On Strongly sgα-Irresolute Functions in Topological Spaces

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Abstract:

In this paper, we introduce and investigate the notion of strongly sgα -irresolute functions. We obtain fundamental properties and characterization of strongly sgα -irresolute functions and discuss the relationships between strongly sgα -irresolute functions and other related functions .

Keywords: strongly sgα -irresolute, strongly α -irresolute, strongly β- sgα irresolute.

Mathematics Subject Classification: 54C05,54C08,54C10.

1.Introduction

N. Levine [16] introduced generalized closed sets (briefly g-closed set) in 1970. N. Levine [15] introduced the concepts of semi-open sets in 1963. Bhattacharya and Lahiri [6] introduced and investigated semi-generalized closed (briefly sg- closed) sets in 1987. Arya and Nour [3] defined generalized semi-closed (briefly gs-closed) sets for obtaining some characterization of s-normal spaces in 1990. O.Njastad in 1965 defined α-open sets [23].

In 1996, Dontchev [12] introduced a new class of functions called contra- continuous functions. A new weaker form of this class of functions called contra semi-continuous function is introduced and investigated by Dontchev and Noiri [13].

In this paper, the notion of sgα-closed sets [8] and contra sgα- continuous Space[9]and characterization of contra sgα-continuous functions in topological spaces [10] is applied to introduce and study a new class of functions called on strongly sgα-Irresolute functions in topological spaces, as a new generalization of strongly α -irresolute functions and strongly β- sgα -irresolute functions, to obtain some of their characterizations and properties. Also the relationships with some other functions are discussed.

2. PRELIMINARIES

Throught this paper (X, τ), (Y, σ) and (Z, η) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and the interior of A are denoted by cl (A) and int (A) respectively. (X, τ) will be replaced by X if there is no chance of confusion. Let us recall the following definitions as pre requests.

A subset A of a topological space X is said to be open if $A \in \tau$. A subset A of a topological space X is said to be closed if the set $X-A$ is open. The interior of a subset A of a topological space X is defined as the union of all open sets contained in A. It is denoted by int(A). The closure of a subset A of a topological space X is defined as the intersection of all closed sets containing A. It is denoted by cl(A).

Definitions 2.1: A subset A of a space (X, τ) is said to be

1. semi open [15] if $A \subseteq cl(int(A))$ and semi closed if $int(cl(A)) \subseteq A$.

2. α-open [23] if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$.

3. β-open or semi pre-open [1] if $A \subseteq cl(int(cl(A)))$ and β-closed or semi pre-closed if $int(cl(int(A))) \subseteq A$.
4. pre-open [11] if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$.

The complement of a semi-open (resp.pre-open, α-open, β-open) set is called semi-closed (resp.pre-closed, α-closed, β-closed). The intersec- tion of all semi-closed (resp.pre-closed, α-closed, β-closed) sets containing A is called the semi-closure (resp.pre-closure, α-closure, β-closure) of A and is denoted by scl(A)(resp. pcl(A), α-cl(A), β-cl(A)). The union of all semi-open (resp.pre-open, α-open, β-open) sets contained in A is called the semi-interior(resp.pre-interior, α-interior, β-interior) of A and is de- noted by sint(A)(resp. pint(A), α-int(A), β-int(A)). The family of all semi- open (resp.pre-open, α-open, β-open) sets is denoted by SO(X)(resp. P O(X), $\alpha - O(X)$, $\beta - O(X)$). The family of all semi-closed (resp.pre-closed, α-closed, β-closed) sets is denoted by SCl(X) (resp. P Cl(X), α-Cl(X), β-Cl(X)).

Definitions 2.2: A subset A of a space (X, τ) is called
1. g-closed[16] if cl (A) ⊆ U, whenever A ⊆U and U is open in (X, τ). The complement of a g-closed set is called g-open set.
2. gs-closed set[7] if scl (A)⊆U, whenever A⊆U and U is open in (X, τ).
3. sg-closed set[6] if scl (A)⊆U, whenever A⊆U and U is semi-open in (X, τ).
4. 4. αg-closed[17] if α (cl (A)) ⊆U, whenever A ⊆U and U is open in (X, τ).
5. gα-closed [18] if α (cl (A)) ⊆U, whenever A⊆U and U is α-open in (X, τ).
6. gp-closed [19] if pcl (A)⊆U, whenever A⊆U and U is open in (X, τ).

Definition 2.3: Let X and Y be topological spaces. A function f: X → Y is said to be

1. continuous [14] if for each open set V of Y the set $f^{-1}(V)$ is an open subset of X .
2. α-continuous [23] if $f^{-1}(V)$ is a α-closed set of (X, τ) for every closed set V of (Y, σ).
3. β-continuous [1] if $f^{-1}(V)$ is a β-closed set of (X, τ) for every closed set V of (Y, σ).
4. pre-continuous [21] if $f^{-1}(V)$ is a pre-closed set of (X, τ) for every closed set V of (Y, σ).
5. semi-continuous [15] if $f^{-1}(V)$ is a semi-closed set of (X, τ) for every closed set V of (Y, σ).

Definition 2.4: A function f: (X, τ) → (Y, σ) is said to be

1. g-continuous [16] if $f^{-1}(V)$ is a g-closed set of (X, τ) for every closed set V of (Y, σ).
2. gs-continuous[7] if $f^{-1}(V)$ is a gs-closed set of (X, τ) for every closed set V of (Y, σ).
3. sg-continuous [6] if $f^{-1}(V)$ is a sg-closed set of (X, τ) for every closed set V of (Y,σ).
4. αg-continuous [17] if $f^{-1}(V)$ is a αg-closed set of (X, τ) for every closed set V of (Y, σ).
5. gα-continuous [18] if $f^{-1}(V)$ is a gα-closed set of (X, τ) for every closed set V of (Y, σ).
6. gp-continuous [19] if $f^{-1}(V)$ is a gp-closed set of (X, τ) for every closed set V of (Y, σ).

Definitions 2.5[22]: A function f: (X, τ) →(Y, σ) is said to be almost continuous if for every open set V of Y , $f^{-1}(V)$is regular open in X.
Definitions 2.6[8]: A subset A of space (X, τ) is called sgα -closed if scl (A) ⊆U , whenever A⊆U and U is α-open in X . The family of all sgα-closed subsets of the space X is denoted by SGαC (X).
Definitions 2.7[8]: The intersection of all sgα-closed sets containing a set A is called sgα-closure of A and is denoted by sgα-cl(A). A set A is sgα-closed set if and only if sgα C l (A) = A.
Definitions 2.8[8]: A subset A in X is called sgα-open in X if A^c is sgα-closed in X. The family of a sgα-open sets is denoted by SGαO(X).
Definitions 2.9[8]: The union of all sg α-open sets containing a set A is called sgα-interior of A and is denoted by sgα-I nt(A). A set A is sgα-open set if and only if sgα I nt (A) = A.
Definition 2.10[9]: A function f :(X, τ)→(Y, σ) is called sgα-continuous if $f^{-1}(V)$ is sgα-closed in (X, τ) for every closed set V of (Y, σ).

Definition 2.11[9]: A function f: X → Y is said to be Contra sgα-Continuous if f^{-1} (V) is sgα-closed in X for each open set V of Y .
Definition 2.12[9]: A space X is called locally sgα-indiscrete if every sgα-open set is closed in X.
Definition 2.13 [9]: If a function f : X → Y is called almost sgα-continuous if for each x ∈ X and each open set V of Y containing f (x), there exists U ∈ SGαO(X, x) such that f (U)⊂I nt (cl (V)).
Definition 2.14[9]: If a function f: X → Y is called quasi sgα-open if image of every sgα-open set of X is open set in Y.
Definition 2.15[9]: If a function f : X → Y is called weakly sgα-continuous if for each x ∈ X and each open set V of Y containing f (x), there exists U ∈ SGαO(X, x) such that f (U) ⊂ scl (V).
Definition 2.16 [9]: Let A be a subset of X. Then sgα-C l (A)-sgα-I nt (A) is called sgα-frontier of A and is denoted by sgα-F r (A).
Definition 2.17 [10]: A subset A of a topological space X is said to be sgα-dense in X if sgα − Cl (A) = X.
Definition 2.18 [10]: A space X is called sgα-connected provided that X is not the union of two disjoint non-empty sgα-open sets.
Definition 2.19 [10]: A subset A of a space (x, τ) is said to be sgα-clopen if A is both sgα-open and sgα-closed.
Definition 2.20 [10]: A topological space X is said to be sgα-T1-space if for any pair of disjoint points x and y, there exist disjoint sgα-open sets G and H such that x∈G and y∈H.
Definition 2.21 [10]: A topological space X is said to be sgα-T2-space if for any pair of disjoint points x and y, there exist disjoint sgα-open sets G and H such that x ∈ G and y∈H.
Definition 2.22 [10]: A space (X, τ) is called sgα-T 1/2 space if every sgα-closed set is semi-closed. The notion of sgα-T 1/2-spaces and T 1/2 spaces are independent of each other.
Definition 2.23 [10]: A topological space X is said to be sgα-Normal if each pair of disjoint closed sets can be separated by disjoint sgα-open sets.
Definition 2.24 [10]: A function f: X → Y is said to be strongly sgα-open (resp. strongly sgα-closed) if image of every sgα-open(resp.sgα-closed) set of X is sgα-open (resp.sgα-closed) set in Y .
Definition 2.25 [10]: A space X is said to be
1. SGα-closed compact if every sgα-closed cover of X has a finite subcover.
2. Countably SGα-closed compact if every countable cover of X by sgα- closed sets has a finite subcover.
3. SGα-Lindeloff if every sgα-closed cover of X has countable subcover.

3. Strongly sgα -irresolute functions.

Definition:3.1
A function f: X→Y is said to be strongly sgα -irresolute if f^{-1}(V) is open in X for every sgα -open set V of Y.

Definition:3.2

A function f: X→Y is said to be strongly α- irresolute if f^{-1}(V) is open in X for every α- open set V of Y.

Theorem:3.3

If f: X→Y is a strongly sgα -irresolute ,then f is strongly α-irresolute.

Proof: Let V be α- open set in Y and hence V is sgα -open in Y. Since f is strongly sgα -irresolute, then f^{-1}(V) is open in X. Therefore f^{-1}(V) is open in X for every α- open set V in Y. Hence f is strongly α-irresolute.

Theorem:3.4

If f: X→Y is a continuous and Y is a sgα -$T_{1/2}$-space , then f is strongly sgα -irresolute.

Proof: Let V be sgα -open in Y. Since Y is sgα -$T_{1/2}$-space, V is α-open in Y and hence open in Y. Since f is continuous, f^{-1}(V) is open in X. Thus,f^{-1}(V) is open in X for every sgα -open set V in Y. Hence f is strongly sgα - irresolute.

Theorem:3.5

If f: X→Y is a sgα -irresolute, X is a sgα -$T_{1/2}$-space , then f is strongly sgα -irresolute.

Proof: Let V be sgα -open in Y. Since f is sgα -irresolute, $f^{-1}(V)$ is sgα -open in X. Since X is a sgα -$T_{1/2}$-space, $f^{-1}(V)$ is α- open in X and hence open in X. Thus, $f^{-1}(V)$ is open in X for every sgα -open set V in Y. Hence f is strongly sgα - irresolute.

Theorem:3.6

Let f: X→Y and g:Y→Z be any functions. Then

(i) g o f: X→Z is sgα -irresolute if f is sgα -continuous and g is strongly sgα -irresolute.
(ii) g o f: X→Z is strongly sgα -irresolute if f is strongly sgα -irresolute and g is sgα –irresolute.

Proof:
(i)Let V be a sgα -open set in Z. Since g is strongly sgα -irresolute, $g^{-1}(V)$ is open in Y. Since f is sgα -continuous, $f^{-1}(g^{-1}(V))$ is sgα -open in X.
⟹ $(g \circ f)^{-1}(V)$ is sgα -open in X for every sgα -open set V in Z.
⟹ (g o f) is sgα -irresolute.

(ii) Let V be a sgα -open set in Z. Since g is sgα -irresolute, $g^{-1}(V)$ is sgα -open in Y. Since f is strongly sgα -irresolute, $f^{-1}(g^{-1}(V))$ is open in X.
⟹$(g \circ f)^{-1}(V)$ is open in X for every sgα -open set V in Z.
⟹(g o f) is strongly sgα -irresolute.

Theorem:3.7

The following are equivalent for a function f: X→Y:
(i) f is strongly sgα -irresolute.
(ii) For each x∈ X and each sgα -open set V of Y containing f(x), there exists an open set U in X containing x such that f(U)⊂V.
(iii) $f^{-1}(V) \subset int(f^{-1}(V))$ for each sgα -open set V of Y.
(iv) $f^{-1}(F)$ is closed in X for every sgα -closed set F of Y.

Proof: (i)⟹(ii):

Let x ∈X and V be a sgα -open set in Y containing f(x). By hypothesis,$f^{-1}(V)$ is open in X and contains x.

Set $U=f^{-1}(V)$. Then U is open in X and f(U)⊂V.

(ii)⟹(iii):

Let V be a sgα -open set in Y and $x \in f^{-1}(V)$.

By assumption, there exists an open set U in X containing x , such that f(U)⊂V.

Then x∈U⊂int(U)

⊂int $(f^{-1}(V))$.
Then $f^{-1}(V) \subset int(f^{-1}(V))$

(iii)⟹(iv):

Let F be a sgα -closed set in Y. Set V= Y – F. Then V is sgα -open in Y.

By (iii), $f^{-1}(V)\subset int(f^{-1}(V))$.

Hence $f^{-1}(F)$ is closed in X.

(iv)$\Longrightarrow$ (i):

Let V be sgα -open set in Y. Let F = Y – V. That is F is sgα -closed set in Y. Then $f^{-1}(F)$ is closed in X,(by (iv)). Then $f^{-1}(V)$ is open in X. Hence f is strongly sgα -irresolute.

Theorem:3.8

A function f: X→Y is strongly sgα -irresolute if A is open in X, then f/A: A→Y is strongly sgα -irresolute.

Proof: Let V be a sgα -open set in Y. By hypothesis, $f^{-1}(V)$ is open in X. But $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is open in A and hence f/A is strongly sgα -irresolute.

Theorem:3.9

Let f: X→Y be a function and $\{A_i: i\in\Lambda\}$ be a cover of X by open sets of (X,τ). Then f is strongly sgα -irresolute if $f/A_i : (A_i,\tau/A_i) \to(Y,\sigma)$ is strongly sgα -irresolute for each $i \in\Lambda$.

Proof: Let V be a sgα -open set in Y. By hypothesis, $(f/A_i)^{-1}(V)$ is open in A_i. Since A_i is open in X, $(f/A_i)^{-1}(V)$ is open in X for every $i\in \Lambda$.

$$f^{-1}(V) = X\cap f^{-1}(V)$$

$$=\cup\{A_i \cap f^{-1}(V): i \in \Lambda\}$$

$$=\cup\{(f/A_i)^{-1}(V): i\in \Lambda\} \text{ is open in X.}$$

Hence f is strongly sgα -irresolute.

Theorem:3.10

Let f: X→Y be a strongly sgα -irresolute surjective function. If X is compact, then Y is SGαO-compact.

Proof: Let $\{A_i: i \in\Lambda\}$ be a cover of sgα -open sets of Y. Since f is strongly sgα -irresolute and X is compact, we get $X \subset\cup\{f^{-1}(A_i): i \in\Lambda\}$.Since f is surjective, $Y = f(X) \subset\cup\{A_i: i\in\Lambda\}$.Hence Y is SGαO-compact.

Theorem:3.11

If f:X → Y is strongly sgα -irresolute and a subset B of X is compact relative to X, then f(B) is SGαO-compact relative to Y.

Proof: Obvious.

Definition: 3.12

A function f: X→Y is said to be

(i)a strongly α- sgα -irresolute function if $f^{-1}(V)$ is α- open in X for every sgα -open set V in Y.
(ii) a strongly β-sgα -irresolute function if $f^{-1}(V)$ is β-open in X for every sgα -open set V in Y.

Theorem:3.13

(i) If f: X→Y is strongly α- sgα -irresolute , then f is strongly sgα -irresolute.

(ii) If f: X→Y is strongly α-sgα -irresolute , then f is strongly β-sgα -irresolute.

Proof: (i)Let f be a strongly α- sgα -irresolute function and let V be a sgα -open set in Y. Then $f^{-1}(V)$ is α- open in X and hence open in X.

⟹$f^{-1}(V)$ is open in X for every sgα -open set V in Y.

Hence f is strongly α- sgα -irresolute.

(ii) Let f be a strongly α- sgα -irresolute function and let V be a sgα -open set in Y. Then

$f^{-1}(V)$ is α-open in X and hence open in X.

⟹$f^{-1}(V)$ is open in X for every sgα -open set V in Y.
⟹ $f^{-1}(V)$ is β-open in X for every sgα -open set V in Y.

Hence f is strongly β- sgα -irresolute.

Remark: 3.14

Converse of the above need not be true as seen in the following examples.

Example: 3.15

(i)Let X= Y = {a,b,c} , τ= {φ, X,{a},{b},{a,b},{a,c}} and σ ={φ, Y,{a},{b},{a,b}}.

Let f:X→Y be an identity map. Here for every sgα -open set V in Y, $f^{-1}(V)$ is open and β-open in X. Hence f is strongly sgα -irresolute and strongly β-sgα -irresolute.

But for every sgα -open set V in Y, $f^{-1}(V)$ is not α- open in X. Thus, f is not strongly α-sgα -irresolute .Hence strongly sgα -irresolute function need not be strongly α- sgα -irresolute function and strongly β-sgα -irresolute function .

Theorem:3.16

If f:X→Y and g:Y→Z , then g o f: X→Z is

(i) strongly sgα -irresolute if f is strongly α- sgα -irresolute and g is sgα -irresolute.
(ii) strongly β-sgα –irresolute if f is strongly sgα - irresolute and g is sgα -irresolute.

Proof: Let V be an sgα -open set in Z. Since g is sgα -irresolute, $g^{-1}(V)$ is sgα -open in Y. Since f is strongly α- sgα -irresolute , $f^{-1}(g^{-1}(V))$ is α- open in X.

⟹$(g \circ f)^{-1}(V)$ is regular open in X and hence open in X.

Hence (g o f) is strongly sgα -irresolute.

(i) Let V be an sgα -open set in Z. Since g is sgα -irresolute, $g^{-1}(V)$ is sgα -open in Y. Since f is strongly sgα -irresolute , $f^{-1}(g^{-1}(V))$ is open in X and hence β-open in X.

⟹$(g \circ f)^{-1}(V)$ is β- open in X for every sgα –open set V in Z.

Hence (g o f) is strongly β- sgα -irresolute.

Theorem:3.17

If f:X→Y and g:Y→Z , then g o f: X→Z is

(i) strongly α- sgα -irresolute if f is regular irresolute and g is strongly α- sgα -irresolute.
(ii) strongly α- sgα -irresolute if f is α- continuous and g is strongly sgα -irresolute.
(iii) strongly β- sgα -irresolute if f is continuous and g is strongly sgα -irresolute.

Proof: Let V be a sgα -open set in Z. Since g is strongly α- sgα -irresolute, $g^{-1}(V)$ is α- open in Y. Since f is α- irresolute , $f^{-1}(g^{-1}(V))$ is α- open in X.

⟹$(g \circ f)^{-1}(V)$ is α- open in X .

Hence (g o f) is strongly α-sgα -irresolute.

(i) Let V be an sgα -open set in Z. Since g is strongly sgα -irresolute, $g^{-1}(V)$ is open in Y. Since f is α- continuous , $f^{-1}(g^{-1}(V))$ is α-open in X.

⟹$(g \circ f)^{-1}(V)$ is α- open in X .

Hence (g o f) is strongly α- sgα -irresolute.

(ii) Let V be an sgα -open set in Z. Since g is strongly sgα -irresolute, $g^{-1}(V)$ is open in Y.

Since f is continuous , $f^{-1}(g^{-1}(V))$ is open in X.

⟹$(g \circ f)^{-1}(V)$ is open in X and hence β-open in X.

Hence (g o f) is strongly β-sgα -irresolute.

Theorem :3.18

The following are equivalent for a function f: X→Y:

(i) f is strongly α-sgα -irresolute.
(ii) For each x ∈X and each sgα -open set V of Y containing f(x), there exists a α- open set U in X containing x such that f(U) ⊂V.
(iii) $f^{-1}(V) \subset Cl(Int(f^{-1}(V)))$ for each sgα -open set V of Y.
(iv) $f^{-1}(F)$ is regular closed in X for every sgα -closed set F of Y.

Proof: Similar to that of Theorem 3.7

Theorem:3.19

The following are equivalent for a function f: X→Y:

(i) f is strongly β- sgα -irresolute.
(ii) For each x ∈X and each sgα -open set V of Y containing f(x), there exists a β- open set U in X containing x such that f(U)⊂ V.
(iii) $f^{-1}(V) \subset Cl(Int(f^{-1}(V)))$ for each sgα -open set V of Y.
(iv) $f^{-1}(F)$ is β-closed in X for every sgα -closed set F of Y.

Proof: Similar to that of Theorem 3.7.

Lemma: 3.20

If f: X→Y is strongly α- sgα -irresolute and A is a α- open subset of X, then f/A : A→Y is strongly α- sgα -irresolute.

Proof:

Let V be a sgα -open in Y . By hypothesis, $f^{-1}(V)$ is α- open in X. But $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is regular open in A. Hence f/A is strongly α- sgα -irresolute.

Theorem:3.21

Let f: X→Y and $\{A_\lambda: \lambda \in \Lambda\}$ be a cover of X by α- open set of (X,τ). Then f is a strongly α- sgα -irresolute function if $f/A_\lambda: A_\lambda \to Y$ is strongly α- sgα -irresolute for each $\lambda \in \Lambda$.

Proof: Let V be any sgα -open set in Y. By hypothesis, $(f/A_\lambda)^{-1}(V)$ is α- open in A_λ. Since A_λ is regular open in X, it follows that $(f/A_\lambda)^{-1}(V)$ is sgα -open in X for each $\lambda \in \Lambda$.

$f^{-1}(V) = X \cap f^{-1}(V)$
$= \cup\{A_\lambda \cap f^{-1}(V): \lambda \in \Lambda\}$
$= \cup\{(f/A_\lambda)^{-1}(V): \lambda \in \Lambda\}$ is regular open in X.

Hence f is strongly α-sgα -irresolute.

Lemma:3.22

If f: X→Y is strongly β- sgα -irresolute and A is a α-open subset of X, then f/A : A→Y is strongly β- sgα -irresolute.

Proof: Let V be a sgα -open in Y . By hypothesis, $f^{-1}(V)$ is β-open in X. But $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is β-open in A.Hence f/A is strongly β- sgα -irresolute.

Theorem:3.23

Let f: X→Y and $\{A_\lambda: \lambda \in \Lambda\}$ be a cover of X by β- open sets of (X,τ). Then f is a strongly β- sgα -irresolute function if $f/A_\lambda: A_\lambda \to Y$ is strongly β- sgα -irresolute for each $\lambda \in \Lambda$.

Proof: Let V be any sgα -open set in Y. By hypothesis, $(f/A_\lambda)^{-1}(V)$ is β- open in A_λ. Since A_λ is β- open in X, it follows that $(f/A_\lambda)^{-1}(V)$ is β-open in X for each $\lambda \in \Lambda$.

$f^{-1}(V) = X \cap f^{-1}(V)$

$= \cup\{A_\lambda \cap f^{-1}(V): \lambda \in \Lambda\}$

$= \cup\{(f/A_\lambda)^{-1}(V): \lambda \in \Lambda\}$ is β- open in X.

Hence f is strongly β- sgα -irresolute.

Theorem:3.24

If a function f: X→Y is strongly β-sgα -irresolute , then $f^{-1}(B)$ is β-closed in X for any nowhere dense set B of Y.

Proof: Let B be any nowhere dense subset of Y. Then Y−B is regular in Y and hence sgα -open in Y. By hypothesis, $f^{-1}(Y-B)$ is β-open in X. Hence $f^{-1}(B)$ is β-closed in X.

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