CRITICAL HINDSIGHT OF PERT NETWORK BY MONTE CARLO SIMULATION AND ANT COLONY OPTIMIZATION ALGORITHM

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Abstract: Complex innovative work ventures can be managed effectively if the project managers have the means to plan and control the schedule and cost of the work required to accomplish their technical performance destinations. The direct methods used for solving project management problems are critical path method (CPM) and project evaluation and review technique (PERT). The effectiveness of direct methods in modeling the complex situation is limited when uncertainty in terms of time duration arise in real world situation. Monte Carlo Simulation process for uncertain time duration of activities will be discussed briefly. Furthermore, Optimization methods yield better approximation for the uncertain real-world situations. The ant colony optimization (ACO) algorithms are applied to solve many complex combination optimization problems, such as traveling salesman problem, the vehicle routing problem, the problem of graph coloring, the quadratic assignment problem etc. Here an ant colony optimization (ACO) approach for the PERT network is presented.

***Index Terms* –PERT, CPM, ACO, Monte-Carlo-Simulation.**

I. INTRODUCTION

A project is well defined set of activities, all of which must be completed to finish the project. Program Evaluation and Review Techniques (PERT) and Critical Path Method (CPM) are the techniques of operations research used for planning, scheduling and controlling of large and complex projects. These techniques are based on representation of the project as a network of activities. A network is a graphical plane consisting of a certain configuration of arrows and nodes for showing the logical sequence of various activities to be performed to achieve project objectives. PERT and CPM both of them share in common determination of critical path available in the network [1], [2].

As we know most of the activities in real world are uncertain so PERT and CPM failure in most of the cases. In this kind of situation one can make approach of simulation of activities of the project. Here we will apply concepts of Monte-Carlo Simulation for Simulation of PERT network [3].

In several situations when we talk about optimization method for batter approximation of activities of the project. Here we present detailed study for application of concepts of Ant Colony Optimization as in [7, 8 and 9] for solving project management problem.

II. MONTE-CARLO SIMULATION

Simulation is the imitation of the operation of a real-world process or system over time [3]. The act of simulating something first requires that a model be developed; this model represents the key characteristics or behaviors/functions of the selected physical or abstract system or process. The model represents the system itself, whereas the simulation represents the operation of the system over time.

In simulation, probability distributions are used to define numerically outcomes in a sample space by assigning a probability to each of possible outcomes. The principal behind the Monte Carlo Simulation technique is representative of the given system under analysis by system described by some known probability distribution and then drawing random samples from probability distribution by means of random numbers. In case it is not possible to describe a system in terms of standard probability distribution such as normal, Poission, exponential, gamma, etc., an empirical probability distribution can be constructed [4] [5].

2.1 Steps for Monte-Carlo Simulation

Following are the steps used for simulation of PERT network by Monte Carlo Simulation:

Step 1: To analyze variables for setting up probability distribution.

Step 2: For each random variable construct a cumulative probability distribution.

Building a cumulative probability distribution for each random variables.

Step 3: Generate random numbers. Assign an appropriate set of random numbers to represent value of interval of values for each random variable.

Step 4: Conduct the simulation experiment by means of random sampling.

Step 5: Repeat Step 4 until the required number of simulation run has been generated.

Step 6: Design and implement a course of action and maintain control.

2.2 Random Numbers

Monte Carlo Simulation requires the generation of a sequence of random numbers. This sequence of random number used for choosing random observations from the given probability distribution. Random Numbers can be generated by Arithmetic computation and by Computer generator [6].`

2.2.1 Arithmetic computation

The n^{th} random number r_n consisting of k-digits generated by congruential method given by

$$r_n \equiv p.r_{n-1}(\text{modulo } m) \quad (1)$$

where p and m are positive integers, p < m, r_{n-1} is a k-digit number and modulo m means that r_n is the remainder when $p.r_{n-1}$ is divided by m. To start the process of generating random numbers, the first random number (also called seed) r_0 is specified by user. Then using above recurrence relation a sequence of k-digit random number with period h < m at which point the number r_0 occurs again can be generated.

2.2.2 Computer generator

The random numbers that are generated by using computer software are uniformly distributed fractions between 0 and 1. The software works on the concept of cumulative distribution function for the random variables for which we are seeking to generate random numbers.

For example, the negative exponential function with density function with density function

$$f(x) = \lambda e^{-\lambda x} dx = 1 - e^{-\lambda x} \quad (2)$$

$$e^{-\lambda x} = 1 - f(x)$$

Taking log on both sides, we have

$$\lambda x = \log[1 - f(x)]$$

$$x = -(1/\lambda)\log[1 - f(x)]$$

If r = f(x) is uniformly distributed random decimal fraction between 0 and 1, then the exponential variable associated with r is given by

$$x_n = -(1/\lambda)\log(1 - r) = (1/\lambda)\log r \quad (3)$$

III. Ant Colony Optimization for PERT Network

Ant Colony Optimization is a meta heuristic technique for hard discrete optimization problems that was first proposed in the early 1990s. One of the problems that was first proposed in the understand how almost blind insects like ants could manage to establish shortest route paths from their colony to feeding sources and back. When searching for food, ants initially explore the area surrounding their nest in a random manner. As soon as an ant find a food source. It evaluates quantity and quality of the food and carries some of the found food to the nest. During the return trip, the ant deposits a chemical pheromone trail on the ground. The quantity of pheromone deposited, which may depend on the quantity and quality of food, will guide other ants to the food source [7]. While an isolated ant moves essentially at random, an ant encountering a previously laid trail can detect it and decide with high probability to follow it, thus reinforcing the trial with its own pheromone. The collective behavior that emerges is a form of automated catalytic behavior where the more the ants following a trial, the more attractive that trial becomes for being followed. The process is characterized. Also pheromone concentration vanishes with time and then the less used path will be much lower in pheromone concentration. The indirect communication between the ants via the pheromone trial allows them to find shortest paths between their nest and food source. In Figure 3.1, "A" shows the real ants following a path between neat and food source [8]. "B" shows an obstacle appears on the path, "C" and "D" shows that the pheromone is deposited faster on the shorter path, and all ants have chosen the shorter path.

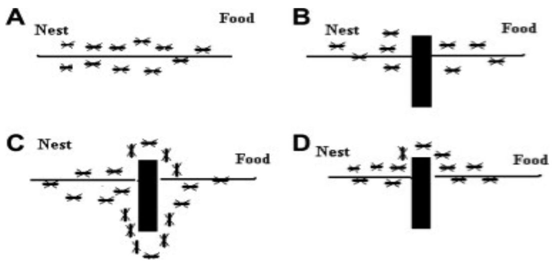

Figure 3.1: Real ants following a path between nest and food source

3.1 ACO based Algorithm

For a given PERT project schedule defined by a graph G = (N, A) with N being the set of nodes (activities) and A being the set of arcs (activity relationships) connecting the subject nodes, the proposed ACO-based procedure for finding the critical path(s) between chosen nodes N1 and N2 can be summarized by the following steps:

1. Initialize all arcs with small amount of pheromone, τ_0. This value can be an inverse line-distance between the nodes N_1 and N_2, or the inverse line-distance of the subject arc.

2. An artificial ant is launched from node N_1 (the start node) pseudo-randomly walking from a node to a successor node via the connecting arcs until it reaches either the end-node (N_2) or a dead end. When at a given node, the artificial ants selection of an arc to follow is probabilistic, based on a stochastic assignment of each i^{th} arc likelihood of selection, as defined by equation 4.

$$p_i = \frac{\tau_i}{\sum_i \tau^i \eta_i^{\beta}} \quad (4)$$

3. In the above equation, τ_i is the pheromone concentration on the i^{th} arc, η_i is a priori available heuristic value for the i^{th} arc and β_i is a parameter determining the relative influence of the heuristic information. The value of η_i can be defined either as the inverse of the length of the arc, or the inverse of the length of the arc plus the line-distance between the subject node and N_2. It should be noted that previously visited arcs are excluded from the selection.

a. Selection is further assisted by the consideration of a randomly generated number, $0 \leq q \leq 1$, which is compared to a predefined value, q_0 and specific to the network topology. If $q_0 \leq q$ then the arc with the highest value p_i will selected. Otherwise, a random selection of an arc is used.

4. Upon crossing each i^{th} arc during the aforementioned solution-constructing phase a local pheromone update rule is applied to update the level of pheromone concentration at the given arc. The updated pheromone level is defined by equation 5.

$$\tau_i = (1-p)\tau_i + \rho\tau_0 \quad (5)$$

Where ρ is another network topology parameter ($0 \leq \rho \leq 1$). The goal of the local updating is to enable exploration of more path/route variations by making already traversed arcs less likely to be chosen again during the randomization of the arc selection process.

5. Steps (2)-(3) are repeated for all ants and most successful ant is used to globally update the network's pheromone trails. The global update rule defined by equation 6.

$$\tau_i = (1-\alpha)\tau_i + \alpha\tau_L \quad (6)$$

where α is yet another network topology parameter ($0 \leq \alpha \leq 1$) whose value determines level of evaporation of pheromone concentrations. The factor τ_L is a value inversely proportional to path length of best solution in case of an arc visited by the best ant or zero for all other ants.

a. The global update rule can be applied by either the "global-best" or by the "iteration best" ant. In the first case, the ant to perform the update is the one that obtained the best solution during the entire optimization process. In the second case the update is performed by the ant reaching the best solution during each iteration of the algorithm.

6. Steps (2)-(4) are repeated for either a fixed number of iterations or until a predefined condition is met, and upon termination of the algorithm the pheromone trail in the graph G=(N,A) is used to determine the solution with highest pheromone concentration.

IV. SOLUTION OF PERT NETWORK

Consider the following example Figure 4.1 of PERT network with given activities and their time duration.

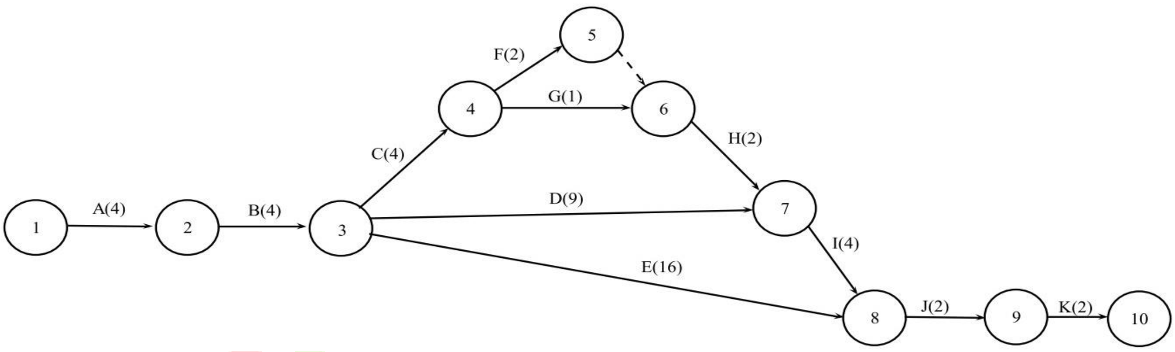

Figure 4.1

Activity denoted by dotted line is called dummy activity. Our target is to find critical path from the given PERT network.

4.1 Solution by Monte-Carlo Simulation

In real world whenever uncertainties arise in the activity time duration can be solved by Monte-Carlo Simulation method. Considering the given example with its probability distribution is shown in Table 4.1.

Table 4.1

Activity	Probability / Day															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A		0.3		0.5			0.2									
B			0.1	0.7					0.2							
C		0.2			0.6			0.2								
D						0.1		0.1	0.7			0.1				
E									0.2	0.2			0.2			0.4
F		0.4	0.6													
G	0.7			0.3												
H		0.6												0.4		
I	0.1			0.3											0.6	
J		0.4	0.6													
K	0.2	0.6		0.2												

The network diagram based on the precedence relationship and expected time duration is shown in Figure 4.2.

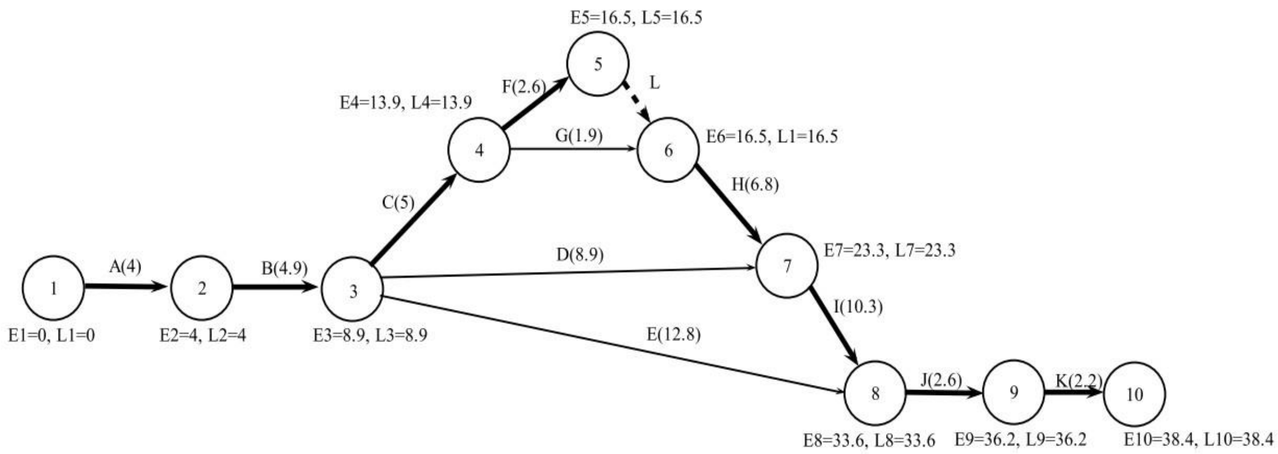

Figure 4.2

The expected completion time of each activity can be calculated by the equation:

$$\text{Expected time} = \sum(\text{Activity Time} \times \text{Probability}) \quad (9)$$

The simulation sheet for simulation run given in Table 4.2.

Table 4.2

Activity	Time	Probability	Cumulative Probability	Random number range
A	2	0.3	0.3	00-29
	4	0.5	0.8	30-79
	7	0.2	1	80-99
B	3	0.1	0.1	00-09
	4	0.7	0.8	00-79
	9	0.2	1	80-99
C	2	0.2	0.2	00-19
	5	0.6	0.8	20-79
	8	0.2	1	80-99
D	6	0.1	0.1	00-09
	8	0.1	0.2	10-19
	9	0.7	0.9	20-89
	12	0.1	1	90-99
E	9	0.2	0.2	00-19
	10	0.2	0.4	20-39
	13	0.2	0.6	40-59
	16	0.4	1	60-99
F	2	0.4	0.4	00-39
	3	0.6	1	40-99
G	1	0.7	0.7	00-69
	4	0.3	1	70-99
H	2	0.6	0.6	00-59
	14	0.4	1	60-99
I	1	0.1	0.1	00-09
	4	0.3	0.4	10-39
	15	0.6	1	40-99
J	2	0.4	0.4	00-39
	3	0.6	1	40-99
K	1	0.2	0.2	00-19
	2	0.6	0.8	20-79
	4	0.2	1	80-99

Simulation worksheet given by Table 4.3 can be generated by using data given in Table 4.2.

Table 4.3

Run	R	A	R	B	R	C	R	D	R	E	R	F	R	G	R	H	R	I	R	J	R	K
1	22	2	7	3	68	5	99	12	15	9	68	3	95	4	23	2	29	4	9	2	85	4
2	92	7	95	9	81	8	9	6	28	10	95	3	6	1	87	14	88	15	61	3	52	2
3	2	2	22	4	57	5	51	9	89	16	24	2	82	4	3	2	7	1	32	2	5	1

4	47	4	99	9	16	2	17	8	92	16	46	3	13	1	9	14	38	4	92	3	30	2
5	56	4	37	4	13	2	85	12	52	13	5	2	90	4	62	14	91	15	27	2	62	2
Sum		19		29		22		47		67		13		14		46		39		12		11
Ave.		3.8		5.8		4.4		9.4		13		2.6		2.8		9.2		7.8		2.4		2.2

Where in Table 3 the column R gives values of random numbers.
For each run the project time is obtained as follows and we have simulation result shown in Table 4.4:

Table 4.4

Simulation Run	Activity Time	Project Duration
1	2+3+5+2+9+2+4	37
2	7+9+8+14+15+3+2	58
3	2+4+5+12+9+2+1	35
4	4+9+2+14+16+3+2	51
5	4+4+4+15+2+2	31

4.2 Solution of a PERT Network by ACO:

Performing first iteration for 10 ants considering all arcs with small amount of pheromone value, for probability equation consider value of $\beta = 1$ and all other values of τi, ηi and probability by equation 4. After applying Ant Colony Optimization Algorithm for 10 ants globally updated pheromone values as in table 4.5 and result of selected path as in Table 4.6.

Table 4.5 Globally updated pheromone values

τ_i	globally updated τ_i	τ_i	globally updated τ_i	τ_i	globally updated τ_i	τ_i	globally updated τ_i
τ_1	0.185713	τ_4	0.077	τ_7	0.7	τ_{10}	0.175
τ_2	0.185713	τ_5	0.054463	τ_8	-	τ_{11}	0.360713
τ_3	0.175	τ_6	0.35	τ_9	0.35	τ_{12}	0.360713

After first iteration of 10 ants the result is shown in tabular list(Table 4.6).

Table 4.6 Result of selected path for 10 ants

Ant	Selected Path	Duration	Ant	Selected Path	Duration
1	1-2-3-7-8-9-10	25	6	1-2-3-8-9-10	28
2	1-2-3-8-9-10	28	7	1-2-3-4-5-6-7-8-9-10	24
3	1-2-3-4-6-7-8-9-10	23	8	1-2-3-8-9-10	28
4	1-2-3-4-6-7-8-9-10	23	9	1-2-3-4-8-9-10	25
5	1-2-3-7-8-9-10	25	10	1-2-3-4-5-6-7-8-9-10	24

In next iteration of 20 ants similarly updated probabilities, global values can be calculated and the result is shown in Table 4.7.

Table 4.7 Result of selected path for 20 ants

Ant	Selected Path	Duration	Ant	Selected Path	Duration
11	1-2-3-4-5-6-7-8-9-10	24	16	1-2-3-7-8-9-10	25
12	1-2-3-7-8-9-10	25	17	1-2-3-8-9-10	28
13	1-2-3-8-9-10	28	18	1-2-3-4-5-6-7-8-9-10	24
14	1-2-3-7-8-9-10	25	19	1-2-3-4-6-7-8-9-10	23
15	1-2-3-4-5-6-7-8-9-10	24	20	1-2-3-8-9-10	28

Repeating this iterations for approximately 1000 ants [10], [11]. We will find conclusion that approximately all ants will choose path 1-2-3-8-9-10 which is critical path. Ant Colony Optimization technique is very efficient when we want to implemented to expert system.

IV. RESULTS AND DISCUSSION

4.1 Results of Monte Carlo Simulation

The method provides an accurate result even during the constraint of the time uncertainty. Table 4.4 shows total project duration of 212 days. The Critical Path is 212/5 = 42.4 days which is close to expected critical path that is 38.4 days.

4.2 Result of ACO Based Algorithm

This method is flexible enough to adjust and study the best possible critical path for any modification carried out during execution of the project. This proposed method over traditional optimization algorithm is an ability of the Ant system to produce accurate solution.

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