic intellect (Supreme personality of Godhead) of individual. But the professional attitude of individuals in human resources is related to the senses, mind or intellect. Hence, it shows that the professional attitude of individuals for three human resources for an action does not depend on the order of the resources. Mathematically, this fact is called the multiplicative associativity.

The 13th chapter, namely *Kshtra-Kshetranja Yoga*, explains the relation between the knowledge and the structure of knowledge. *Kshetra* means the body of individuals whereas *Kshetranja* means that absolute cosmic knowledge of the body. The relation between the two is explained (13.01-13.12) and it is further modeled as the relation between the senses (*Indriya*) and the sensory knowledge (13.13-13.19), the mind (*Chittam*) and the mental knowledge (13.20-13.24) as well as the intellect (*Buddhi*) and the intellectual knowledge (13.25-13.25) about the cosmic intellect (Supreme personality of Godhead) of the individual. Thus all these types of experience are of the same structure. But the professional attitude of individuals is related to the senses, mind or intellect. Hence, the *Bhakti Yoga* (Professional attitude) of human resources can be adjoined in any order of the resources. Mathematically, it is called the associativity property of the binary operation, *Bhakti Yoga*.

2.8. Multiplicative Identity Property (*Guna Traya Vibhaga Yoga*):

The 14th chapter explains the 3-tier classification (*Guna traya vibhaga yoga*) of the attitude of individuals based on the personal taste and instinct (*Guna*). The three classifications of the attitude are Virtuous (*Satvik*), Passionate (*Rajas*) and Ignorant (*Tamas*). The Virtuous (Satvik) Personality (attitude) is explained as the best among them and a person can identify the nature and orientation of one's attitude to actions/decisions by knowing the effects of these three quality types in individuals (14.05-14.18). These three quality types bind the individuals within the material body and distracting them from realizing the cosmic intellect within them so that individuals should think and understand beyond these types of quality of human resources to realize the cosmic intellect (14.19-14.20). The chapter concludes with the explanation of the nature and advantages of the persons, who could surpass the qualitative divisions of human resources (*GunaAtheethan*) (14.22-14.27). This attitude has the property that when it adjoins to any human resource, that human resource does not change. Mathematically, this is called the Multiplicative identity element of the binary operation, *Bhakti Yoga*.

2.9. Multiplicative Inverse Property (*Purushottama Yoga*):

The 15th chapter begins with a structural explanation of the cosmic intellect (15.01-15.06) and explains the knowledge and the nature of the cosmic intellect within the individuals (15.07-15.19), thereby imparting a significant attitude for the Human beings towards the realization of the cosmic intellect within them. Upon the realization of the cosmic intellect within the individuals, any human resource is beyond all types of quality divisions. That is, it becomes another human resource which negates all qualitative attitudes of individuals and results in a state called as *GunaAtheethan*, which was the identity element of the binary operation, *Bhakti Yoga*. Hence, the latter is called the inverse of the former human resource.

2.10. Multiplicative Commutativity (*Akshara Brahma yoga*)

The 8th chapter, namely *Akshara Brahma Yoga*, describes the most comprehensive knowledge about the Cosmic Intellect. It begins with description of physical human resources to attain the cosmic intellect (8.01-8.08) and explains Pranayama breathing exercises and Brahmacharya lifestyle practices for stimulating cosmic intellect in oneself (8.09-8.16). Further, it reminds the great nature of cosmic intellect (8.17-8.24) and concludes with the significance of constant mind (*ekagra chittan*) towards the cosmic intellect (8.25-8.28). Thus in the *Bhakta-Yoga,* the human resources can be integrated to create a professional attitude for one's actions/decisions, where the order of the resources does not matter. This is precisely the Mathematical definition of multiplicative Commutativity.

2.11. Distributivity Property (*Raja Guhya Yoga*)

The 9th chapter, namely *Raja Guhya Yoga*, describes the knowledge of Cognizant Systems, there we can see the interplay between the *Karma-yoga* and the *Bhakti-yoga*. As both of them depends and demands the cosmic intellect, the chapter contains the detailed explanation of the cosmic intellect: the cosmic intellect contains all beings interior to it (9.01-9.08), the cosmic intellect is the cause for the creation of all individuals (9.09-9.16) and that the cosmic intellect determines the yoga of the dedicated devotees (9.17-9.24) and the cosmic intellect can be attained only by maintaining a constant mind (*ekagra chiittan*) (9.25-9.34). In (9.22), it is made clear that:

Ananyaschintayanto mam ye janah paryupasate
Thesham nithyabhiyuktanam yogakshemam vahamyaham

"I am directly controlling the yoga of the people with controlled senses; mind and intellect and who always think of me, always remember me in all actions". So, Bhakti-yoga of controlled individuals (Professionals) with Karma-yoga is controlled by the cosmic intellect, which means that Bhakti-yoga determines and multiplies the Karma-yoga of the controlled individuals (Professionals). Mathematically, it is called the Distributivity property, which states that *Bhakti-yoga* of the *Karma-yoga* of two human resources is the *Karma-yoga* of the *Bhakti-yoga* of the respective human resources.

Thus, (F, +, x) satisfies all the eleven properties of an algebraic field and hence it is a Field. The Mathematical formulation of the Human Resource Management principles mentioned in the Bhagavat Gita is an active research area that can extend this work. Some of these like the Bloom's Taxonomy levels of the cognizant domain of the learners and the continuous quality improvement principle of Ishikava were discussed here. On the other way, we can consider the abstract algebraic Field theory concepts such as Extension Fields as applied to the Human resources to investigate the resulting HRM principles which will be the extensions or variations of the HRM principles mentioned in the Bhagavat Gita.

REFERENCES:

[1] Prabhupada, Bhagavad Bhagavat Gita as it is, Bhakti-Vedanta Book Trust, Mumbai, India, 1981.

[2] J.B. Fraleigh, A first course in abstract algebra (Seventh Edition), Pearson, 2013.

[3] Bimal Chandra Gogoi, Karmayoga of Bhagavad Gita - A Philosophical Analysis, International Journal of Humanities, Arts, Medicine and Sciences, Vol. 2, Issue 10, Oct 2014, 71-74.

[4] Ishikawa Kaoru, Guide to Quality Control. Asian Productivity Organization, 1976.

[5] Suresh Kumar. J, Algebraic Modeling of Bhagavat Gita: Group of Human Resources for Aptitude, International Journal for Creative Research Thoughts (IJCRT), April, 2020.

[6] Suresh Kumar.J, Algebraic Modeling of Bhagavat Gita: Group of Human Resources for Attitude, International Journal for Research in Applied Science and Engineering Technology, April, 2020.