STRUCTURE OF FUZZY IDEALS IN ORDERED SEMIRINGS

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Abstract

In this paper, the concept of fuzzy set is applied to define fuzzy subgroups and fuzzy ideals with an identity element and ideal in semirings have been changed into fuzzy ideal in semiring and we have given examples for left and right ideal in ordered ideal.

Keywords

Fuzzy ideals, fuzzy left and right ideals, semirings, ordered semirings, hemirings

Introduction

In this paper L.A Zadeh introduced the concept of a fuzzy set in 1965.For the first time and this concept was applied by rosenfeled to define fuzzy subgroups and fuzzy ideals .On the other hand, No and wang defined some kinds of fuzzy ideals generated by fuzzy subsets in a semiring with an identity element.Semirings which provide a common generalization of rings and distributive lattices arise naturally in such diverse areas combinatorics, functional, analysis, graph theory, automata theory, mathematical modeling and parallel computation system etc. Semirings have also been proved to be an important algebraic tool in theoretical computer science. Many of the semirings, such as N, have an order structure in addition to their algebraic structure and indeed the most interesting results concerning them make use of the interplay between these two structures . Ideals of semirings play an important role in the structure theory of ordered semirings and useful for many purposes.

Semirings, ordered Semirings and hemirings appear in a natural manner in some applications to the theory of automata and formal languages. The concepts of Universal algebras generalizing an associative ring (R,+).The second type of those algebras (S,+,.), called Semirings(and for sometimes halfrings),share the same properties as a ring except that (S,+) is assumed to be a semigroup rather than a commutative group.Ideals of semirings play a central role in the structure theory and are useful for many purposes.However, they do not in general coincide with the usual ring ideals if S is a ring and, for this reason, their use is some what limited in trying to obtain analogues of ring theorems for Semirings .Indeed, many results in rings apparently have no analogues in Semirings using only ideals. We can see an h-ideal of the hemiring S. The properties of h-ideals hemirings were thoroughly investigated by La Torre and by using the h-ideals, La Torre established some analogous ring theorems for hemirings.

Finally, the concept of h-hemiregularity as a generalization of the regularity in rings. Next we describe Prime fuzzy h-ideals of hemirings and characterize prime fuzzy h-ideals of h-hemiregular hemirings by fuzzy h-ideals. Also the properties of normal and maximal fuzzy left h-ideals of hemirings.

Definition

A non empty set R with binary operation + and . If the following axiom hold,

i. $(\boldsymbol{R},+)$ **is** an abelian group
ii. For all a,b,c ∈ R such that

(R, .) semigroup. Then (R, +, .) is called a Ring.

Definition

A semiring is a system consisting of a non-empty set S on which operations addition and multiplication (denoted in the usual manner).Have been defined such that (s, +) is a semigroup (s, .) is a semigroup and multiplications distributes over addition from either side.

A zero element of a semiring S is an element 0 such that

$0.x = x.0 = 0$ and $0 + x = x + 0 = x$,for all $x \in S$.

A semiring S is zero sum free if and only if $S + S = 0$

$$\Rightarrow S = S = 0.$$

Definition

A left ideal I of semiring S is non-empty subset of S satisfying the following conditions

i. $a, b \in I$, then $a + b \in I$
ii. If $a \in I$ and $s \in S$, then $\mu(s\ a) \in I$

An right ideal of S is defined in an analogous manner and an ideal of S is a non-empty subset which is both a left and right ideal of S.

Definition

A non-empty subset A of S is said to be an interior ideal if it is closed under addition,

$$A^2 \subseteq A \text{ and } SAS \subseteq A$$

Definition

An ordered semirings is a S equipped with a partial order $\leq$ such that the operation is monotonic and constant 0 is least element of S.

Definition

A left (resp. right) ideal I of S is called a left (resp. right) ordered ideal if for any $a \in S$, $b \in I$, $a \leq b$ implies $a \in I$.

(i.e) $(I \subseteq I)$.

I is called an ordered ideal of S.

If it is both a left and a right ordered ideal of S.

Example

Let $S=([0,1], \vee, . \circ)$ where [0, 1] is the unit interval $a \vee b = \max [a ,b]$ and $a.b = (a+b - 1) \vee 0$ for $a ,b \in [0,1]$. Then it is easy to verify that S equipped with the usual ordering $\leq$ is an ordered semiring and $I = [0 , \frac{1}{2}]$ is an ordered ideal of S.

Definition

The union and intersection of two fuzzy subsets μ and σ of a set S,

denoted by $\mu \cup \sigma$ and $\mu \cap \sigma$ respectively , are defined as,

$(\mu \cup \sigma)(x) = \max\{\mu(x), \sigma(x)\}$ for all $x \in S$,

$(\mu \cap \sigma)(x) = \min\{\mu(x), \sigma(x)\}$ for all X∈ S.

Definition

Let μ and v be two fuzzy subsets of an ordered semirings S and

x, y, z ∈ S. we define composition and sum of μ and γ as follows:

$$\mu \circ_1 \gamma(x) = Sup_{x \leq yz}\{\min\{\mu(y), \gamma(z)\}\}$$

$= 0$, if cannot be expressed as x≤yz

and,

$$\mu +_1 \gamma(x) = Sup_{x \leq y+z}\{\min\{\mu(y), \gamma(z)\}\}$$

$= 0$, if cannot be expressed as x≤y+z.

Proposition

For any fuzzy subset μ of an ordered semiring S,

$(\chi_s \circ_1 \mu(x) \geq (\chi_s \circ_1 \mu)(y)(resp. (\chi_s +_1 \mu)(x) \geq (\chi_s +_1 \mu)(y)\ \forall x, y \in S$ with x≤ y.

Proof

Let μ be a fuzzy subset of an ordered semiring S and x,y∈ S with x≤y.

If y cannot be expressed as y≤ $y_1 y_2$ for $y_1, y_2 \in S$,

Then the proof is trivial.

Let y have such an expression. Then,

$$(\chi_s \circ_1 \mu)(y) = Sup_{y \leq y_1 y_2}\{\min\{\chi_s(y_1), \mu(y_2)\}\}$$

$$(\chi_s \circ_1 \mu)(y) = Sup_{y \leq y_1 y_2}\{\mu(y_2)\}$$

Since, x≤y≤ $y_1 y_2$

we have

$$(\chi_s \circ_1 \mu)(x) = Sup_{x \leq x_1 x_2}\{\min\{\chi_s(x_1), \mu(x_2)\}\}$$

$$= Sup_{x \leq x_1 x_2}\{\min\{\chi_s(y_1), \mu(y_2)\}\}$$

$$= Sup_{y \leq y_1 y_2}\{\mu(y_2)\}$$

$$(\chi_s \circ_1 \mu)(x) = (\chi_s \circ_1 \mu)(y).$$

Similarly, For x≤ y, we can prove that

$$(\chi_s +_1 \mu)(x) \geq (\chi_s +_1 \mu)(y).$$

Definition

Let μ be a non-empty fuzzy subset of an ordered semiring S .

(i.e),$\mu(x) \neq 0$ for some x$\in S$).

Then μ is called a fuzzy left ideal (resp.fuzzy right ideal) of S if,

i. $\mu(x+y) \geq \min\{\mu(x),\mu(y)\}$

ii. $\mu(xy) \geq \mu(y)[resp.\mu(xy) \geq \mu(x)]$and

iii. x≤y implies μ(x)≥ $\mu(y)$ *for all* $x,y \in S$.

By a fuzzy ideal we mean,

It is both a fuzzy left ideal as well as a fuzzy right ideal.

Definition

Let μ be a fuzzy subset of an ordered semiring S and a∈ S. we denote $I_a$the subset of S defined as follows. $I_a = \{b \in S/\mu(b) \geq \mu(a)$.

Proposition

Let S be an ordered semiring and μ be a fuzzy right (resp.left) ideal

of S. Then I_a is a right (resp.left) ideal of S for every a∈ S .

Proof

Letμ be a fuzzy right ideal of S and a∈ S.

Then $I_a \neq \emptyset$ because a∈ I_a for every a∈ S.

Let b, c∈ I_a and $x \in S$.

Since,

b, c∈ I_a ,

$\mu(a) \geq \mu(b)$ *and* $\mu(c) \geq \mu(a)$

Now,

$\mu(b+c) \geq \min\{\mu(b),\mu(c)\}$ [∴ μ is a fuzzy right ideal]

$\mu(b+c) \geq \mu(a)$ which implies b+c∈ I_a.

Also $\mu(bx) \geq \mu(b) \geq \mu(a)$

(i.e) b $x \in I_a$

Let b∈ I_a and S∋ $x \leq b$.

Then $\mu(x) \geq \mu(b) \geq \mu(a) \Rightarrow x \in I_a$.

Thus I_a is a right ideal of S.

Similarly,

We can prove the result for left ideal also.

The converse of the above proposition is not possible which can be seen

By the following example

Example

Let S={0,a,b,c} with the ordered relation 0<c<b<a.

Define operation on S by following

$\oplus$	0	a	b	c
0	0	a	b	c
a	a	a	a	a
b	b	a	a	a
c	c	a	a	a

$\odot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	c	c	c

Then (S,$\oplus$,$\odot$) forms an ordered semiring. Now suppose μ be a fuzzy subset of S defined by

$\mu(0) = 1$,$\mu(c) = 0.3$,$\mu(b) = 0.2$ and $\mu(a) = 0.1$.

Then $I_0 = \{0\}, I_c = \{0, c\}$ $I_b = \{0, c, b\} and I_a = \{0, c, b, a\}$ – all are right ideal of S.

But μ is not a fuzzy right ideal.

Since,

$$\mu(b + c) = \mu(a) = 0.1 \ngeq 0.2 = \min\{0.2,0.3\} = \min\{\mu(b), \mu(c)\}$$

Proposition

Intersection of a non-empty collection of fuzzy right (resp.left) ideals is also fuzzy right (resp.left) ideal of S.

Proof

Let $\{\mu_i / i \in I\}$ be a non-empty family of fuzzy right ideals of S and $x,y \in S$.

Then,
$$\begin{aligned} \cap_{i\in I} \mu_i(x + y) &= inf_{i\in I} \{\mu_i(x + y)\} \\ &\geq \inf\{\min\{\mu_i(x), \mu_i(y)\}\} \\ &= \min\{inf_{i\in I}\mu_i(x), inf_{i\in I}\mu_i(y)\} \\ \cap_{i\in I} \mu_i(x + y) &= \min\{\cap_{i\in I} \mu_i (x), \cap_{i\in I} \mu_i (y)\} \end{aligned}$$

Again,
$$\begin{aligned} \cap_{i\in I} \mu_i (xy) &= inf_{i\in I}\{\mu_i(xy)\} \\ &\geq inf_{i\in I}\{\mu_i(x)\} \\ \cap_{i\in I} \mu_i (xy) &= \cap_{i\in I} \mu_i (x). \end{aligned}$$

Suppose $x \leq y$

Then $\mu_i(x) \geq \mu_i(y)$ for all $i \in I$

Which implies,

$$\cap_{i \in I} \mu_i(x) \geq \cap_{i \in I} \mu_i(y)$$

Hence,

$\cap_{i \in I} \mu_i$ is a fuzzy right ideal of S.

We can prove the result for fuzzy left ideal also.

Proposition

Let f: R→S be a morphism of ordered semiring. (i.e) semiring homomorphism satisfying additional condition $a \leq b \Rightarrow f(a) \leq f(b)$.Then if $\emptyset$ is a fuzzy left ideal of S. Then $f^{-1}(\emptyset)$ is also a fuzzy left ideal of R.

Proof

Let f : R→S be a morphism of ordered semiring and $\emptyset$ is a fuzzy left ideal of S.

Now $f^{-1}(\emptyset)(0_R) = \emptyset(0_S) \geq \emptyset(x) \neq 0$ for some $x \in S$.

$\therefore f^{-1}(\emptyset)$ is non –empty

Now, for any $r, s \in R$,

$$f^{-}(\emptyset)(r+s) = \emptyset(f(r+s))$$

$$= \emptyset(f(r)+f(s))$$

$$\geq \min\{\emptyset(f(r)), \emptyset(f(s))\}$$

$$f^{-1}(\emptyset)(r+s) = \min\{f^{-1}(\emptyset)(r), (f^{-1(}\emptyset)(s)\}.$$

Again,

$$(f^{-1}(\emptyset))(rs) = \emptyset(f(rs))$$

$$= \emptyset(f(r)f(s)$$

$$\geq \emptyset(f(s))$$

$$(f^{-1}(\emptyset))(rs) = (f^{-1}(\emptyset))(s)$$

Also,

If $r \leq S$, then $f(r) \leq f(s)$.

Then,

$$(f^{-1}(\emptyset))(r) = \emptyset(f(r)) \geq \emptyset(f(s))$$

$$(f^{-1}(\emptyset))(r) = (f^{-1}(\emptyset)(s)$$

Thus,

$f^{-1}(\emptyset)$ is a fuzzy left ideal of R.

Definition

Let μ and γ be fuzzy subsets of x. The Cartesian product of μ and γ

Is defined by $(\mu \times \gamma)(x, y) = \min\{\mu(x), \gamma(y)\}$ for all x,y∈ X.

Theorem

Let μ be a fuzzy subset of an ordered semiring S. Then μ is a fuzzy left ideal of S IFF $\mu x \mu$ is a fuzzy left ideal of SxS

Proof

Assume that is a fuzzy left ideal of S.

Then by theorem $\mu x \mu$ is a fuzzy left ideal of SxS

Conversely,

Suppose that $\mu x \mu$ is a fuzzy left ideal of SxS.

Let $x_1, x_2, y_1, y_2 \in S$

Then,

$$\min\{\mu(x_1 + y_1), \mu(x_2 + y_2)\} = (\mu \times \mu)(x_1 + y_1, x_2 + y_2)$$

$$= (\mu \times \mu)((x_1, x_2) + (y_1, y_2))$$

$$\geq \min\{\mu \times \mu)\ (x_1, x_2), (\mu \times \mu)(y_1, y_2)\}$$

$$\min\{\mu(x_1 + y_1), \mu(x_2 + y_2)\} = \min\{\min\{\mu(x_1), \mu(x_2)\}, \min\{\mu(y_1), \mu(y_2)\}\}$$

Now,

Putting $x_1 = x, x_2 = 0, y_1 = y \ and \ y_2 = 0$

In this inequality and nothing that $\mu(0) \geq \mu(x) for \ all \ x \in S$

We obtain $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$

Next we have,

$$\min\{\mu(x_1 y_1), \mu(x_2 y_2)\} = (\mu \times \mu)(x_1 y_1, x_2 y_2)$$

$$= (\mu \times \mu)(x_1, x_2)(y_1, y_2)$$

$$\geq (\mu \times \mu)(y_1, y_2)$$

$$\min\{\mu(x_1 y_1), \mu(x_2 y_2)\} = \min\{\mu(y_1), \mu(y_2)\}$$

Taking, $x_1 = x, y_1 = y \ and \ y_2 = 0$

$\mu(xy) \geq \mu(y)$

Also if, $(x_1, x_2) \leq (y_1, y_2)$

Then, $\min\{\mu(x_1),\mu(x_2)\} \geq \min\{\mu(y_1),\mu(y_2)\}$

Now,

Putting $x_1 = x, x_2 = 0, y_1 = y \ and \ y_2 = 0$

In this inequality we have

$$\mu(x) \geq \mu(y)$$

Hence μ is a fuzzy left ideal of S.

Proposition

For any three fuzzy subset μ_1, μ_2, μ_3 of an ordered semiring S,$\mu_1 \circ_1 (\mu_2 +_1 \mu_3) = (\mu_1 \circ_1 \mu_2) +_1 (\mu_1 \circ_1 \mu_3)$

Proof

Let μ_1, μ_2 be any three fuzzy subset of an ordered semiring S and

$x \in$ S

Then, $\mu_1 \circ_1 (\mu_2 +_1 \mu_3) = Sup_{x \leq yz}\{\min(\mu_1(y), (\mu_2 +_1 \mu_3)(z)\}$

$= Sup_{x \leq yz}\{\min\{\mu_1(y), Sup_{x \leq a+b}\{\min\{\mu_2(a), \mu_3(b)\}\}\}$

$= Sup\{\min\{Sup\{\min\{\mu_1(y), \mu_2(a)\}\}, Sup\{\min\{\mu_1(y), \mu_3(b)\}\}$

$\leq Sup_{x \leq ya+yb}\{\min\{\mu_1 \circ_1 \mu_2)(ya), (\mu_1 \circ_1 \mu_3)(ya)\}\}$

$\mu_1 \circ_1 (\mu_2 +_1 \mu_3) \leq ((\mu_1 \circ_1 \mu_2) +_1 (\mu_1 \circ_1 \mu_3))(x).$

Also,

$((\mu_1 \circ_1 \mu_2) +_1 (\mu_1 \circ_1 \mu_3))(x) = Sup_{x \leq x_1 + x_2}\{\min\{\mu_1 \circ_1 \mu_2)(x_1), (\mu_1 \circ_1 \mu_3)(x_3)\}\}$

$= Sup_{x \leq x_1 + x_2}\{\min\{Sup_{x_1 \leq c_1 d_1}\{\min\{\mu_1(c_1), \mu_2(d_1)\}\},$

$Sup_{x_2 \leq c_2 d_2}\{\min\{\mu_1(c_1), \mu_3(d_2)\}\}\}\}$

$\leq Sup_{x \leq x_1 + x_2 \leq c_1 d_1 + c_2 d_2 < (c_1 + c_2)(d_1 + d_2)} \{\min\{(c_1 + c_2), Sup\{min\{\mu_2(d_2), \mu_3(d_2)\}\}\}\}$

$\leq Sup_{x \leq cd}\{\min\{\mu_1(c), (\mu_2 +_1 \mu_3)(d)\}\}$

$((\mu_1 \circ_1 \mu_2) +_1 (\mu_1 \circ_1 \mu_3))(x) = (\mu_1 \circ_1 (\mu_2 +_1 \mu_3))(x)$

Therefore $\mu_1 \circ_1 (\mu_2 +_1 \mu_3) = (\mu_1 \circ_1 \mu_2) +_1 (\mu_1 \circ_1 \mu_3).$

Theorem

If μ_1, μ_2 be any two fuzzy ideals of an ordered semiring S then $\mu_1 +_1 \mu_2$ is also so.

Proof

Assume that μ_1, μ_2 are any two fuzzy ideals of an ordered semiring S and x,y∈S.

Then,

$$(\mu_1 +_1 \mu_2)(x+y) = Sup_{x+y \le c+d}\{\min\{\mu_1(c), \mu_2(d)\}\}$$

$$\ge Sup_{x+y \le (a_1+b_1)+(a_2+b_2)=(a_1+a_2)+(b_1+b_2)}\{min\{\mu_1(a_1+a_2), \mu_2(b_1+b_2)\}\}$$

$$\ge Sup\{\min\{\mu_1(a_1), \mu_1(a_2), \mu_2(b_1), \mu_2(b_2)\}\}$$

$$\ge \min\left\{Sup\{\min\{\mu_1(a_1), \mu_2(b_1)\}\}, Sup_{y \le a_2+b_2}\{\min\{\mu_1(a_1), \mu_2(b_2)\}\}\right\}$$

$$(\mu_1 +_1 \mu_2)(x+y) = \min\{(\mu_1 +_1 \mu_2)(x), (\mu_1 +_1 \mu_2)(y)\}$$

Now assume,

μ_1, μ_2 are as fuzzy right ideals and we have,

$$(\mu_1 +_1 \mu_2)(xy) = Sup_{xy \le c+d}\{\min\{\mu_1(c), \mu_2(d)\}\}$$

$$\ge Sup_{xy \le (x_1+x_2)y}\{\min\{\mu_1(x_1 y), \mu_2(x_2 y)\}\}$$

$$\ge Sup_{x \le x_1+x_2}\{\min\{\mu_1(x_1), \mu_2(x_2)\}\}$$

$$(\mu_1 +_1 \mu_2)(xy) = (\mu_1 +_1 \mu_2)(x)$$

Similarly,

Assuming μ_1, μ_2 are as fuzzy left ideal .

we can Show that,

$$(\mu_1 +_1 \mu_2)(xy) \ge (\mu_1 +_1 \mu_2)(y)$$

Now suppose, $x \le y$

Then $\mu_1(x) \ge \mu_1(y)$ and $\mu_2(x) \ge \mu_2(y)$

$$(\mu_1 +_1 \mu_2)(x) = Sup_{x \le x_1 +_1 x_2}\{\min\{\mu_1(x_1), \mu_2(x_2)\}\}$$

$$\ge Sup_{x \le y \le y_1+y_2}\{\min\{\mu_1(y_1), \mu_2(y_2)\}\}$$

$$= Sup_{y \le y_1+y_2}\{\min\{\mu_1(y_1), \mu_2(y_2)\}\}$$

$$(\mu_1 +_1 \mu_2)(x) = (\mu_1 +_1 \mu_2)(y).$$

Hence, $\mu_1 +_1 \mu_2$ is a fuzzy ideal of S

Theorem

If μ_1, μ_2 be any two fuzzy ideals of an ordered semiring S, then $\mu_1 \circ_1 \mu_2$ is also so.

Proof

Then, $(\mu_1 \circ_1 \mu_2)(x+y) = Sup_{x+y \le cd}\{\min\{\mu_1(c), \mu_2(d)\}\}$

$$\geq Sup_{x+y\leq c_1d_1+c_2d_2<(c_1+c_2)(d_1+d_2)}\{\min\{\mu_1(c_1+c_2),\mu_2(d_1+d_2)\}\}$$

$$\geq Sup\ \{\min\{\mu_1(c_1),\mu_1(c_2),\mu_2(d_1),\mu_2(d_2)\}\}$$

$$\geq \min\left\{Sup_{x\leq c_1d_1}\{\min(c_1),\mu_2(d_1)\}\right\},Sup_{y\leq c_2d_2}\{\min\{\mu_1(c_2),\mu_2(d_2)\}\}\}$$

$$(\mu_1\circ_1\mu_2)(x+y)=\min\{(\mu_1\circ_1\mu_2)(x),(\mu_1\circ_1\mu_2)(y)\}$$

Now,

Assume μ_1,μ_2 are as fuzzy right ideals and

we have,

$$(\mu_1\circ_1\mu_2)(xy)=Sup_{xy\leq cd}\{\min\{\mu_1(c),\mu_2(d)\}\}$$

$$\geq Sup_{xy\leq(x_1x_2)y}\{\min\{\mu_1(x_1),\mu_2(x_2y)\}\}$$

$$\geq Sup_{x\leq x_1x_2}\{\min\{\mu_1(x_1),\mu_2(x_2)\}\}$$

$$(\mu_1\circ_1\mu_2)(xy)=(\mu_1\circ_1\mu_2)(x)$$

Similarly,

Assuming μ_1,μ_2 are as fuzzy left ideal.

We can show that,

$$(\mu_1\circ_1\mu_2)(xy)\geq(\mu_1\circ_1\mu_2)(y)$$

Now suppose, $x\leq y$

Then $\mu_1(x)\geq\mu_2(y) and\ \mu_2(x)\geq\mu_2(y)$

$$(\mu_1\circ_1\mu_2)(x)=Sup_{x\leq x_1x_2}\{\min\{\mu_1(x_1),\mu_2(x_2)\}\}$$

$$\geq Sup_{x\leq y\leq y_1y_2}\{\mu_1(y_1),\mu_2(y_2)\}\}$$

$$=Sup_{y\leq y_1y_2}\{\min\{\mu_1(y_1),\mu_2(y_2)\}\}$$

$$(\mu_1\circ_1\mu_2)(x)=(\mu_1\circ_1\mu_2)(y)$$

Hence,

$(\mu_1\circ_1\mu_2)$ is a fuzzy ideal of S.

Conclusion

In this paper we have concluded ideal semirings into fuzzy ideal semirings. We have proved few examples for left and right ideal in semirings. Also we have proved examples for left and right fuzzy ideals in ordered semirings. By using two fuzzy ideals of an ordered semirings we have proved addition and multiplication operation.

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