

A STUDY OF HEXAGONAL FUZZY NUMBER IN NEURAL NETWORK

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ABSTRACT:

In this paper, we propose an application of hexagonal fuzzy number neural network. Neural network is used to be a principle component of mathematics education. Finally, we use fuzzy number to find the best machine for factory by feed forward neural network and given a numerical example to verify the proposed approach.

Keywords:

Fuzzy set, fuzzy number, fuzzy membership function, hexagonal fuzzy number neural network, feed forward neural network.

INTRODUCTION:

Fuzzy sets were introduced by L.A.Zadeh in 1965 to manipulate data and information possessing non-statistical uncertainties. A fuzzy number is a generalization of a regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. A fuzzy number is thus a special case of a convex, normalized fuzzy set of the real line calculations with fuzzy numbers allow the incorporation of uncertainly on parameters, properties geometry, initial conditions, etc.

Neural network (NN) is a kind of designed for control task is presented. It is called fuzzy neural network (FNN). The structure of the network can be interpreted terms of a fuzzy controller. It has a three – layered architecture and uses fuzzy sets as its weight. We present some arithmetic operations such as addition, subtraction, scalar multiplication of hexagonal fuzzy numbers, then we use feed-forward model and procedure derive the problem. Finally, numerical example is to illustrate approaches.

PRELIMINARIES:

Definition 1: (Fuzzy set)

A fuzzy set A in the universe of discourse X is defined as the set of ordered pairs

$$A = \{(x, \mu_A(x); x \in X)\}$$

Here, $\mu_A : X \rightarrow [0,1]$ is called the membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A .

Definition 2: (support)

Let A be a fuzzy subset of X ; the support of A , denoted $\text{supp}(A)$, is the crisp subset of X whose element all have non-zero membership grades in A .

$$\text{Supp}(A) = \{x \in X / \mu_A(x) > 0\}$$

Definition 3: (fuzzy number)

A fuzzy number “ A ” is a convex normalized fuzzy set on real line \mathbb{R} such that:

- There exist at least one $x_0 \in \mathbb{R}$ with $\mu_A(x_0) = 1$
- $\mu_A(x)$ is piecewise continuous

Definition 4 : (Hexagonal fuzzy number)

A fuzzy number \tilde{A}_H is a hexagonal fuzzy number denoted by $\tilde{A}_H(a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below.

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{1}{2} \left(\frac{x-a_1}{a_2-a_1} \right) & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x-a_2}{a_3-a_2} \right) & \text{for } a_2 \leq x \leq a_3 \\ 1 & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left(\frac{x-a_4}{a_5-a_4} \right) & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2} \left(\frac{a_6-x}{a_6-a_5} \right) & \text{for } a_5 \leq x \leq a_6 \\ 0 & \text{for } x > a_6 \end{cases}$$

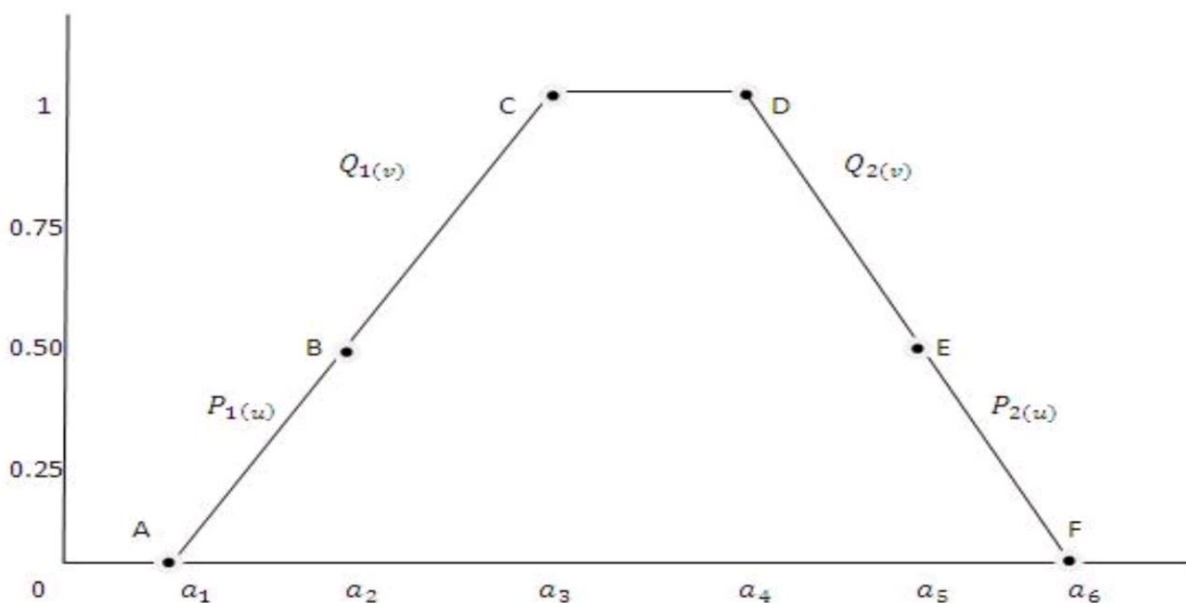


Fig 1 Graphical representation of a normal hexagonal fuzzy number for $x \in [0, 1]$

Definition 5:

An Hexagonal fuzzy number denoted by \tilde{A}_H is defined as $\tilde{A}_H = (P_1(u), Q_1(v), Q_2(v), P_2(u))$ for $u \in [0,0.5]$ and $v \in [0.5,w]$ where,

- i. $P_1(u)$ is a bounded left continuous non decreasing function over $[0,0.5]$
- ii. $Q_1(v)$ is a bounded left continuous non decreasing function over $[0.5,w]$
- iii. $Q_2(v)$ is a bounded continuous non increasing function over $[w,0.5]$
- iv. $P_2(u)$ is a bounded left continuous non increasing function over $[0.5,0]$

Arithmetic Operations on Hexagonal Fuzzy Number (HFN)

Let us consider two hexagonal fuzzy numbers $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and

$\tilde{I}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$. then the basic arithmetic operations are defined as :

a) **Addition**

$$\tilde{A}_H (+) \tilde{I}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$$

b) **Subtraction**

$$\tilde{A}_h(-) \tilde{I}_h = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6)$$

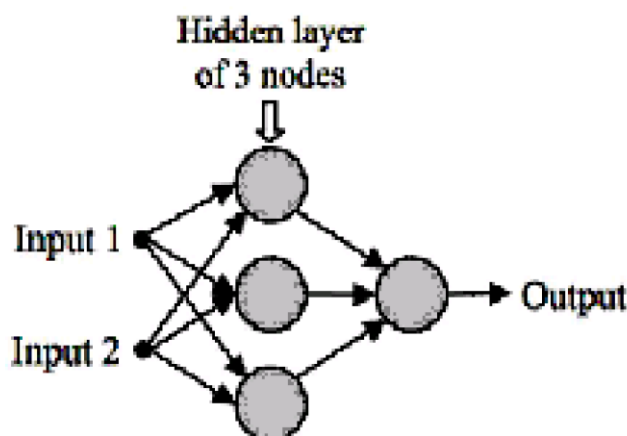
c) **Multiplication**

$$\tilde{A}_h(*) \tilde{I}_h = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$$

NEURAL NETWORK (NN):

A Neural network is a system composed of many simple processing elements operating in parallel which can acquire, store, and utilize experimental knowledge. Artificial neural networks (ANN) are powerful tools that can be used to manage knowledge and solve problems. Fuzzy Neural Network (FNN) is an architecture that combines standard MLP network with fuzzy logic in one system. The FNN consists of 3 layers of neurons.

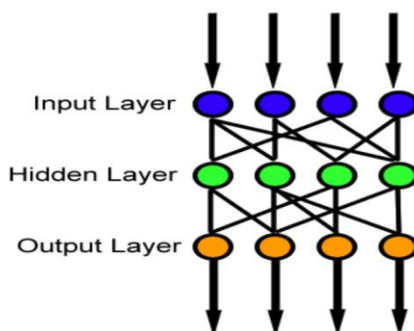
5.



The input layer of neurons represents the input variables as crisp values and outputs from the condition layer are propagated to the rule layer. The rule layer is identical in its structure and operation to a hidden layer of a standard multi layer perceptron (MLP) network.

Feed Forward Neural Network (FFNN)

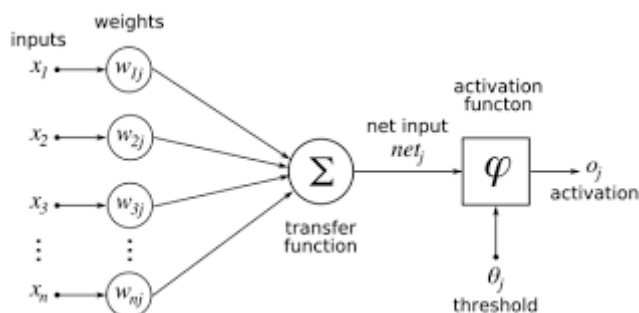
FFNN is an artificial neural network wherein connections between the nodes do not form a cycle. The feed forward neural network was the first and simplest type of artificial neural network devised. In this network, the information moves in only one direction, forward, from the input nodes, through the hidden nodes (if any) and to the output nodes. There are no cycles or loops in network. In feed forward fuzzy neural network consists three types of fuzzy neural networks depending on the type of fuzzification of inputs, outputs and weights.



In a feed forward network information always moves one direction it never goes backwards.

Activation function

It's just a think (node) that you add to the output end of any neural network. It is also known as transfer function and it can also be attached in between two neural networks. It is used to determine the output of neural network like yes or no. It maps the resulting values in between 0 to 1 or -1 to 1.



Therefore sigmoid is especially used for models where we have to predict the probability of anything exists only between the range of 0 and 1 sigmoid is right choice.

ALGORITHM

- Step 1: Estimate the problem of hexagonal fuzzy number.
- Step 2: Change the hexagonal fuzzy number matrix into its membership.
- Step 3: Consider the hexagonal fuzzy number is fuzzy weight.

Step 4: Assume the input 0 and 1

Step 5: Calculate the weight is

$$m_i = \sum_j W_{j,i} x_j = W_i x_i$$

Step 6: Output of a neuron (S) is a function $S=f(m)$

Step 7: Calculate the sigmoid function by

$$f(m) = 1.0 / (1.0 + \exp(-m))$$

Step 8: Find the minimum value of $f(m)$.

Numerical example

Let us assume that there are three spinning machine in a factory m_1, m_2, m_3 . Let the possible elements to the machines $W = (a, b, c, d, e, f)$ where a, b, c, d, e, f represent the time period, power consumption, spinning and weaving, maintenances and servicing and dyeing and finishing respectively. Calculate the hexagonal fuzzy number to finish the work.

Step 1: $W_1 = (1, 2, 3, 4, 5, 2)$ $W_2 = (2, 3, 6, 7, 1, 2)$ $W_3 = (3, 4, 5, 6, 4, 5)$

Step 2: $W_1 = (0.1, 0.2, 0.3, 0.4, 0.5, 0.2)$ $W_2 = (0.2, 0.3, 0.6, 0.7, 0.1, 0.2)$ $W_3 = (0.3, 0.4, 0.5, 0.6, 0.4, 0.5)$

Step 3: $w_{11} = 0.1, w_{12} = 0.2, w_{13} = 0.3, w_{14} = 0.4, w_{15} = 0.5, w_{16} = 0.2$

$w_{21} = 0.2, w_{22} = 0.3, w_{23} = 0.6, w_{24} = 0.7, w_{25} = 0.1, w_{26} = 0.2$

$w_{31} = 0.3, w_{32} = 0.4, w_{33} = 0.5, w_{34} = 0.6, w_{35} = 0.4, w_{36} = 0.5$

Step 4: let assume $(1, 1, 0, 0, 1, 1)$

Step 5: $m_i = \sum_j W_{ji} x_j = W_i x_i$; where $i, j = 1, 2, 3, 4, 5, 6$

$$\begin{aligned} m_1 &= W_{11}x_1 + W_{12}x_2 + W_{13}x_3 + W_{14}x_4 + W_{15}x_5 + W_{16}x_6 \\ &= (0.1)(1) + (0.2)(1) + (0.3)(0) + (0.3)(0) + (0.5)(1) + (0.2)(1) \\ &= 0.1 + 0.2 + 0 + 0 + 0.5 + 0.2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} m_2 &= W_{21}x_1 + W_{22}x_2 + W_{23}x_3 + W_{24}x_4 + W_{25}x_5 + W_{26}x_6 \\ &= (0.2)(1) + (0.3)(1) + (0.6)(0) + (0.7)(0) + (0.1)(1) + (0.2)(1) \\ &= 0.2 + 0.3 + 0 + 0 + 0.1 + 0.2 \\ &= 0.8 \end{aligned}$$

$$\begin{aligned} m_3 &= W_{31}x_1 + W_{32}x_2 + W_{33}x_3 + W_{34}x_4 + W_{35}x_5 + W_{36}x_6 \\ &= (0.3)(1) + (0.4)(1) + (0.5)(0) + (0.6)(0) + (0.4)(1) + (0.5)(1) \\ &= 0.3 + 0.4 + 0 + 0 + 0.4 + 0.5 \\ &= 1.6 \end{aligned}$$

Step 6: output

$$\begin{aligned} s_1 &= f(m_1) = 1 \\ s_2 &= f(m_2) = 0.8 \\ s_3 &= f(m_3) = 1.6 \end{aligned}$$

Step 7 : calculate the sigmoid function

$$\begin{aligned} f(m) &= 1.0 / (1.0 + \exp(-m)) \\ f(m_1) &= 1.0 / (1.0 + \exp(-1)) \\ &= 1.0 / (1.0 + \exp(-1)) \\ &= 1.0 / (1.0 + 0.3678) \\ &= 1.0 / 1.3678 \\ &= 0.7311 \\ f(m_2) &= 1.0 / (1.0 + \exp(-m_2)) \\ &= 1.0 / (1.0 + \exp(-0.8)) \\ &= 1.0 / (1.0 + 0.4493) \\ &= 1.0 / 1.4493 \\ &= 0.6899 \\ f(m_3) &= 1.0 / (1.0 + \exp(-m_3)) \\ &= 1.0 / (1.0 + \exp(-1.6)) \\ &= 1.0 / (1.0 + 0.2019) \end{aligned}$$

$$=1.0/1.2019$$

$$=0.8320$$

Step 8: verify the minimum value

$$f(m_1)=0.7311$$

$$f(m_2)=0.6899$$

$$f(m_3)=0.8320$$

The minimum value is 0.6899. So, m_2 is the best machine.

Conclusion

In this paper, a hexagonal fuzzy number is utilized to study the arithmetic operations on fuzzy numbers in neural network. We proposed this arithmetic operation to make some problems in numerical example to find the solution of hexagonal fuzzy numbers in neural network. The numerical example for verify the given the best machine and this method is very useful for to selection of best result.

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