INQUIRIES IN LINEAR ALGEBRA: ON SYSTEMIC PERSPECTIVE AND THE STRATEGIES IN E-LEARNING

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Abstract: There is presented a teaching experience described with the help of some theoretic elements of the onto semiotic way in. We offer a number, order, group, line of problem-situations of discovery sort which have need of a complete way in to intra-mathematical looks into, with observations ends, which have to do with applications of theoretical makings and methods on the joints in quality example directions, in another general condition. Our secret design is to design a number, order, group, line of theoretical questions, which lead to operation of making observations operation, and to keep the students toward systemic view of mathematical practices through the practices of unitary view. We take into account the onto based on observations about signs way in as an enough theoretical framework which lets us to observe the learning process in the e-modality of having an effect equal to the input algebra direction in order to order the activities so that the students would get done the deeper mind's power to see clearly of the most important ideas of a quality common to a group and their common (to 2 or more) relations.

***IndexTerms* -Epistemic Configuration; Systemic Perspective; Linear Algebra; E-Learning Modality**

I. INTRODUCTION

The most important ideas in linear algebra course to be learned by students are related to the concepts of vector (linear) spaces and linear transformations, which presents certain difficulties due to the abstract nature of the matter. Here we describe a series of activities which will allow the students to achieve the deeper comprehension of the most important concepts and their mutual relations.

Preliminary consideration of problems in the linear algebra course

Analyzing lists of problems in linear algebra courses and the results in recent researches in mathematics education, we noted that there is a lack of problems which would require creative applications of the methods and techniques traditionally taught in these courses. It is rather difficult to find the problems which would indicate the direction of applications of the theoretical facts beyond the themes covered in standard courses, as well as such sort of problems which would involve interdisciplinary considerations. Here we propose some tasks of exploration type which require an integral approach to intra-mathematical inquires with investigation objectives that involve applications of theoretical constructions and methods on the junctions in standard courses at the undergraduate level.

Towards intra and extra-mathematical applications within standard themes of linear algebra courses

In having an effect equal to the input algebra directions, students should be got used to with some able to work methods related to vector (having an effect equal to the input) spaces and having an effect equal to the input great changes. We suggest some first stage problem-situations of in addition or intra-mathematical applications at the early stages of the direction, in order to have to do with students in the mathematical activities of personal cognitions and to outline their attention to a possibly taking place in addition point of view on the same mathematical place, position. For example, normally the lines of numbers come into view as an useful apparatus for making or put right things to represent having an effect equal to the input great changes of guide spaces, nevertheless some groups of lines of numbers may be gave attention to in another general condition, where the close relation geometry, topology and given to getting details methods should be had to do with.

We offer the putting into effect of the open meeting place stage place in order to have to do with all ones taking part of e-modality direction into the discursive experience of problem-situations had a part in to the range of qualities of having an effect equal to the input great changes being like (in some way) to like in size lines of numbers and to antisymmetric ones which are most important in applications to trained workmen using machines, as well the orthogonal and unimodular lines of numbers which make, be moving in geometric great changes used in applications (specially in the knowledge processing machine 3d act or power of seeing). We take into account related to a putting into effect of web technologies such as a web quest which is taken into account as an inquiry-oriented teachings form and size, as well as open meeting place of general discussion of deep ideas of a quality common to a group.

II. Background Information

We consider the Theory of Ontosemiotic Approach as an adequate framework which allows us to analyze the learning process in the e-modality of Linear Algebra course in order to arrange the activities so that the students would achieve the deeper comprehension of the most important concepts and their mutual relations.

Theoretical background of the Ontosemiotic Approach

The Ontosemiotic way in (OSA), taken into account as a joined framework to the work-place of the cognitive and instructional events, emphasizes the part of mathematical operation which is designed to be copied in terms of systems of practices (operative and discursive, adjustment to events to problems answer), forms of first things and processes (Godino, font, Wilhelmi& De Castro, 2009; font, Planas& Godino, 2010). In the process of being able to get money for of these practices there take part different types of first things (questions, language, Arguments, ideas of a quality common to a group, statements, suggestions and ways), which are put into order in knowledge-wise or cognitive forms (being dependent on whether the organization O personal level is taken into thoughts), which in turn produce others mathematical things of higher level being complex. The problem-situations give a higher position to and contextualize the mathematical operation; languages (special signs, system of naming and giving clear, full picture) represent the other things and give note in law as apparatus for making or put right things for acting; Arguments make even the ways, meanwhile statements, suggestions have a relation with the ideas of a quality common to a group. On the other hand the things that come into view as in mathematical operation and those, of more complex nature, which come out of from these experiences, be dependent on the language play activity wittgenstein 1953) in which they take part and might be taken into account from the five faces, sides of dualities offered in OSA. It is important to take into account that selecting something into thought as a first one is rather one in family relation, there is no unlimited of note (Godino, Batanero& font, 2007), because all of them are able to use things and are related with respect to ready, without fear languages (organization frameworks and framing senses of surprise). To get at the details of the mathematical operation got money for by the students in the process of decision of questions, within the OSA theory there taken into account not only the practices Y forms but also the processes formed from the application of the process-product duality to the forms together with other on the point processes such that picture (Godino, Cajaraville, fernandez Gonzato, 2012), idealization ((stone) basin& Contreras, 2008), argumentation, algoritmization, and so on. We use the small useful things of knowledge-wise forms in Onosemiotic way in order to get money for an observations of the mathematical practices done by the students in the process of problems getting answer to, way out of.

Description of the inquiry-oriented proposal

From the view took up in this work the decision of problem-situations is taken into account as the last end, purpose of the mathematical operation. Our secret design is to give effect to a number, order, group, line of theoretical problem-situations, designed with a making a point of on observations operation, and to keep the students toward systemic view of mathematical practices through the practices of unitary view (decision of quality example of a certain sort problems normally given work in the having an effect equal to the input algebra directions). In other words, with each problem-situation there is connected an experience and a knowledge-wise form which permits its being able to get money for. Being rather different in chief quality, there is presented some articulations between them so that it is possible to make certain some relations that be dependent on the level of the generality. taking into account unitary-systemic duality, the relations of putting things into a given form, with the help of the process of generality, give knowledge of the being complex of the mathematical purpose under thought (the lines of numbers in our example) and the range of the spaces where they play important part in the intra-mathematical material fact. (Rondero& font, 2015).

The opening of the complexity-articulation dialectic 8 in the learning process of a mathematical purpose (with the help of ideas of knowledge-wise form and of putting things into a given form) gives a chance to produce criteria of the quality of mathematics in the learning process of the purpose. From one hand, the being complex let to produce a rule for testing of the Representative (taking into thought the order of tasks as a sample of Representative of the being complex of the purpose that is to be did teaching to), from the other hand, the processes of putting things into a given form let concertize the process of connection, which is one of the processes that let to say, talk of the that making a thing beautiful of mathematics had to do with in some order of the given to works.

The process of problems-solving has need of the effect on one another of other processes: argumentations (in each of the unitary practices and between them), reification (in order to put into an orderly way the going past, through from unitary things to the systemic one) and algorithmization (ways). In order to get done a systemic do of higher being complex through the process of reification the students have to get money for the range of unitary experiences. The argumentation processes make certain relations between these unitary practices with the help of thought of certain properties (put clearly within the statements, suggestions). From the systemic view these unitary practices should be put into order as the orders in an order said words to be taken down in writing by the algorithmic processes, thus doing the systemic experience, which permits the students to get done the mind's power to see clearly of the complete sense, value of the deep ideas of a quality common to a group of having an effect equal to the input algebra in intra-mathematical Contexts 11. In general, the systemic do gives go higher to the knowledge-wise form which takes into account all the things which take part in each unitary experience.

3. Epistemic configuration for the task concerned the set of invertible matrices

In this section we would like to illustrate how primary mathematical objects may be organized in epistemic configurations. For example, the object *proposition* is encountered among the links (arguments) which relate one practice with another, meanwhile each step, realized within the object *procedure*, corresponds to every one of the practices organized in the sequences.

Traditionally the epistemic configurations are presented as a table in full page format that require more printed space.

Linear transformations and the group structure in the set of invertible matrix

Problem-situation1:

Consider the set of matrices corresponding to the linear transformations group $GL(n,R)$ as a subset of $M(n,R)$ and describe the variety of mathematical constructions for R^N such as coordinates, metric, topology, neighborhood, regions, series, convergence, etc., in the new environment of $M(n,R)$ and $GL(n,R)$.

Language: vector space, basis, Euclidean structure, matrices, linear transformations, invertible matrix, group structure, denotations (notational representations), topological structure, metric, open set (unit ball). Definitions (Concepts): axioms of n-dimensional vector space, vector space, linear combinations, base, coordinates, linear transformation, general linear group of transformations, isomorphism, metric, open set (open ball), matrix series.

Propositions: (P1) The space of all n X n matrices, denoted as $M(n,R)$, is isomorphic to R^N **,** N= n**.** (P2) Defining properties of the metric. (P3) Relations between the Euclidean metric and topological structures. (P4) Criterion of the convergence of series in metric spaces.

Arguments:(A1.1) According to the proposition (P1) the vector space $M(n,R)$ is isomorphic to R^N , N= n^2, therefore the properties of this standard vector space permit the traditional assigning of coordinates to a vector and its Euclidean norm. Thus for any matrix element

$A = (a_j^i)$, $i,j = 1,2,\dots n,$ the Euclidean metric could be defined as $|A|^2 = \sum_{i,j}|a_j^i|^2$, where a_j^i are the coordinates of an element A in the Standard basis E(ij). (A1.2) In order to define an open set, the interply between the topological structure and Euclidean metric should be employed: Since M(n,R) is a linear space of square matrices A = (a_j^i) , a topological structure is determined by the Euclidean metric, which permits the definition of the unit ball in the space M(n,R):$|X| < 1$, X = (x_j^i) .

Unitary practice 1 (UP1): this unitary practice contains the process of assigning coordinates with respect to the chosen base, introducing the Euclidean metric, verification of the axioms of metric defining open sets.

Arguments: (A2) Now the serious obstacles should be overcome if we would like to transfer the same structures to the subset of invertible matrices GL(n, R) regarding its group structure: namely, how one should determine a neighborhood of unit element, and furthermore special attention should be paid to assure that all the matrices from a chosen neighborhood be still invertible. To guarantee that any matrix of a neighbourhood of unit element be invertible, the interplay with the higher algebra, using the division algorithm of unity I by (A - I), should be involved giving rise to matrix series which provides the inverse matrix. This algorithm suggests considering the product of matrices: AB = (I+X) (I - X + X^2 - X^3 + ... + $(-1)^n X^n$ + ...) = I. This leads to introduce the inverse matrix in the form

A^{-1} = B. Nevertheless, special caution should be taken to demonstrate that the series.

I - X + X^2 - X^3 + ... + $(-1)^n X^n$ + ... converge, in order to guarantee the inverse matrix be well defined. The question of converging of this matrix series plays a crucial role. A sequence of partial sums in the matrix space should be considered so that one could demonstrate that it is the fundamental sequence, i.e., the series converge. This will prove that the matrix A = (I + X) is invertible when $|X| < 1$ and as result it is shown that, for $|X| < 1$, A = (I +X) ∊ GL (n, R).

Unitary Practice (UP2). The corresponding unitary practice requires construction of the corresponding open unit ball in the neighbourhood of the unit matrix, in order to preserve the property of invertibility of matrices within such neighbourhood. The new coordinates (x_j^i) of the matrix A= (a_j^i), i,j = 1,2, ...n, in the neighborhood $|A - E|$< 1 of unit element E ∊ GL(n,R), are determined as x_j^i(A) = a_j^i - δ_j^i, giving x_j^i(E) = 0 as desired.

Arguments (A3). Naturally a question arises about the coordinates in a neighbourhood U_D centered at arbitrary element of $D \in GL(n,R)$. The group structure permits multiply every element of U_D byD^{-1}, so that the whole neighborhood U_D will be displaced to the neighborhood of the unit element, where the coordinates have been just determined.

Unitary Practice (UP3). The corresponding unitary practice involves interplay with the group structure of GL(n,R) to construct the so-called local coordinate system. The real importance of the geometric nature will be disclosed in the next unitary practice.

Velocity of the curves in GL(n,R)

Problem-Situation 2.

Since GL(n,R) is an open set in M(n,R) in the topology introduced in previous considerations it is possible to define curves and try to determine the corresponding velocity vector, as in the traditional three-dimensional space.

Unitary practice (UP4) consists of description of the notions which are characteristic for R^N, such as continuity, differentials, curves, tangent vectors, superficies, in the new environment of M(n,R)y GL(n,R). A curve in GL(n,R), may be considered as an element whose coordinates are differentiable functions of one parameter, say A(t). We require A(0) = E, which means that the curve passes through the unit element when the parameter value is 0. As in the case of R^N, there should exist tangent vector (velocity), which in our new context is the derivative of the matrixA(t), denoted as $A'(t)|_{t=0}$.

It is always a surprise for students that any matrix from M(n,R) serves as the *tangent* to some element of GL(n,R) (although they can visualize that any vector serves as velocity to some trajectory). The geometric meaning of this is that all vectors tangent to GL(n,R) at unit element belong to the space of all matrices of order n.

4. PROBLEMS RELATED TO GEOMETRY

Naturally, similar questions there arise for the tangent spaces of SL(n,R) and O(n,R), where O(n,R)and SL(n,R) are subgroups of GL(n,R) determined by specifications of the properties of its elements, dictated primarily by geometric considerations, and expressed through certain algebraic relations.

Geometry of the sets of matrices SL(n,R) and O(n,R)

We give a brief description of exploration and geometric interpretations of the matrix groups SL(n,R) and O(n,R) and their tangent spaces**.**

Problem-Situation 3.

Describe the tangent spaces at unit element of the surfaces formed by the subsets SL(n,R)and O(n,R) in GL(n,R).

Definitions: (D1) in the Euclidean space the system $f_{i(u_1,u_2,\dots,u_m)}, i = 1,\dots,k$ describes a regular surface of dimension d = m-r if the Jacobi matrix has rank r. (D2) The set SL(n,R) is determined by the requirement set A=1, i.e., there is only one equation (functional relation) which determines the surface of dimension n^2-1,which is called hypersurface, by definition (the rang of Jacobi matrix is equal to 1 here). (D3) The set O(n,R) is determined by the requirement on its elements:A^t A =E.

Theorem: a surface is regular at a given point if its tangent space has dimension d = m-r , where the Jacobi matrix has a rank r.

Unitary practice (UP5) consists of counting dimensions of subspaces and finding tangent objects (using interplay with the continuity and differentiations). In particular, the set O(2,R) of such matrices is characterized by three relations: they could be denoted $f_i(a,b,c,d) = 0, i = 1,2,3.$ And in the neighborhood of E three variables can be expressed in terms of the one variable: $a^2 + b^2 = 1$,

$c^2 + d^2 = 1, (ac + bd) = 0.$

The geometrical meaning is that this set represents a one dimensional subset (a curve) because the Jacobi matrix has rank 3.
The analogous consideration for SL(n,R) gives a three dimensional surface in M(n,R).

Construction of vector spaces tangent to the superficies of SL(n,R) and O(n,R)at unit element

The tangent vector to an orthogonal matrix A, A^tA =I is just a skew symmetric matrix. The dimensions coincide.In the case of unimodular matrices the tangent vectors are the matrices with the zero trace. Both discoveries need corresponding neat treatment which bring a surprise to be able to differentiate a matrix and furthermore a determinant.

The unitary practice (UP6) establishes that SL(n,R) is a regular superficies in the space of all matrices.

Procedure: First of all we demonstrate that the unit element (point E in SL(n,R) is regular point of this hyper surface. It is sufficient to show that the tangent space for SL(n,R) at this point has the dimension exactly n^2 - 1. Students have to consider an arbitrary curve A(t)$\in SL(n,R)$, passing through unit element when t is equal to zero: A(0) = E. Then this curve satisfy the polynomial relation det(A(t)) =1. Thus for the tangent element $X = \frac{d}{dt}\,(\det A(t))|_{t=0}$ we obtain the relation Sp(X) = 0. (Sp denotes the trace of matrix.) This result can be obtained by the direct differentiation of determinant taking into account that the coefficients of the equations are the elements of the Jacobi matrix $\partial\,(\det A)/\partial a_j^i$, A = (a_j^i) , when A=E. Thus the equation Sp(X) = 0 determines tangent space to SL(n,R) at the unit point. It determines all the matrices with trace zero, and the dimension of this space is just n^2 - 1.Thus, it is proved that the unit point E is regular.

Students should demonstrate that any point B is regular (as in the (UP2).

Further algebraic constructions in the vector space of matrices

Problem-Situation 4 (applications to computer 3D Vision).

It easy to see that any 3d skew symmetric matrix is related to 3d-vector, and the vector product is related to Lie bracket AB - BA in this case, thus our 3d vector space with the vector product is a Lie Algebra. The natural question arises, what is the corresponding local Lie group. This gives a simple way to introduce quaternions and obtain an algebraic operation on the sphere, which is norm preserving. Such sort of questions are considered in optional courses of Differential Manifolds, Lie Groups and Lie Algebras serving as trivial examples, to which usually a reference is made without close considerations, leaving such tasks as individual work, because there are many special questions to treat within such courses.

CONCLUSION

The opening of the unitary-systemic duality permits the reformulation of the bright act or power of seeing that there is the same mathematical purpose with the different pictures of (as the lines of numbers in our example) (Rondero& (stone) basin, 2015). In fact, there is just a complex system of practices which let to get answer to the problems where the mathematical purpose matrix does not come into view as clearly, with detail. There come into view as the pictures of the lines of numbers, different clear outlines of the lines of numbers (not only as the simple agreement of numbers), statements, suggestions and properties of them, procedures and techniques which are sent in name for to the special groups of lines of numbers and the being like (in some way) argumentations. As an outcome of the process of the of, based on history development there have been undergone growth different knowledge-wise forms which have to do with the lines of numbers so that some classes of them can be taken into account as a generality of the first idea of a quality common to a group. Under this view, we take into account the Theory of onto based on observations about signs way in as an enough framework which lets us to get at the details of the learning process in the e-modality of having an effect equal to the input algebra direction in order to order the activities so that the students would get done the deeper mind's power to see clearly of the most important ideas of a quality common to a group and their common (to 2 or more) relations. We use the small useful things of knowledge-wise forms in On based on observations about signs way in order to get money for an observations of the mathematical practices done by students in the process of problems getting answer to, way out of. The number, order, group, line of activities which we have designed for our students have need of higher level of having thoughts, where the join by heating of theoretical points to be taken into account, given to getting details methods and operative techniques is needed. These activities put ball in play as a first stage experience in operation of making observations before their thesis undertakings.

REFERENCES

[1] Font, V. & Contreras, Á. (2008). The problem of the particular and its relation to the general in mathematics education. Educational Studies in Mathematics, 69(1), pp. 33-52.
[2] Font, V., Planas, N., & Godino, J. D. (2010). Modelo para el análisis didáctico en educación matemática. Infancia y Aprendizaje, 33(1), pp. 89- 105.
[3] Godino, J. D., Batanero, C., & Font, V. (2007). The ontosemiotic approach to research in mathematics education. ZDM-The International Journal on Mathematics Education, 39(1-2), 127-135.
[4] Godino, J. D., Font,V., Wilhelmi, M. R., & De Castro,C. (2009). Aproximación a la dimensión normativa en didáctica de las matemáticas desde un enfoque ontosemiótico. *Enseñanza de las Ciencias*, 27(1), pp. 59–76.
[5]Godino, J. D., Cajaraville, T., Fernández, T. & Gonzato, M. (2012). Una aproximación ontosemiótica a la visualización en educación matemática. *Enseñanza de las ciencias*, 30(2), pp. 109-130.
[6]Rondero, C., & Font, V. (2015). Articulación de la complejidad matemática de la media aritmética. *Enseñanza de las Ciencias*, *33*(2), 29-49. Wittgenstein, L. (1953). *Philosophical investigations*. New York, NY: The MacMillan Company
[7]K. Shima, M. Tsuda, Mod. Phys. Lett.A 23 (2008) 3149.
[8]K. Shima, M. Tsuda, Phys. Lett. B 666 (2008) 410.
[9]M. Tsuda, Commutator-based linearization of N=1nonlinear supersymmetry, arXiv:1609.07210 [hep-th].
[10] A.A. Zheltukhin, Linear space of spinor monomials and realization of the Nambu–Goldstone fermion in the Volkov–Akulov and Komargodski–Seiberg La-grangians, arXiv:1012.2568 [hep-th].