On New Class of βg^* Continuous Functions In Topological Spaces

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Abstract: The purpose of this paper is to introduce strongly and perfectly βg * Continuous Maps, totally βg * continuous functions and slightly βg * continuous functions are investigated .Additionally, we relate and compare these functions with some other functions in topological spaces.

Keywords and phrases: strongly βg ***** continuous functions, perfectly βg ***** continuous functions, totally βg * continuous and slightly βg * continuous.

I. Introduction

In 1960, Levine [5] introduced strong continuity in topological spaces. Also, in 1982 Malghan [6] introduced the generalized closed mappings Recently, C. Dhanapayam and K. Indirani [3]introduced βg * set in topological spaces. RC Jain [7] introduced the concept of totally continuous functions and slightly continuous for topological spaces. In this paper, we define totally βg * continuous functions and slightly βg * continuous functions and basic properties of these functions are investigated and obtained. In this paper we introduce and investigate a new class of functions called strongly βg ***** continuous functions, perfectly βg ***** continuous functions, totally βg * continuous and slightly βg * continuous functions are investigated and obtained..

II. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) or X , Y , Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. The power set of X is denoted by P(X). If A is βg * open and βg * closed , then it is said to be βg * clopen.

Definition 2.1: A subset A of a topological space X is said to be a βg *closed [3] if gcl (A)⊆ U, A⊆U, U is semi pre open (β ***open***)in X.

Definition 2.2: A subset A of a topological space X is said to be β ***open*** [1] if A⊆ cl (int (cl(A))).

Definition 2.3: A function f: (X, τ) ⟶ (Y, σ) is called a **strongly continuous** [6] if $f^{-1}(O)$ is both open and closed in (X, τ)for each subset O in (Y, σ).

Definition 2.4: A function f: (X, τ) ⟶ (Y, σ) is called a βg *** continuous** [2] if $f^{-1}(O)$ is a βg *open set of (X, τ)for every open set O of (Y, σ).

Definition 2.5: A function f: (X, τ) ⟶ (Y, σ) is called a **perfectly continuous** [6] if $f^{-1}(O)$ is both open and closed in (X, τ)for every open set O in (Y, σ).

Definition 2.6: A Topological space X is said to be βg ***** $\mathbf{T_{1/2}}$ **space** if every βg * open set of X is open in X.

Definition 2.7: A function f : (X, τ) ⟶(Y, σ) is called ***totally continuous*** [4] if $f^{-1}(V)$ is clopen set in X for each open set V of Y .

Definition 2.8: A function f: (X, τ) ⟶ (Y, σ) is called a ***slightly continuous***[4] if the inverse image of every clopen set in Y is open in X.

Definition 2.9: A function f: (X, τ) ⟶ (Y, σ) is called a ***contra continuous*** [4] if $f^{-1}(O)$ is closed in (X, τ) for every open set O in (Y, σ).

Definition 2.10: A function f: (X, τ) ⟶ (Y, σ) is called ***Contra*** βg * ***continuous functions*** if f^{-1} (O) is βg * closed in (X, τ) for every open set O in (Y, σ).
Definition 2.11: A topological space X is called a βg * ***connected*** if X cannot be expressed as a disjoint union of two non-empty βg * open sets.
Definition 2.12: A map f : (X, τ) ⟶(Y, σ) is said to be ***pre*** βg * ***open*** if the image of every βg * open set of X is βg * open in Y.
Definition 2.13: A topological space X is said to be **connected** if X cannot be expressed as the union of two disjoint nonempty open sets in X.
Definition 2.14: A Topological space X is said to be βg * ***T1/2 space*** *or* βg * ***space*** if every βg * open set of X is open in X.
Definition 2.15: A space (X, τ) is called a ***locally indiscrete space*** if every open set of X is closed in X.
Theorem 2.16[3]**:** Every open set is βg * - open and every closed set is βg * -closed set

III. Strongly βg *Continuous Function

We introduce the following definition.

Definition 3.1: A function f: (X, τ) ⟶ (Y, σ) is called a strongly βg * continuous if the inverse image of every βg *open set in (Y, σ) is open in (X, τ).

Theorem 3.2: If a map f: X ⟶ Y from a topological spaces X into a topological spaces Y is strongly βg * continuous then it is continuous.

Proof: Let O be a open set in Y. Since every open set is βg *open, O is βg *open in Y. Since f is strongly βg *continuous, f^{-1}(O) is open in X. Therefore f is continuous.

Remark 3.3: The following example supports that the converse of the above theorem is not true in general.

Example 3.4: Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, {b, c}, X},
σ = {ϕ, {c}, {a, c}, {b, c}, Y}.
βg * o(Y, σ) = {ϕ, {c},{a, c}, { b ,c} , Y}.
Let g: (X, τ) ⟶(Y, σ) be defined by g (a) = a, g(b) = b, g(c) = c.
Clearly, g is not strongly βg *continuous.
Since {c} is βg * open set in Y but g^{-1}({c}) = {c} is not a open set of X.
Clearly, g is not strongly βg *continuous.
Let g: (X, τ) ⟶(Y, σ) be defined by g (a) = a, g(b) = c, g(c) = b.
Since {a}, {b}, {a, b} is closed set in Y but g^{-1}({a}) = {a}, g^{-1}({b}) = {c}, g^{-1}({a, b}) = {a, c}, is closed subset of X.
However, g is continuous.
Theorem 3.5: A map f: X ⟶ Y from a topological spaces X into a topological spaces Y is strongly βg * continuous if and only if the inverse image of every βg * closed set in Y is closed in X.
Proof: Assume that f is strongly βg * continuous. Let O be any βg * closed set in Y. Then O^c is βg *open in Y. Since f is strongly βg * continuous, $f^{-1}(O^c)$ is open in X. But $f^{-1}(O^c)$= X/ f^{-1}(O) and so f^{-1}(O) is closed in X.
Conversely, assume that the inverse image of every βg * closed set in Y is closed in X . Then O^c is βg *closed in Y. By assumption, $f^{-1}(O^c)$ is closed in X,
but $f^{-1}(O^c)$= X/ f^{-1}(O) and so f^{-1}(O) is open in X. Therefore, f is strongly βg * continuous.

Theorem 3.6: If a map f: X ⟶ Y is strongly continuous then it is strongly βg *continuous. **Proof:** Assume that f is strongly continuous. Let O be any βg *open set in Y. Since f is strongly continuous, f^{-1}(O) is open in X. Therefore, f is strongly βg *continuous.

Remark 3.7: The converse of the above theorem need not be true.

Example 3.8: Let X = Y= {a, b, c}, τ = {∅,{a}, {b},{a, b},{ac}, X} and σ = {∅, {a},{a, c},{a, b},Y}.

βg * O(Y, σ) = {∅, {a},{a, c}, {a, b} , Y}.

Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a, f(b) = b , f(c) = c.
since {a} , { a, c} , { a, b} is βg * open set in Y
But f^{-1}({a}) = a, f^{-1}({a, c}) = {a, c}, f^{-1}({a, b}) = {a, b} is open in X,
clearly, f is strongly βg *continuous.
Since {a, b} is subset of Y.
But f^{-1}({a, b}) = {a, b} is open in X, not closed in X.
Therefore f is not strongly continuous.

Theorem 3.9: If a map f: X ⟶ Y is strongly βg * continuous then it is βg *continuous.

Proof: Let O be an open set in Y. By [3] O is βg * open in Y. Since f is strongly βg * continuous ⟹ f^{-1}(O) is open in X. By [2] f^{-1}(O) is βg * open in X. Therefore, f is βg * continuous.

Remark 3.10: The converse of the above theorem need not be true.

Example 3.11: Let X = Y= {a, b, c}, τ = {,∅ {a}, {b}, {a, b}, {b, c}, X} and

σ = {,∅ {c}, {a, c}, {b, c}, Y},

βg * O(Y, σ) = {∅, {c},{a, c}, {b, c} , Y}.

βg * O(X, τ) = {,∅ {a}, {b}, {c},{a, b}, {a, c}, {b, c} , X}.

Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a, f(b) = b , f(c) = c.
Since {c}, {a, c}, {b, c} is open set in Y
But f^{-1}({c}) = c, f^{-1}({a, c}) = {a, c}, f^{-1}({b, c}) = {b, c} is βg * open set in X,
Clearly, f is βg *continuous.
Since {c} is βg * O(Y, σ)
But f^{-1}({c}) = {c} is not open in X.
Therefore f is not strongly βg *continuous.

Theorem 3.12: If a map f: X ⟶ Y is strongly βg * continuous and a map g: Y ⟶ Z is βg *continuous then g ∘ f: X ⟶ Z is continuous.

Proof: Let O be any open set in Z. Since g is βg * continuous, g^{-1}(O) is βg * open in Y. Since f is strongly βg * continuous f^{-1}(g^{-1}(O)) is open in X. But $(g \circ f)^{-1}$ (O) = f^{-1}(g^{-1}(O)).
Therefore, g ∘f is continuous.

Theorem 3.13: If a map f: X ⟶ Y is strongly βg * continuous and a map g: Y ⟶ Z is βg *irresolute , then g ∘ f: X ⟶ Z is strongly βg * continuous.

Proof: Let O be any βg *open set in Z. Since g is βg * irresolute, g^{-1}(O) is βg * open in Y. Also, f is strongly βg * continuous f^{-1}(g^{-1}(O)) is open in X. But $(g \circ f)^{-1}$ (O) = f^{-1}(g^{-1}(O)) is open in X. Hence, g ∘ f: X ⟶ Z is strongly βg * continuous.

Theorem 3.14: If a map f: X ⟶ Y is βg * continuous and a map g: Y ⟶ Z is strongly βg *continuous , then g ∘ f: X ⟶ Z is βg * irresolute.

Proof: Let O be any βg *open set in Z. Since g is strongly βg * continuous, g^{-1}(O) is open in Y. Also, f is βg *continuous, f^{-1}(g^{-1}(O)) is βg *open in X. But $(g \circ f)^{-1}$ (O) = f^{-1}(g^{-1}(O)).
Hence, g ∘ f: X ⟶ Z is βg * irresolute.

Theorem 3.15: Let X be any topological spaces and Y be a βg *$T_{1/2}$ space and f: X ⟶ Y be a map. Then the following are equivalent

1) f is strongly βg * continuous
2) f is continuous

Proof: (1) ⟹(2) Let O be any open set in Y. By [3] O is βg * open in Y. Then f^{-1}(O) is open in X. Hence , f is continuous.

(2) ⟹(1) Let O be any βg * open in (Y, σ). Since, (Y, σ) is a βg *$T_{1/2}$ space,O is open in(Y, σ). Since, f is continuous. Then f^{-1}(O) is open in (X, τ) . Hence, f is strongly βg * continuous.

Theorem 3.16: Let f: (X, τ) ⟶ (Y, σ) be a map. Both (X, τ) and (Y, σ) are βg *$T_{1/2}$ space. Then the following are equivalent.

1) f is βg * irresolute

2) f is strongly βg * continuous
3) f is continuous
4) f is βg * continuous

Proof: The proof is obvious.

Theorem 3.17: The composition of two strongly βg* continuous maps is strongly βg * continuous.

Proof: Let O be a βg * open set in (Z, η). Since, g is strongly βg * continuous, we get $g^{-1}(O)$ is open in (Y, σ). By[3] $g^{-1}(O)$ is βg *open in (Y, σ). As f is also strongly βg * continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is open in (X, τ). Hence, (g ∘ f) is strongly βg * continuous.

Theorem 3.18: If f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z, η) be any two maps .Then their composition g ∘ f : (X, τ) ⟶ (Z, η) is strongly βg * continuous if g is strongly βg * continuous and f is continuous.

Proof: Let O be a βg * open in (Z, η) . Since, g is strongly βg * continuous, $g^{-1}(O)$ is open in (Y, σ). Since f is continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is open in (X, τ). Hence, (g ∘ f) is strongly βg * continuous.

IV. Perfectly βg *Continuous Function

Definition 4.1: A map f: (X, τ) ⟶ (Y, σ) is said to be perfectly βg * continuous if the inverse image of every βg *open set in (Y, σ) is both open and closed in (X, τ) .

Theorem 4.2: If a map f: (X, τ) ⟶ (Y, σ) from a topological space (X, τ) into a topological space (Y, σ) is perfectly βg * continuous then it is strongly βg * continuous.

Proof: Assume that f is perfectly βg * continuous. Let O be any βg *open set in (Y, σ) . Since, f is perfectly βg * continuous, $f^{-1}(O)$ is open in (X, τ) . Therefore, f is strongly βg* continuous.

Remark 4.3: The converse of the above theorem need not be true.

Example 4.4: Let X = Y= {a, b, c}, τ = {∅,{a},{b},{a, b},{a, c}, X} and
σ = {∅, {a},{a, b},{a, c},Y}.
βg * O(Y, σ) = {∅, {a},{a, b}, {a, c} , Y}.
Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a, f(b) = b , f(c) = c .
since {a} , { a, b} , { a, c} is βg * open set in Y
But f^{-1}({a}) = a, f^{-1}({a, b}) = {a, b}, f^{-1}({a, c}) = {a, c} is open set in X,
clearly, f is strongly βg *continuous.
Since {a} is βg *open set in (Y, σ)
But f^{-1}({a}) = {a} is open in X, but not closed in X.
Therefore f is not perfectly βg* continuous.

Theorem 4.5: If a map f: (X, τ) ⟶ (Y, σ) from a topological space (X, τ) into a topological space (Y, σ) is perfectly βg * continuous then it is perfectly continuous .

Proof: Let O be an open set in Y. By[3] O is an βg *open set in (Y, σ). Since f is perfectly βg * continuous, $f^{-1}(O)$ is both open and closed in (X, τ) . Therefore, f is perfectly continuous.

Remark 4.6 : The converse of the above theorem need not be true.

Example 4.7: Let X = Y= {a, b, c}, τ = {∅,{a},{b},{a, c}, {a, b}, X} and
σ = {∅, {b},Y}.
βg * O(Y, σ) = {∅, {a},{b},{c},{a, b}, {b, c} , Y}.
Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a , f(b) = b , f(c) = c.
Since {b} is open set in (Y, σ)
f^{-1}({b}) = {b} is both open and closed in X.
clearly, f is perfectly continuous.
Since { a}is βg*open set in (Y, σ)

$f^{-1}(\{a\}) = \{a\}$ is open and not closed in X.
Therefore f is not perfectly βg * continuous.

Theorem 4.8: A map f: (X, τ) ⟶ (Y, σ) from a topological space (X, τ) into a topological space (Y, σ) is perfectly βg * continuous if and only if $f^{-1}(O)$ is both open and closed in (X, τ) for every βg *closed set O in (Y, σ).

Proof: Let O be any βg *closed set in (Y, σ).Then O^c is βg *open in (Y, σ). Since, f is perfectly βg * continuous , $f^{-1}(O^c)$ is both open and closed in (X, τ) . But $f^{-1}(O^c) = X/ f^{-1}(O)$ and so $f^{-1}(O)$ is both open and closed in (X, τ) .

Conversely, assume that the inverse image of every βg *closed set in (Y, σ) is both open and closed in (X, τ). Let O be any βg *open in (Y, σ). Then O^c is βg *closed in (Y, σ).By assumption $f^{-1}(O^c)$ is both open and closed in (X, τ) . But $f^{-1}(O^c) = X/ f^{-1}(O)$ and so $f^{-1}(O)$ is both open and closed in (X, τ).Therefore, f is perfectly βg * continuous.

Theorem 4.9: Let (X, τ) be a discrete topological space and (Y, σ) be any topological space. Let f: (X, τ) ⟶(Y, σ) be a map, then the following statements are true.
1) f is strongly βg * continuous
2) f is perfectly βg * continuous

Proof: (1) ⟹(2) Let O be any βg * open set in (Y, σ) . By hypothesis, $f^{-1}(O)$ is open in (X, τ). Since (X, τ) is a discrete space, $f^{-1}(O)$ is closed in (X, τ). $f^{-1}(O)$ is both open and closed in (X, τ). Hence, f is perfectly βg* continuous.

(2) ⟹(1) Let O be any βg * open set in (Y, σ). Then, $f^{-1}(O)$ is both open and closed in (X, τ). Hence, f is strongly βg * continuous.

Theorem 4.10: If f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z, η) are perfectly βg * continuous , then their composition g ◦ f : (X, τ) ⟶ (Z, η) is also perfectly βg * continuous .

Proof: Let O be a βg * open set in (Z, η) . Since, g is perfectly βg * continuous.
We get that $g^{-1}(O)$ is open and closed in (Y, σ). By thm [ ] $g^{-1}(O)$ is βg * open in (Y, σ).
Since f is perfectly βg * continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is both open and closed in (X, τ) . Hence, g ◦ f is perfectly βg * continuous.

Theorem 4.11: If f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z, η) be any two maps . Then their composition is strongly βg * continuous if g is perfectly βg * continuous and f is continuous.

Proof: Let O be any βg * open set in (Z, η). Then, $g^{-1}(O)$ is open and closed in (Y, σ).
Since, f is continuous. $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is open in (X, τ) .
Hence, g ◦ f is strongly βg * continuous.

Theorem 4.12: If a map f: (X, τ) ⟶ (Y, σ) is perfectly βg * continuous and a map g: (Y, σ) ⟶ (Z, η) is strongly βg * continuous then the composition g ◦ f : (X, τ) ⟶ (Z, η) is perfectly βg * continuous.

Proof: Let O be any βg * open set in (Z, η). Then, $g^{-1}(O)$ is open in (Y, σ).By thm [ ]$g^{-1}(O)$ is βg *open in (Y, σ). By hypothesis, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is both open and closed in (X, τ) . Therefore, g ◦ f is perfectly βg * continuous.

V. Totally βg * continuous functions

Definition 5.1: A function (X, τ) ⟶(Y, σ) is called **totally** βg * **continuous functions** if the inverse image of every open set of (Y, σ) is both βg * open and βg * closed subset of (X, τ).

Example 5.2: Let X = Y ={a, b, c, d}, τ = {ϕ,{c},{d}, {c, d}, X}, σ = {ϕ, {b},{a,b},{ b, c},{a, b, c},Y} ,βg * O(X, τ) = {ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c}, {b, d}, {c, d},{b, c, d},{a, c, d},X} and βg * C(X,τ) = {ϕ , {a} ,{b},{a, c}, {a, b},{a, d} ,{b, c},{b, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d},X}. Let *g:* (X, τ) ⟶(Y, σ) be defined by g (a) = b , g(b) = d , g(c) = a, g(d) = c.Therefore , $g^{-1}(\{b\}) = \{a\}$, $g^{-1}(\{a,b\}) = \{c,a\}$, g^{-}

^{1}({b, c}) = {a, d}, g^{-1}({a, b, c}) = {a, c, d}.Hence the inverse image of every open set of (Y, σ) is both βg * open and βg * closed subset of (X, τ). Therefore, g is totally βg * continuous.

Theorem 5.3: Every totally βg * continuous functions is βg * continuous.
Proof: Let f: (X, τ) ⟶(Y, σ) be totally βg * continuous and O be an open set of (Y, σ). Since, f is totally βg * continuous functions, f^{-1}(O) is both βg * open and βg * closed in (X, τ). Therefore, f is βg * continuous.

Remark 5.4: The converse of above theorem need not be true.

Example 5.5: Let X = Y = {a, b, c, d}, τ = {ϕ, {c},{d}, {c, d}, X}, σ = {ϕ, {a},{b},{a, b},Y}.
βg * O(X, τ) = { ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c}, {b, d},{c, d},{b, c, d},{a, c, d} ,X} and βg * C(X, τ) ={ ϕ,{a} , {b} , {a, c} , {a, b},{a, d} ,{b, c} ,{b, d} ,{a, b, c} ,{a, b, d},{a, c, d}, {b, c, d} ,X} .
Let *g:* (X, τ) ⟶(Y, σ) be defined by g (a) = a , g(b) = c, g(c) = b , g(d) = d.
Clearly, g is not totally βg * continuous since g^{-1}({b}) = {c} is βg * open in X but not βg * closed.
However, g is βg * continuous.

Theorem 5.6: Every totally continuous function is totally βg * continuous.
Proof: Let f: (X, τ) ⟶(Y, σ) be the totally βg * continuous and O be an open set of (Y, σ). Since, f is totally continuous functions, f^{-1}(O) is both open and closed in (X, τ).
Since every open set is βg * open and every closed set is βg * closed. f^{-1}(O) is both βg * open and βg * closed in (X, τ). Therefore, f is totally βg * continuous.
Remark 5.7: The converse of above theorem need not be true.

Example 5.8: Let X = Y = {a, b, c, d}, τ = {ϕ, {c},{d}, {c, d} , X},
$τ^{c}$ = {ϕ, {a, b, d }, { a, b, c },{a, b}, X} , σ = {ϕ, {b},{b, d}, Y}.
βg * O(X, τ) = { ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c}, {b, d},{c, d},{b, c, d},{a, c, d} ,X} andβg * C(X, τ) ={ ϕ,{a} , {b} , {a, c} , {a, b},{a, d} ,{b, c} ,{b, d} ,{a, b, c} ,{a, b, d},{a, c, d},{b, c, d} , X} . Let f*:* (X, τ) ⟶(Y, σ) be defined by f (a) = c , f (b) = b , f (c) = a , f (d) = d,since , f^{-1}({b}) = {b} , f^{-1}({b, d}) = {b, d} is both βg * open and βg * closed in X.
Clearly, f is totally βg * continuous.
But f is not totally continuous as the inverse image of the open set {b} of (Y, σ) is not a clopen set in (X, τ), and hence βg * continuous.
Theorem 5.9: Every perfectly βg * continuous map is totally βg * continuous.
Proof: Let f : (X, τ) ⟶(Y, σ) be a perfectly βg * continuous map.
Let O be an open set of (Y, σ). Then O is βg * open in (Y, σ).
Since f is perfectly βg * continuous, f^{-1}(O) is both open and closed in (X, τ), implies f^{-1}(O) is both βg * open and βg * closed in (X, τ).
Therefore, f is totally βg * continuous.

Remark 5.10: The converse of above theorem need not be true.

Example 5.11: Let X = Y = {a, b, c,d}, τ = {ϕ, {c},{d}, {c, d}, X},
$τ^{c}$ = {ϕ, {a, b, d }, { a, b, c },{a, b}, X} , σ = {ϕ, { b}, {a, c, d},Y}.
Let *g:* (X, τ) ⟶(Y, σ) be defined by g (a) = a , g(b) = b, g(c) = c , g(d) = d.
βg * O(X, τ) = { ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c}, {b, d},{c, d},{b, c, d},{a, c, d} ,X} andβg * C(X, τ) ={ ϕ,{a} , {b} , {a, c} , {a, b},{a, d} ,{b, c} ,{b, d} ,{a, b, c} ,{a, b, d},{a, c, d},{b, c, d} , X} . since , g^{-1}({b}) = {b} , g^{-1}({a, c, d}) = {a, c, d} is both βg * open and βg * closed in X.Clearly, g is totally βg * continuous.
βg * O(Y, σ) = {ϕ, {b},{c},{d},{b, c }, {b, d }, {c, d }, {a, b, c},{b, c, d }, Y} .
But g^{-1}({c,d}) = {c,d} is open in X but not closed in X.
Therefore, g is not perfectly βg * continuous.

Remark 5.12: The concept of totally βg * continuous and strongly βg * continuous are independent of each other.

Example 5.13: Let X = Y = {a, b, c,d}, τ = {ϕ, {c},{d}, {c, d}, X},
τ^c ={ϕ,{a, b, d },{ a, b, c },{a, b}, X} , σ = {ϕ, { b}, {a, c, d},Y}.
Let *g:* (X, τ) ⟶(Y, σ) be defined by g (a) = a , g(b) = b, g(c) = c , g(d) = d.
βg * O(X, τ) = { ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c}, {b, d},{c, d},{b, c, d},{a, c, d} ,X} andβg * C(X, τ) ={ ϕ,{a} , {b} , {a, c} , {a, b},{a, d} ,{b, c} ,{b, d} ,{a, b, c} ,{a, b, d},{a, c, d},{b, c, d} , X} . Since , g^{-1}({b}) = {b} , g^{-1}({a, c, d}) = {a, c, d} is both βg * open and βg * closed in X. Clearly, g is totally βg * continuous.βg * O(Y, σ) = {ϕ, {b},{c},{d},{b, c }, {b, d }, {c, d }, {a, b, c},{b, c, d }, Y} . Clearly, g is totally βg * continuous but g^{-1}({a, b, c}) = {a, b, c} is not open in X .Therefore, g is not strongly βg * continuous.

Theorem 5.14: If f: X→Y is a totally βg * continuous map, and X is βg * connected, then Y is an indiscrete space.
Proof: Suppose that Y is not an indiscrete space.
Let A be a non-empty open subset of Y.
Since, f is totally βg * continuous map, then f^{-1} (A) is a non-empty βg * clopen subset of X.
Then X = f^{-1} (A) ∪ $(f^{-1}(A))^c$.
Thus, X is a union of two non-empty disjoint βg * open sets which is contradiction to the fact that X is βg * connected.
Therefore, Y must be an indiscrete space

Theorem 5.15: Let f: X → Y and g: Y → Z be functions. Then g∘f : X → Z
(i)If f is βg * irresolute and g is totally βg *continuous then g ∘ f is totally βg * continuous
(ii) If f is totally βg * continuous and g is continuous then g ∘ f is totally βg * continuous.
Proof:
(i) Let O be an open set in Z. Since g is totally βg * continuous, g^{-1} (O) is βg * clopen in Y.
Since f is βg * irresolute, f^{-1}(g^{-1} (O)) is βg * open and βg * closed in X.
Since, $(g \circ f)^{-1}$ (O) = f^{-1}(g^{-1} (O)). Therefore, g ∘ f is totally βg * continuous.
(ii) Let O be an open set in Z. Since g is continuous, g^{-1} (O) is open in Y.
Since, f is totally βg * continuous, f^{-1}(g^{-1} (O)) is βg * clopen in X. Hence, g ∘ f is totally βg * continuous.

VI. Slightly βg * continuous functions

Definition 6.1:A function f: (X, τ) ⟶(Y, σ) is called slightly βg * continuous at a point x∈X if for each clopen subset V of Y containing f(x), there exists a βg * open subset U in X containing x such that f(U)⊆V. The function f is said to be slightly βg * continuous if f is slightly βg * continuous at each of its points.
Definition 6.2: A function (X, τ) ⟶(Y, σ) is said to be **slightly** βg * **continuous** if the inverse image of every clopen set in Y is βg * open in X

Example 6.3: Let X = Y = {a, b, c,d}, τ = {ϕ,{c},{d}, {c, d}, X},
σ = σ^c = {ϕ, {a, d},{b, c},Y}.
βg * O(X, τ) = { ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c}, {b, d},{c, d},{b, c, d},{a, c, d} ,X}
Let *g:* (X, τ) ⟶(Y, σ) be defined by g (a) = b , g(b) = a, g(c) = c , g(d) = d.
since , g^{-1}({a, d}) = {b, d}, g^{-1}({b, c}) = {a, c}is βg * open in X.
Therefore, g is **slightly** βg * **continuous** .

Proposition 6.4: The definitions 6.1 and 4.6 are equivalent.
Proof: Suppose the definition 6.1 holds. Let O be a clopen set in Y and x ∈ f^{-1} (O).
Then f(x) ∈ O and thus there exists a βg * open set U x such that x ∈ U x ⊆ f^{-1} (O) and
f^{-1} (O) = ∪ x ∈f^{-1}(O) U x .
Since, arbitrary union of βg * open set is βg * open.

f^{-1} (O) is βg * open in X.
Therefore, f is slightly βg * continuous.
Suppose the definition 6.2 holds.
Let f(x) ∈ O where, O is a clopen set in Y. Since, f is slightly βg * continuous f^{-1}(O) is βg * open in X, x ∈ f^{-1} (O)
Let U = f^{-1} (O). Then U is βg * open in X, x ∈ U, and f(U) ⊆ O.

Theorem 6.5: For a function f: (X, τ) ⟶(Y, σ) , the following statements are equivalent.
(i) f is slightly βg * continuous.
(ii) The inverse image of every clopen set O of Y is βg * open in X.
(iii) The inverse image of every clopen set O of Y is βg * closed in X.
(iv) The inverse image of every clopen set O of Y is βg * clopen in X.
Proof:
(i)⇒ (ii): Follows from the proposition 6.4
(ii)⇒(iii): Let O be a clopen set in Y which implies O^c is clopen in Y.
By (ii), f^{-1} (O^c) =(f^{-1}(O))^c is βg * open in X.
Therefore, f^{-1} (O) is βg * closed in X.
(iii) ⇒ (iv): By (ii) and (iii), f^{-1} (O) is βg * clopen in X.
(iv) ⇒ (i): Let O be a clopen set in Y containing f(x), by (iv) f^{-1} (O) is βg * clopen in X.
Take U = f^{-1} (O), then f(U) ⊆ O.
Hence, f is slightly βg * continuous .

Theorem 6.6: Every slightly continuous function is slightly βg * continuous.
Proof: Let f: (X, τ) ⟶(Y, σ) be a slightly continuous function.
Let O be a clopen set in Y.
Then, f^{-1} (O) is open in X. Since, every open set is βg * open.
Hence, f is slightly βg * continuous .
Remark 6.7: The converse of the above theorem need not be true as can be seen from the following example.
Example 6.8: Let X = Y = {a, b, c, d}, τ = {ϕ,{c},{d}, {c, d}, X}, σ = $σ^c$ ={ϕ, {a,d},{b, c},Y}.
βg * O(X, τ) = { ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c}, {b, d},{c, d},{b, c, d},{a, c, d} ,X}
Let *g:* (X, τ) ⟶(Y, σ) be defined by g (a) = b , g(b) = a, g(c) = c , g(d) = d.
since , g^{-1}({a, d}) = {b, d}, g^{-1}({b, c}) = {a, c}is βg * open in X.
Clearly, g is slightly βg * continuous.
But not slightly continuous.
Since, g^{-1}(a, d) = { b, d }where {a , d } is clopen in Y but {b, d } is not open in X.
Theorem 6.9: Every βg * continuous function is slightly βg * continuous.
Proof: Let f: (X, τ) ⟶(Y, σ) be a βg * continuous function.
Let O be a clopen set in Y.
Then, f^{-1} (O) is βg * open in X and βg * closed in X.
Hence, f is slightly βg * continuous.
Remark 6.10: The converse of the above theorem need not be true as can be seen from the following example.
Example 6.11: Let X = Y = {a, b, c, d}, τ = {ϕ,{c},{d}, {c, d}, X}, σ = $σ^c$ ={ϕ, {a, d},{b, c},Y}.βg * O(X, τ) = {ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c}, {b, d}, {c, d},{b, c, d},{a, c, d},X}Let f*:* (X, τ) ⟶(Y, σ) be defined by f(a) = b , f(b) = a, f(c) = c, f(d) = d
since , f^{-1}({a, d}) = {b, d}, f^{-1}({b, c}) = {a, c}is βg * open in X.
The function f is slightly βg * continuous.Let *g:* (X, τ) ⟶(Y, σ) be defined by g(a) = a , g(b) = d, g(c) = c, g(d) = bBut not βg * continuous,since, g^{-1} (a,d) = {a,b} is not βg * open in X.

Theorem 6.12: Every contra βg * continuous function is slightly βg * continuous.
Proof: Let f: (X, τ) ⟶(Y, σ) be a contra βg * continuous function.
Let O be a clopen set in Y.
f^{-1} (O) is βg * open in X as f is contra βg * continuous.

Hence, f is slightly βg * continuous.

Remark 6.13: The converse of the above theorem need not be true as can be seen from the following example.

Example 6.14: Let X = Y = {a, b, c,d}, τ = {ϕ,{c},{d}, {c, d}, X}, τ^c = { ϕ,{a, b, d},{a, b, c}, {a, b}, X }, σ = σ^c ={ϕ, {a, d},{b, c},Y}.

βg * O(X, τ) = {ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c}, {b, d}, {c, d},{b, c, d},{a, c, d},X}

βg * C(X, τ) ={ ϕ,{a} , {b} , {a, c} , {a, b},{a, d} ,{b, c} ,{b, d} ,{a, b, c} ,{a, b, d},{a, c, d}, {b, c, d} , X} .

Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = b , f(b) = a, f(c) = c, f(d) = d

since , f^{-1}({a, d}) = {b, d}, f^{-1}({b, c}) = {a, c}is βg * open in X.

The function f is slightly βg * continuous.

Remark 6.15: Composition of two slightly βg * continuous need not be slightly βg * continuous as it can be seen from the following example.

Example 6.16: Let X=Y={a,b,c,d} , Z = {a,b,c} and the topologies are

τ = { ϕ ,{c} ,{d}, {c, d } ,X} and σ = { ϕ , {a, d} , {b, c} ,Y } and η = {ϕ ,{a, b},{c, d},Z}.

Define f: (X, τ) ⟶(Y, σ) by f(a) = b ,f(b) = a, f(c) = c, f(d)=d,

βg * O (X,τ)={ ϕ , {a} , {b} , {c} , {d}, {a, c} , {a, d},{b, c} , {b, d}, {c, d}, {a, c, d},{b, c, d},X } since f^{-1} (a, d) = (b, d), f^{-1} (b, c) =(a, c) is βg * open in X.

Clearly, f is slightly βg * continuous.Define g: (Y, σ) ⟶ (Z , η) by g(a) = a, g(b) = b , g (c) = d , g(d) = c,

βg * O (Y, σ) = { ϕ , {a} , {b} , {c} ,{d}, {a, b }, {a, c} , {a, d},{b, c} , {b, d}, {c, d}, {a, b, c},{a, c, d},{b, c, d} , {a, b, d}, Y } . since g^{-1} (a, b) = (a, b), g^{-1} (c, d) =(c, d) is βg * open in Y.Clearly, g is slightly βg * continuous.

But (g ∘ f): (X, τ) ⟶ (Z , η) is not slightly βg * continuous,

since $(g \circ f)^{-1}$ ({a, b}) = f^{-1}(g^{-1}{a, b}) = f^{-1}({ a, b}) = {a, b} is not a βg * open in (X, τ).

Theorem 6.17: Let f: X → Y and g: Y → Z be functions. Then the following properties hold:

(i) If f is βg * irresolute and g is slightly βg * continuous then (g ∘ f) is slightly βg * continuous.

(ii) If f is βg * irresolute and g is βg * continuous then (g ∘ f) is slightly βg * continuous.

(iii) If f is βg * irresolute and g is slightly continuous then (g ∘ f) is slightly βg * continuous.

(iv) If f is βg * continuous and g is slightly continuous then (g ∘ f) is slightly βg * continuous.

(v) If f is strongly βg * continuous and g is slightly βg * continuous then (g ∘ f) is slightly continuous.

(vi) If f is slightly βg * continuous and g is perfectly βg * continuous then (g ∘ f) is βg * irresolute.

(vii) If f is slightly βg * continuous and g is contra continuous then (g ∘ f) is slightly βg * continuous.

(viii) If f is βg * irresolute and g is contra βg * continuous then (g ∘ f) is slightly βg * continuous.

Proof:

(i) Let O be a clopen set in Z. Since, g is slightly βg * continuous, g^{-1}(O) is βg * open in Y.
Since, f is βg * irresolute, $f^{-1}(g^{-1}(O))$ is βg * open in X.
Since, $(g \circ f)^{-1}$ (O) = $f^{-1}(g^{-1} (O))$, g ∘ f is slightly βg * continuous .

(ii) Let O be a clopen set in Z. Since, g is βg * continuous, g^{-1}(O) is βg * open in Y.
Since, f is βg * irresolute, $f^{-1}(g^{-1}(O))$ is βg * open in X.
Hence, g ∘ f is slightly βg * continuous.

(iii)Let O be a clopen set in Z.
Since, g is slightly continuous, g^{-1}(O) is open in Y .
Since, f is βg * irresolute, $f^{-1}(g^{-1}(O))$ is βg * open in X.
Hence, g ∘ f is slightly βg * continuous.

(iv)Let O be a clopen set in Z.
Since, g is slightly continuous, g^{-1}(O) is open in Y .
Since, f is βg * continuous, $f^{-1}(g^{-1}(O))$ is βg * open in X.
Hence, g ∘ f is slightly βg * continuous.

(v) Let O be a clopen set in Z. Since, g is slightly βg * continuous, g^{-1}(O) is βg * open in Y.
Since, f is strongly βg * continuous, $f^{-1}(g^{-1}(O))$ is open in X.
Therefore, g ∘ f is slightly continuous.

(vi)Let O be a βg * open in Z.

Since, g is perfectly βg * continuous, $g^{-1}(O)$ is open and closed in Y.
Since, f is slightly βg * continuous, $f^{-1}(g^{-1}(O))$ is βg * open in X.
Hence, g ∘ f is βg * irresolute.

(vii) Let O be a clopen set in Z.
Since, g is contra continuous, $g^{-1}(O)$ is open and closed in Y.
Since, f is slightly βg *continuous, $f^{-1}(g^{-1}(O))$ is βg * open in X.
Hence, g ∘ f is slightly βg * continuous.

(viii) Let O be a clopen set in Z.
Since, g is contra βg * continuous, $g^{-1}(O)$ is βg * open and βg * closed in Y.
Since, f is βg * irresolute, $f^{-1}(g^{-1}(O))$ is βg * open and βg * closed in X.
Hence, g ∘ f is slightly βg * continuous.

Theorem 6.18: If the function f: (X, τ) ⟶(Y, σ) is slightly βg * continuous and (X, τ) is βg * $T_{1/2}$ space, then f is slightly continuous.
Proof: Let O be a clopen set in Y. Since, f is slightly βg * continuous, $f^{-1}(O)$ is βg * open in X. Since, X is βg * $T_{1/2}$ space, $f^{-1}(O)$ is open in X. Hence, f is slightly continuous.
Theorem 6.19: Let f: (X, τ) ⟶(Y, σ) and g: (Y, σ) ⟶ (Z , η) be functions. If f is surjective and pre βg * open and (g ∘ f): (X, τ) ⟶ (Z , η) is slightly βg * continuous, then g is slightly βg * continuous.
Proof: Let O be a clopen set in (Z , η).
Since, (g ∘ f): (X, τ) ⟶ (Z , η) is slightly βg * continuous, $f^{-1}(g^{-1}(O))$ is βg * open in X.
Since, f is surjective and pre βg * open $f(f^{-1}(g^{-1}(O))) = g^{-1}(O)$ is βg * open in Y.
Hence, g is slightly βg * continuous.
Theorem 6.20: Let f: (X, τ) ⟶(Y, σ) and g: (Y, σ) ⟶ (Z , η) be functions.
If f is surjective , pre βg * open and βg * irresolute, then (g ∘ f): (X, τ) ⟶ (Z , η) is slightly βg * continuous if and only if g is slightly βg * continuous.
Proof: Let O be a clopen set in (Z , η). Since, (g ∘ f): (X, τ) ⟶ (Z , η) is slightly βg * continuous, $f^{-1}(g^{-1}(O))$ is βg * open in X.
Since, f is surjective and pre βg * open $f(f^{-1}(g^{-1}(O))) = g^{-1}(O)$ is βg * open in Y.
Hence, g is slightly βg * continuous.
Conversely, let g is slightly βg * continuous.
Let O be a clopen set in (Z , η), then $g^{-1}(O)$ is βg * open in Y.
Since, f is βg * irresolute, $f^{-1}(g^{-1}(O))$ is βg * open in X.
Hence, (g ∘ f): (X, τ) ⟶ (Z , η) is slightly βg * continuous.
Theorem 6.21: If f is a slightly βg * continuous from a βg * connected space (X, τ) onto a space (Y, σ) then Y is not a discrete space.
Proof: Suppose that Y is a discrete space. Let O be a proper non-empty open subset of Y. Since, f is slightly βg * continuous, $f^{-1}(O)$ is a proper non-empty βg * clopen subset of X which is contradiction to the fact that X is βg * connected.
Theorem 6.22: If f: (X, τ) ⟶(Y, σ) is a slightly βg * continuous surjection and X is βg * connected, then Y is connected.
Proof: Suppose Y is not connected, then there exists non-empty disjoint open sets U and V such that Y = U∪V.
Therefore, U and V are clopen sets in Y.
Since, f is slightly βg * continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are non-empty disjoint βg * open in X and $X= f^{-1}(U) \cup f^{-1}(V)$.
This shows that X is not βg * connected.
This is a contradiction and hence, Y is connected.
Theorem 6.23: If f: (X, τ) ⟶(Y, σ) is a slightly βg * continuous and (Y, σ) is a locally indiscrete space then f is βg * continuous.
Proof: Let O be an open subset of Y.
Since, (Y, σ) is a locally indiscrete space, O is closed in Y. Since, f is slightly βg * continuous, $f^{-1}(O)$ is βg * open in X.
Hence, f is βg * continuous.

Theorem 6.24: If $f: (X, \tau) \longrightarrow (Y, \sigma)$ is a slightly βg * continuous and A is an open subset of X then the restriction $f|A : (A, \tau A) \longrightarrow (Y, \sigma)$ is slightly βg * continuous.

Proof: Let V be a clopen subset of Y. Then $(f|A)^{-1}(V) = f^{-1}(V) \cap A$.

Since $f^{-1}(V)$ is βg * open and A is open, $(f|A)^{-1}(V)$ is βg * open in the relative topology of A. Hence, $f|A : (A, \tau_A) \longrightarrow (Y, \sigma)$ is slightly βg * continuous.

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