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A REPAIRABLE RETRIAL $M^X/G/1$ QUEUEING SYSTEM UNDER MULTIPLE ADAPTED VACATION POLICY AND BERNOULLI SCHEDULE VACATION

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Abstract: The present paper deals with the batch arrival single server retrial queueing system in which the arrivals occur in batches following Poisson process and service time is generally distributed. It is assumed that the server takes vacations during idle period according to the Multiple Adapted Vacation (MAV) policy and single vacation between two consecutive services during busy period. Unexpected interruptions during service time are also considered and the interrupted service will be resumed from the point of interruption as soon as the system is fixed. The Markovian structure of the model is obtained by introducing the remaining time of service time, vacation time and repair time as supplementary variables. The queue size probabilities and mean queue lengths when the system is in different states are calculated. Since the MAV policy generalizes many other vacation policies, the steady-state results of various vacation queueing models including non-vacation case are deduced by establishing the stochastic decomposition property for vacation queues.

***Index Terms* – Supplementary variables, Multiple Adapted Vacation policy, Bernoulli Schedule vacation, breakdown.**

I. INTRODUCTION

Server vacation model was first discussed by Levy and Yechiale (1975). As far as the vacation queueing models existing in the literature are concerned, various authors analysed the vacation queueing models by considering different types of vacation as independent characteristics. Baba (1986) considered the batch arrival queue with multiple vacations. Aissani (1998) discussed the retrial $M^X/G/1$ queueing models with exhaustive vacations. Later, Choudhury (2002) modelled batch arrival queueing system with a single vacation. Multiple vacation retrial models were analysed by Krishna Kumar and Pavai Madheswari (2003). A new vacation policy for $M^X/G/1$ queueing system where the server may leave for at most J vacations was proposed by Ke and Chu (2006) and Ke et al. (2010). Mytalas and Zazanis (2015) considered a more general MAV policy controlled by a sequence of probabilities. Queueing model under MAV policy combines various idle vacation types into a single model so that the results for the other vacation type queueing models, including the non-vacation type can be deduced from the single model. Keilson and Servi (1986) introduced the vacation policy between services in which after the completion of a service to a customer, the server may take a vacation with probability p or continue to serve the next customer with the probability (1-p). Nawel ARRAR et al (2017) explained the decomposition property for retrial single server vacation queueing model.

Gaver (1962) seems to be the first to study the effect of service interruptions on the distribution of busy period, queue length and waiting time for $M^X/G/1$ queueing model. The breakdowns are generally assumed to occur only when the server is busy and are considered to be independent of each other. Keilson (1962), Yue and Tu (2001) and many others have contributed a lot to the queueing models with server breakdown. Fiems et al. (2008) fixed the probability for repeat / resumption of service whereas Krishnamoorthy et al. (2009) provided specific rule to decide whether to repeat / resume an interrupted service. The most recent works on queueing models with interruptions may be found in the survey paper of Krishnamoorthy et al. (2012).

II. MATHEMATICAL ANALYSIS OF THE SYSTEM

2.1 Model Description

Arrival Pattern: Customers arrive in batches at the system according to a time homogeneous Poisson process with group arrival rate λ. The batch size **X** is a random variable with probability distribution $\Pr(X = k) = g_k$, $k = 1,2,\ldots$ and $\sum_{k=1}^{\infty} g_k = 1$. There is no waiting space in front of the server. When the server is idle, the customer at the head of the pre-ordered arriving batch, turns on the service immediately and the other customers of the batch leave the service area and enter into the orbit according to **FCFS** discipline. On completion of the service, the customer at the head of the retrial queue competes with potential primary customers to decide which customer will enter the next service. If a batch of primary customers arrives first, the retrial customer will cancel its attempt for service and returns to its position. The retrial time (A) of a customer in the orbit is generally distributed with distribution function **A(t)**, density function **a(t)** . Further it is assumed that the retrial times begin only when the server is freely available in the system.

Multiple Adapted Vacation (MAV) Policy: A cycle starts whenever the system becomes empty and the server is deactivated. The deactivated server either remains idle in the system with probability $\mathbf{1-\beta_0}$ or takes a (first) vacation with probability $\boldsymbol{\beta_0}$. Upon returning from each vacation **j** (j = 1,2,…), if the server finds at least one customer waiting in the orbit, then he immediately joins the system and waits for the retrial of the customer. Otherwise if there is no customer found waiting in the queue, then the server either joins the system with probability $\mathbf{1-\beta_j}$ or takes the next vacation with probability $\boldsymbol{\beta_j}$. This process continues until the queue becomes non-empty and the server starts a new busy cycle. The time during which the server is either on vacation or idle in the system is called idle period.

Busy and Breakdown Period: Busy period starts at the end of each idle period. During busy period, the server provides service to the customers one at a time. The server may breakdown at any time while servicing the customers, according to the Poisson process with rate α and sent for repair immediately. The service channels will not function for a short interval of time. The interrupted service of the customer is resumed as soon as the system is fixed. The service times (S) and repair times (R) of the server are assumed to be independent identically distributed random variables having distributions **S(t)** and **R(t)** and density functions **s(t)** and **r(t)** respectively.

Bernoulli Schedule Vacation: After the completion of each service, the server may take a Bernoulli schedule vacation (VB) with the probability **p** before starting the next service or continue to serve the next customer with probability **(1-p)**. The server can take at most one vacation between two services. The vacation times **(VI)** during idle period and vacation time between services **(VB)** are assumed to be independent identically distributed random variables with corresponding distributions **VI(t)** and **VB(t)** and density functions **vI(t)** and **vB(t)**. This model is denoted by $\mathbf{M^X/G/1/MAV/breakdown/BSV}$.

If f(x) is the density function of the probability distribution F(x), then the LST is $F^*(\theta) = \int_0^\infty e^{-\theta x}\, d(F(x))$. The LST of the random variables A, VI, VB, S and R are denoted by $A^*(\theta)$, $VI^*(\theta)$, $VB^*(\theta)$, $S^*(\theta)$ and $R^*(\theta)$ respectively. The system is analysed using Supplementary variable technique, by introducing the remaining times of the random variables as supplementary variables.

Let $A^0(t)$, $VI^0(t)$, $VB^0(t)$, $S^0(t)$ and $R^0(t)$ denote the remaining times of the random variables namely retrial time, idle vacation time, busy vacation time, service time and repair time at time t respectively. Further different states of the server at time t are denoted by Y(t) = {0,1,2,3,4} which respectively denotes idle state, vacation state during idle & busy period, busy state and breakdown state. The supplementary variables are introduced in order to obtain a bivariate Markov process $\{N(t), \delta(t)\}$ where N(t) denotes the queue size random variable and $\delta(t) = (A^0(t), VI^0(t), VB^0(t), S^0(t), R^0(t))$ according as Y(t) = (0,1,2,3,4) respectively.

Let $PI_n(w,t)dt = \Pr\{N(t) = n, w \le A^0(t) \le w + dt, Y(t) = 0, n \ge 1\}$ be the joint probability that at time t, there are n customers in the retrial orbit, the server is idle and the remaining retrial time of the server is between w and w + dt, where $n \ge 1$ and $PI_0(t) = \Pr\{N(t) = 0, Y(t) = 0\}$ be the probability that the server is idle at time t, and there is no customer in the retrial orbit.

Let $QI_{n,j}(x,t)dt = \Pr\{N(t) = n, x \le VI^0(t) \le x + dt, Y(t) = 1, n \ge 0\}$ be the joint probability that at time t, there are n customers in the retrial orbit, the server is in j^{th} vacation and the remaining vacation time of the server is between x and x + dt, where $n \ge 0$.

Let $QB_n(x,t)dt = \Pr\{N(t) = n, x \le VB^0(t) \le x + dt, Y(t) = 2, n \ge 1\}$ be the joint probability that at time t, there are n customers in the retrial orbit, the server is in busy vacation and the remaining vacation time of the server is between x and x + dt, where $n \ge 1$.

Let $P_n(x,t)dt = \Pr\{N(t) = n, x \le S^0(t) \le x + dt, Y(t) = 3, n \ge 1\}$ be the joint probability that at time t, there are n customers in the retrial orbit, the server is busy, a customer is being served in service and the remaining service time lies between x and x + dt, where $n \ge 1$.

Let $B_n(x,y,t)dt = \Pr\{N(t) = n, S^0(t) = x, y \le R^0(t) \le y + dt, Y(t) = 4, n \ge 1\}$ be the joint probability that at time t, there are n customers in the retrial orbit, the server is under repair, the remaining service time of the server is equal to x and the server is being repaired with the remaining repair time between y and y + dt, where $n \ge 0$.

Let $E(Y^k)$ (k=1,2…) denote the k^{th} moment of the random variable Y.

Assuming that the steady state probabilities exist, we have as $t \to \infty$, the probabilities are independent of time t. Let $PI_n(w)$, $P_n(x)$, $B_n(x,y)$, $QB_n(x)$ and $QI_{n,j}(x)$ respectively denote that the server is idle, busy, under repair, on vacation during busy & idle period at steady-state .Let $PI_n(\theta)$, $P_n(\theta)$, $B_n(\theta,\theta')$, $QB_n(\theta)$ and $QI_{n,j}(\theta)$ denote LST of the corresponding probabilities. Thus the LST of the Steady State Queue Size Equations are obtained for the model**.** Various stochastic processes involved in the queueing system are assumed to be independent of each other. Using Supplementary Variable Technique, the equations under the steady state condition are analyzed and the PGF of the queue size is also obtained. The following partial generating functions are introduced to analyse the model.

$PI^*(z,\theta) = \sum_{n=1}^{\infty} PI_n^*(\theta)\, z^n$ $\qquad PI(z,0) = \sum_{n=1}^{\infty} PI_n(0)\, z^n$

$P^*(z,\theta) = \sum_{n=0}^{\infty} P_n^*(\theta)\, z^n$ $\qquad P(z,0) = \sum_{n=0}^{\infty} P_n(0)\, z^n$

$QI_j^*(z,\theta) = \sum_{n=0}^{\infty} QI_{n,j}^*(\theta)\, z^n$ $\qquad QI_j(z,0) = \sum_{n=0}^{\infty} QI_{n,j}(0)\, z^n \qquad j \geq 1$

$QB^*(z,\theta) = \sum_{n=1}^{\infty} QB_n^*(\theta)\, z^n$ $\qquad QB(z,0) = \sum_{n=1}^{\infty} QB_n(0)\, z^n$

$B^{**\prime}(z,\theta,\theta') = \sum_{n=0}^{\infty} B_n^{**\prime}(\theta,\theta')\, z^n$ $\qquad B^*(z,0,0) = \sum_{n=0}^{\infty} B_n^*(0,0)\, z^n$

2.2 LST of the Steady State Queue Size Equations

$$\lambda\, PI_0 = \sum_{j=1}^{\infty}(1-\beta_j)\, QI_{0,j}(0) + P_0(0)\,(1-\beta_0) \qquad (2.1)$$

$$\theta PI_n^*(\theta) - PI_n(0) = \lambda PI_n^*(\theta) - \sum_{j=1}^{\infty} QI_{n,j}(0)\, A^*(\theta) - (1 - p)\, P_n(0)\, A^*(\theta) - QB_n(0)\, A^*(\theta), \quad n \geq 1 \qquad (2.2)$$

$$\theta P_0^*(\theta) - P_0(0) = (\lambda + \alpha)\, P_0^*(\theta) - PI_1(0)\, S^*(\theta) - \lambda g_1\, PI_0 S^*(\theta) - B_0^*(\theta,0) \qquad (2.3)$$

$$\theta P_n^*(\theta) - P_n(0) = (\lambda + \alpha)\, P_n^*(\theta) - PI_{n+1}(0)\, S^*(\theta) - \lambda S^*(\theta) \int_0^{\infty} \sum_{k=1}^{n} PI_{n-k+1}(w)\, dw\, g_k - \lambda \sum_{k=1}^{n} P_{n-k}^*(\theta)\, g_k$$
$$- \lambda g_{n+1}\, PI_0\, S^*(\theta) - B_n^*(\theta,0), \qquad n \geq 1 \qquad (2.4)$$

$$\theta QI_{0,1}^*(\theta) - QI_{0,1}(0) = \lambda QI_{0,1}^*(\theta) - \beta_0\, P_0(0)\, VI^*(\theta) \qquad (2.5)$$

$$\theta QI_{0,j}^*(\theta) - QI_{0,j}(0) = \lambda QI_{0,j}^*(\theta) - \beta_{j-1}\, QI_{0,j-1}(0)\, VI^*(\theta), \qquad j \geq 2 \qquad (2.6)$$

$$\theta QI_{n,j}^*(\theta) - QI_{n,j}(0) = \lambda QI_{n,j}^*(\theta) - \lambda \sum_{k=1}^{n} QI_{n-k,j}^*(\theta)\, g_k, \qquad n \geq 1,\ j \geq 1 \qquad (2.7)$$

$$\theta QB_1^*(\theta) - QB_1(0) = \lambda QB_1^*(\theta) - p\, P_1(0)\, VB^*(\theta) \qquad (2.8)$$

$$\theta QB_n^*(\theta) - QB_n(0) = \lambda QB_n^*(\theta) - p\, P_n(0)\, VB^*(\theta) - \lambda \sum_{k=1}^{n} QB_{n-k}^*(\theta)\, g_k, \qquad n \geq 2 \qquad (2.9)$$

$$\theta' B_0^{**\prime}(\theta,\theta') - B_0^*(\theta,0) = \lambda B_0^{**\prime}(\theta,\theta') - \alpha P_0^*(\theta) R^{*\prime}(\theta') \qquad (2.10)$$

$$\theta' B_n^{**\prime}(\theta,\theta') - B_n^*(\theta,0) = \lambda B_n^{**\prime}(\theta,\theta') - \lambda \sum_{k=1}^{n} B_{n-k}^{**\prime}(\theta,\theta')\, g_k - \alpha P_n^*(\theta) R^{*\prime}(\theta'), \qquad n \geq 1 \qquad (2.11)$$

2.3 Probability Generating Functions

The partial probability generating functions of the queue size probabilities at arbitrary epoch when the server is in different states obtained in terms of $P_0(0)$ are obtained through algebraic operations and are listed below:

$$B^{**\prime}(z,0,0) = \alpha P_0(0)\, Q(z)\, \frac{1-S^*(h_\alpha(w_x(z)))}{h_\alpha(w_x(z))}\, \frac{1 - R^{*\prime}(w_x(z))}{w_x(z)} \qquad (2.12)$$

$$QB^*(z,0) = p\, P_0(0)(Q(z)\, S^*(h_\alpha(w_x(z))) - 1)\frac{1 - VB^*(w_x(z))}{w_x(z)} \qquad (2.13)$$

$$QI^*(z,0) = P_0(0) \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right) \frac{1-VI^*(w_x(z))}{w_x(z)} \qquad (2.14)$$

$$PI^*(z,0) = P_0(0)\, Y(z)\, \frac{1-A^*(\lambda)}{\lambda} \qquad (2.15)$$

$$P^*(z,0) = P_0(0)\, Q(z)\frac{1-S^*(h_\alpha(w_x(z)))}{h_\alpha(w_x(z))} \qquad (2.16)$$

where $w_x(z) = \lambda\,(1- X(z))$

$VI^*(w_x(z)) = \sum_{n=0}^{\infty} \alpha_n z^n$; $\alpha_n = \int_0^{\infty} e^{-\lambda t} \sum_{i=0}^{\infty} \frac{(\lambda t)^i g_n^{(i)}}{i!} d(vI(t))$ [where $g_n^{(i)}$ = Pr{n customers in orbit at the end of i^{th} batch arrival}]

$$\varphi = \sum_{j=1}^{\infty}(1-\beta_j)\alpha_0^j \left(\prod_{i=0}^{j-1} \beta_i\right) + (1-\beta_0) \qquad (2.17)$$

$$I_0(z) = \varphi + \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right)(1 - VI^*(w_x(z))) + p\,(VB^*(w_x(z)) - 1) \qquad (2.18)$$

$$M_1(z) = A^*(\lambda) + X(z)\,(1 - A^*(\lambda)) \qquad (2.19)$$

$h_\alpha(w_x(z)) = w_x(z) + \alpha(1 - R^{*\prime}(w_x(z)))$

$$HBV^*(w_x(z)) = S^*(h_\alpha(w_x(z)))\, M_1(z)\, (1 - p + p\, VB^*(w_x(z))) \quad (2.20)$$

$$Q(z) = \frac{-w_x(z)}{z - HBV^*(w_x(z))}\, IVR(z) \quad (2.21)$$

$$IVR(z) = A^*(\lambda)\,\frac{\varphi}{\lambda} + M_1(z)\, IV_0(z) \quad (2.22)$$

$$IV_0(z) = \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right) \frac{1 - VI^*(w_x(z))}{w_x(z)} + p\,\frac{VB^*(w_x(z)) - 1}{w_x(z)} \quad (2.23)$$

$$Y(z) = S^*(h_\alpha(w_x(z)))Q(z)(1 - p + p\, VB^*(w_x(z))) - I_0(z) \quad (2.24)$$

The probability generating function $P_{comp}(z)$ when the system is either in busy state or in breakdown state is

$$P_{comp}(z) = P^*(z,0) + B^{**\prime}(z,\theta,0) = P_0(0)Q(z)\frac{1 - S^*(h_\alpha(w_x(z)))}{w_x(z)} \quad (2.25)$$

The probability generating function $P_{idle}(z)$ when the server is idle is given by

$$P_{idle}(z) = QB^*(z,0) + QI^*(z,0) + PI^*(z,0) + PI_0 = P_0(0)\,\frac{Q(z)}{w_x(z)}\,[S^*(h_\alpha(w_x(z)) - z] \quad (2.26)$$

The total probability generating function PBR(z) of the queue size probabilities of the model is given by,

$$PBR(z) = P^*(z,0) + B^{**}(z,0,0) + QB^*(z,0) + QI^*(z,0) + PI^*(z,0) + PI_0 = P_0(0)\, Q(z)\,\frac{1 - z}{w_x(z)} \quad (2.27)$$

III. PERFORMANCE MEASURES

The steady state queue size probabilities when the system is in different states and the corresponding mean queue length are calculated for the proposed model. The following results obtained from (2.18) to (2.24) are used to derive these measures.

$I_0(1) = \varphi$; $\quad I_0'(1) = \lambda\, E(X)[p\, E(VB) - \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right) E(VI)]$

$M_1(1) = 1$; $\quad M_1'(1) = E(X)\,(1 - A^*(\lambda))$; $\quad M_1''(1) = E(X(X-1))\,(1 - A^*(\lambda))$

$[HBV^*(w_x(z))]\,|_{z=1} = 1$; $\quad [HBV^*(w_x(z))]'|_{z=1} = \lambda\, E(X)[E(H) + \frac{1 - A^*(\lambda)}{\lambda} + p\, E(VB)] = \rho$;

$[HBV^*(w_x(z))]''|_{z=1} = \lambda\, E(X(X-1))\, E(HBV) + (\lambda\, E(X))^2\, E(HBV^2)$

where $E(HBV) = E(H) + \frac{1 - A^*(\lambda)}{\lambda} + p\, E(VB)$

and $E(HBV^2) = \alpha\, E(S)\, E(R^2) + E(S^2)\,(1 + \alpha\, E(R))^2 + 2p\, E(H)\, E(VB) + p\, E(VB^2) + 2\,\frac{1 - A^*(\lambda)}{\lambda}(E(H) + p\, E(VB))$

with $E(H) = E(S)\,(1 + \alpha\, E(R))$

$Q(1) = \frac{\lambda\, E(X)}{1 - \rho} IVR(1)$; $\quad Q'(1) = \frac{\lambda}{1 - \rho}[IVR'(1)\, E(X) + \frac{IVR(1)}{2}(E(X(X-1)) + \frac{E(X)}{1 - \rho}(\lambda\, E(X(X-1))\, E(HBV) + (\lambda\, E(X))^2\, E(HBV^2)))]$

$IVR(1) = A^*(\lambda)\,\frac{\varphi}{\lambda} + \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right) E(VI) - p\, E(VB)$;

$IVR'(1) = E(X)\, [\sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right)((1 - A^*(\lambda))\, E(VI) + \frac{\lambda}{2}\, E(VI^2)) - p\,((1 - A^*(\lambda))\, E(VB) + \frac{\lambda}{2}\, E(VB^2))]$

$IV_0(1) = \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right) E(VI) - p\, E(VB)$; $\quad IV_0'(1) = \frac{\lambda}{2}\, E(X)\, [\sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right) E(VI^2) - p\, E(VB^2)]$

$Y(1) = Q(1) - \varphi$; $\quad Y'(1) = Q'(1) + \lambda\, E(X)\, [E(H) + p\, E(VB)]\, Q(1) - I_0'(1)$

3.1 The Steady State Queue Size Probabilities

Let P_{VI}, P_{VB}, P_{br}, P_I and P_{busy} denote the probability that the server is in idle vacation, busy vacation, breakdown, idle and busy states respectively. Then the above results are used in obtaining the steady state probabilities when the system is in different states from the equations (2.12) to (2.16) at z = 1.

(i) $P_{VI} = \lim_{z\to 1} QI^*(z,0) = P_0(0) \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right) E(VI)$

(ii) $P_{VB} = \lim_{z\to 1} QB^*(z,0) = p\, P_0(0)\,(Q(1)-1)\,E(VB)$

(iii) $P_{br} = \lim_{z\to 1} B^{**'}(z,0,0) = \alpha\, P_0(0)\, Q(1)\, E(S)\, E(R)$

(iv) $P_I = \lim_{z\to 1} PI^*(z,0) + PI_0 = P_0(0) \left(Y(1)\frac{1-A^*(\lambda)}{\lambda} + \frac{\varphi}{\lambda}\right)$

(v) $P_{busy} = \lim_{z\to 1} P^*(z,0) = P_0(0)\, Q(1)\, E(S)$

$P_0(0)$ can be evaluated using the normalizing condition PBR(1) = 1and is given by: $P_0(0) = \frac{\lambda\, E(X)}{Q(1)}$.

3.2 Mean Queue Size

Let L_{VI}, L_{VB}, L_{br}, L_I and L_{busy} denote the expected queue size when the server is in idle vacation, busy vacation, breakdown, idle and busy state respectively. Then the mean queue sizes corresponding to different states of the system are the derivatives of the equations (2.12) to (2.16) at z =1.

(i) $L_{VI} = \frac{d}{dz} QI^*(z,0)|_{z=1} = P_0(0)\frac{\lambda}{2} E(X)\, E(VI^2) \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^{j} \beta_i\right)$

(ii) $L_{VB} = \frac{d}{dz} QB^*(z,0)|_{z=1} = p\, P_0(0)\, [Q'(1)\, E(VB) + \frac{\lambda}{2} E(X)\, Q(1)\, (2\, E(H)\, E(VB) + E(VB^2)) - \frac{\lambda}{2} E(X)\, E(VB^2)]$

(iii) $L_{br} = \frac{d}{dz} B^{**'}(z,0,0)\, |_{z=1} = \alpha\, P_0(0)\, [Q'(1)\, E(S)\, E(R) + \frac{\lambda}{2} E(X)\, Q(1)\, (E(S)\, E(R^2) + E(R)\, E(S^2)\, (1 + \alpha\, E(R)))]$

(iv) $L_I = \frac{d}{dz} PI^*(z,0)|_{z=1} = P_0(0)\, Y'(1)\, \frac{1-A^*(\lambda)}{\lambda}$

(v) $L_{busy} = \frac{d}{dz} P^*(z,0)|_{z=1} = P_0(0)\, [Q'(1)\, E(S) + \frac{\lambda}{2} E(X)\, E(S^2)\, (1 + \alpha\, E(R))\, Q(1)]$

The total expected queue size for the proposed model can be evaluated as follows:

$$L = L_{VI} + L_{VB} + L_{br} + L_I + L_{busy} = P_0(0)\, \frac{IVR(1)}{1-\rho} \left[\frac{IVR'(1)}{IVR(1)} + \frac{\lambda\, E(X(X-1))E(HBV) + (\lambda\, E(X))^2\, E(HBV^2)}{2\,(1-\rho)}\right] \quad (3.1)$$

To justify the computation, it is also verified that the total average queue length $L = \frac{d}{dz} PBR(z)|_{z=1}$ obtained by differentiating the total PGF (2.27) at z = 1 also gives the same result as in (3.1).

IV. PARTICULAR CASES

4.1 Decomposition Property

The total probability generating function PBR(z) can be rewritten as

$$PBR(z) = \frac{(z-1)(1-\rho_{br})}{z - S^*(h_\alpha(w_x(z)))} \frac{P_{idle}(z)}{P_{idle}(1)} = = \frac{(z-1)(1-\rho)}{z - H^*(w_x(z))} \frac{IVR(z)}{IVR(1)} \quad (4.1)$$

where $\frac{P_{idle}(z)}{P_{idle}(1)} = P_0(0)\, \frac{Q(z)}{w_x(z)} \frac{S^*(h_\alpha(w_x(z))) - z}{1-\rho_{br}}$; $\rho_{br} = \lambda E(X)E(H)$

The equation (4.1) shows that, the probability generating function of the queue size of the model under consideration is decomposed into the product of two probability generating functions, one of which is the probability generating function of number of customers in the unreliable retrial $M^X/G/1$ queueing system without server vacation and the other is the probability generating function of the conditional queue size distribution $\frac{P_{idle}(z)}{P_{idle}(1)}$ during the server idle period.

4.2 Total probability generating function for different vacation policies

The decomposition property in equation (4.1) shows that, the total probability generating function corresponding to other vacation policies differ only by the term IVR(z) . The suitable selection of β_j's gives the expressions of IVR(z), for each case.

Single Vacation Model ($\beta_0 = 1$, $\beta_j = 0\ \forall j \geq 1$):

$$IVR(z) = A^*(\lambda)\,\frac{\alpha_0}{\lambda} + M_1(z)\left[\frac{1 - VI^*(w_x(z))}{w_x(z)} + p\,\frac{VB^*(w_x(z)) - 1}{w_x(z)}\right]; \quad IVR(1) = A^*(\lambda)\,\frac{\alpha_0}{\lambda} + E(VI) - pE(VB)$$

Multiple Vacation Model ($\beta_j = 1\ \ \forall j \geq 0$):

$$IVR(z) = M_1(z)\left[\frac{1}{1-\alpha_0}\frac{1-VI^*(w_x(z))}{w_x(z)} + p\,\frac{VB^*(w_x(z))-1}{w_x(z)}\right]; \ IVR(1) = \frac{1}{1-\alpha_0}\,E(VI) - pE(VB)$$

Non-Vacation Model ($\beta_j = 0\ \forall j \geq 0$):

$$IVR(z) = IVR(1) = \frac{A^*(\lambda)}{\lambda}$$

J-Vacation Model ($\beta_0 = 1$, $\beta_j = \bar{p}$ (a constant) $\forall 1 \leq j \leq J\text{-}1$, $\beta_j = 0\ \forall j \geq J$):

$$IVR(z) = \frac{A^*(\lambda)}{\lambda}\left(\alpha_0(1\text{-}\bar{p})\,\frac{1-(\alpha_0\bar{p})^{J-1}}{1-\alpha_0\bar{p}} + \alpha_0^J\bar{p}^{J-1}\right) + M_1(z)\left[\frac{1-(\alpha_0\bar{p})^J}{1-\alpha_0\bar{p}}\,\frac{1-VI^*(w_x(z))}{w_x(z)} + p\,\frac{VB^*(w_x(z))-1}{w_x(z)}\right]$$

$$IVR(1) = \frac{A^*(\lambda)}{\lambda}\left(\alpha_0(1\text{-}\bar{p})\,\frac{1-(\alpha_0\bar{p})^{J-1}}{1-\alpha_0\bar{p}} + \alpha_0^J\bar{p}^{J-1}\right) + \frac{1-(\alpha_0\bar{p})^J}{1-\alpha_0\bar{p}}\,E(VI) - pE(VB)$$

4.3 Classical repairable M^X/G/1 queueing model with vacation

It is shown that the total probability generating function PBR(z) of the queue size probabilities in equation (2.27) of the retrial model will be reduced to the PGF of the corresponding classical model (PBr(z)) under the condition $A^*(\lambda) \to 1$ and is given by:

$$PBr(z) = P_0(0)\,\frac{z-1}{z-S^*(h_\alpha(w_x(z)))\,(1 - p + p\,VB^*(w_x(z)))}\left[\frac{\varphi}{\lambda} + \sum_{j=0}^{\infty}\alpha_0^j\left(\prod_{i=0}^{j}\beta_i\right)\frac{1 - VI^*(w_x(z))}{w_x(z)} + p\,\frac{VB^*(w_x(z)) - 1}{w_x(z)}\right]$$

V. NUMERICAL ANALYSIS

In this section, the queue size probabilities and the mean queue lengths are calculated corresponding to different parameters of different distribution of random variables. The distribution of each random variable and their measures used for the numerical computations of both the models are listed in the following table:

Random Variable (Y)	Distribution	Mean E(Y)	Second order moments E(Y^2)
Retrial Time (A)	Exponential (ν_1)	$\frac{1}{\nu_1}$	$\frac{2}{{\nu_1}^2}$
Vacation Time during Idle period (VI)	Erlang 8-type (8,η_1) $\eta_1 = 0.2$	$\frac{1}{\eta_1}$	$\frac{9}{8\eta_1^2}$
Vacation Time during Busy period (VB)	Gamma 2-type (2,η) $\eta = 3$	$\frac{2}{\eta}$	$\frac{6}{\eta^2}$
Repair Time (R)	Exponential (rI) rI = 4	$\frac{1}{rI}$	$\frac{2}{rI^2}$
Batch Size (X)	Geometric (p_1) $p_1 = 0.7$	$\frac{1}{1-p_1}$	$\frac{2\,p_1}{(1-p_1)^2}$
Service Time (S)	Erlang 5-type (5,μ_I) $\mu_I = 3$	$\frac{1}{\mu}$	$\frac{6}{5\mu^2}$

Figures 1 & 2 give the queue size probabilities and mean system size when the system is in different states respectively for the proposed model. The following observations are made. As the group arrival rate (λ) increases,

(i) queue size probability when the system is empty (PI_0) and in idle vacation state (P_{VI}) along the mean queue length during idle vacation period (L_{VI}) decrease and

(ii) queue size probability during busy vacation (P_{VB}), breakdown (P_{br}), idle (P_I) and busy (P_{busy}) states and their corresponding mean queue lengths L_{VB}, L_{br}, L_I and L_{busy} with the total mean queue length (L) increase.

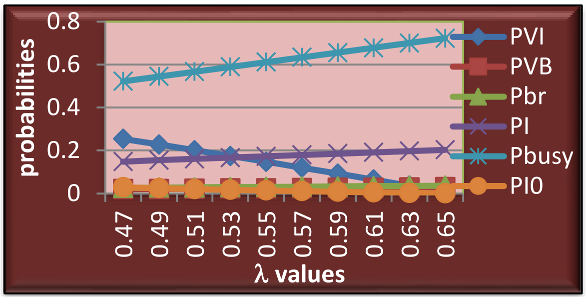

Figure 1

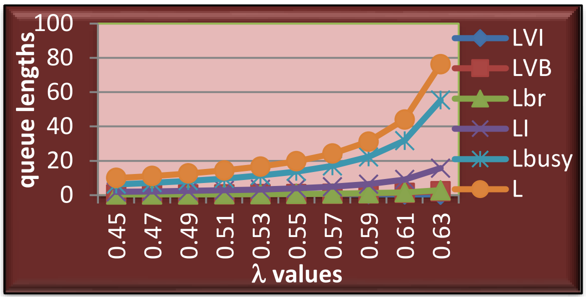

Figure 2

The effects of the breakdown rate (α), mean busy vacation time E(VB), probability that the server takes vacation between two successive services (p) and mean repair time E(R) are noted in figures 3 to 6. The figures show that,

(i) As α increases, P_{VB}, P_{br} & P_I increase and P_{VI} & PI_0 decrease,

(ii) As E(VB) decreases, P_{VB} & P_I decrease and P_{VI} & PI_0 increase,

(iii) As p increases, P_{VB} & P_I increase and P_{VI} & PI_0 decrease and

(iv) As E(R) decreases, P_{VB}, P_{br} & P_I decrease and P_{VI} & PI_0 increase.

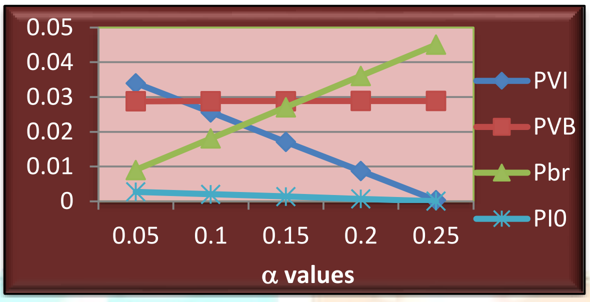

Figure 3

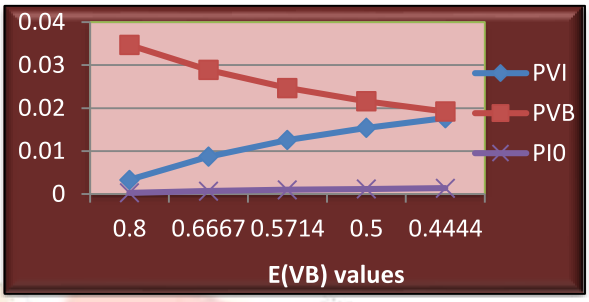

Figure 4

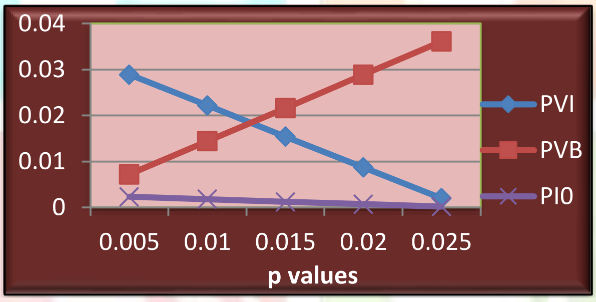

Figure 5

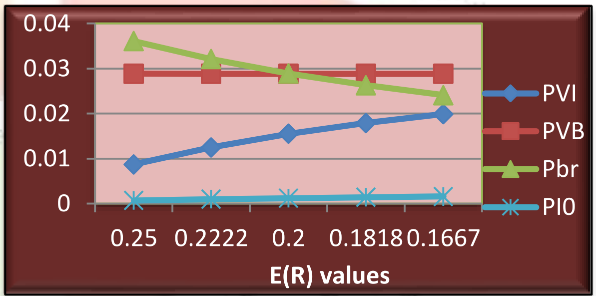

Figure 6

VI. CONCLUSION

Researchers in general treat vacation queueing models with respect to each vacation policy independently. The main purpose of the present work is to present a retrial unreliable bulk arrival queueing model with MAV policy so that the results corresponding to different vacation (single, repeated and J-vacation) policies including non-vacation case can be deduced from it. The numerical computations for the performance measures are calculated to justify the findings. The total PGF of the queue size for the corresponding classical queueing model is also derived by allowing $A^*(\lambda) \to 1$.

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