A New Operation for Intuitionistic Fuzzy Sets

Divya S L

Research Scholar, Department of Mathematics, St Albert's College, Ernakulam, Kochi, India

Abstract : In this paper, we define a new operation connecting two intuitionistic fuzzy sets where one is dominant over the other. This new operation can be applied to many real life situations.

Index Terms **- Fuzzy set, Intuitionistic fuzzy set.**

I. Introduction

The notion of fuzzy sets was introduced by L A Zadeh[1], to handle uncertainty and vagueness. Fuzzy sets which are extensions of classical sets, facilitate gradual transitions from membership to non-membership and vice versa. However in real life situations, it may not be always practical to identify the membership degree and non-membership degree[2]. There may be some hesitation degree also.

The concept of intuitionistic fuzzy set (IFS) which includes the degree of hesitation was first introduced by Krassimir T Atanassov[3,4]. Intuitionistic fuzzy sets (IFSs)[4] is a generalization of fuzzy sets and is a powerful tool to deal with vagueness. IFS is more accurate compared to fuzzy set, as it considers the hesitation margin in addition to the membership degree and non-membership degree.

Some basic relations and operations on IFSs already defined are mentioned in this paper. Also we introduce a new operation which is applicable to many real life situations.

II. Preliminaries

In this section, we mention some elementary concepts[4,5].

Definition 2.1 :

Let X be a non-empty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x) : X \to [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition 2.2 :

Let $A \in X$ be an IFS. Then $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$.

ie., $\pi_A(x) : X \to [0,1]$ and $0 \le \pi_A \le 1$ for every $x \in X$.

$\pi_A(x)$ express the lack of knowledge of whether $x \in X$ belongs to IFS A or not.

Example:

Consider an intuitionistic fuzzy set A with $\mu_A(x) = 0.6$ & $\nu_A(x) = 0.3$. Then

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$
$$= 1 - 0.6 - 0.3 = 0.1$$

It implies that the degree that the object x belongs to IFS A is 0.6, the degree that the object x does not belong to IFS A is 0.3 and the degree of indeterminacy is 0.1.

Definition 2.3 :

$\partial_A(x) = \mu_A(x) + \pi_A(x)\mu_A(x)$ is called the degree of favour of $x \in A$.

$\eta_A(x) = \nu_A(x) + \pi_A(x)\nu_A(x)$ is called the degree of against of $x \in A$.

Example:

Let A be an intuitionistic fuzzy set with $\mu_A(x) = 0.5$ & $\nu_A(x) = 0.2$. Then

$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) = 1 - 0.5 - 0.2 = 0.3.$

$$\partial_A(x) = \mu_A(x) + \pi_A(x)\mu_A(x) = 0.5 + (0.3 * 0.5) = 0.65.$$
$$\eta_A(x) = \nu_A(x) + \pi_A(x)\nu_A(x) = 0.2 + (0.3 * 0.2) = 0.26.$$

It can be interpreted as the degree that the object x belongs to IFS A is 0.5, the degree that the object x does not belong to IFS A is 0.2, the degree of hesitancy or indeterminacy of x belonging to IFS A is 0.3, the degree of favour of x belonging to IFS A is 0.65 and the degree of against of x not belonging to IFS A is 0.26.

2.4 Some basic relations and operations[4,5] on IFSs

1. Inclusion : $A \subseteq B \Rightarrow \mu_A(x) \le \mu_B(x)$ & $\nu_A(x) \ge \nu_B(x)$, $\forall x \in X$.

2. Equality : $A = B \Rightarrow \mu_A(x) = \mu_B(x)$ & $\nu_A(x) = \nu_B(x), \forall x \in X$.
3. Complement : $A^C = \{\langle x, \nu_A(x), \mu_A(x) \rangle : x \in X\}$.
4. Union : $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle : x \in X\}$.
5. Intersection : $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle : x \in X\}$.
6. Addition : $A \oplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x) \rangle : x \in X\}$.
7. Multiplication : $A \otimes B = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x) \rangle : x \in X\}$.
8. Difference : $A - B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)) \rangle : x \in X\}$.

III. NEW OPERATION

Here we introduce another operation which can be applied to many real life situations. We call this operation as D operation for easiness in future mentioning. The set obtained after this operation satisfies all the conditions for an IFS.

Definition :

Let A and B be two IFSs. Then the IFS $B|A$ is defined as

$B|A = \{\langle x, \mu_A(\mathrm{x}) \min(\mu_A(x), \mu_B(x)), \nu_A(\mathrm{x}) \max(\nu_A(x), \nu_B(x)) \rangle : x \in X\}$.

where $\mu_A(\mathrm{x}) \min(\mu_A(x), \mu_B(x))$ is the degree of membership of x to the IFS $B|A$ and $\nu_A(\mathrm{x}) \max(\nu_A(x), \nu_B(x))$ is the degree of non-membership of x.

$ie., \mu_{B|A}(x) = \mu_A(\mathrm{x}) \min(\mu_A(x), \mu_B(x))$ &

$\nu_{B|A}(x) = \nu_A(\mathrm{x}) \max(\nu_A(x), \nu_B(x))$

Result

From the above definition we have the following result.

If $\mu_A(\mathrm{x}) \geq \nu_A(\mathrm{x})$, then $\mu_{B|A}(x) \geq \nu_{B|A}(x)$.

Example :

Consider two IFSs A and B.

1. Let $\mu_A(\mathrm{x}) = 0.5$ & $\nu_A(\mathrm{x}) = 0.3$

$\mu_B(\mathrm{x}) = 0.2$ & $\nu_B(\mathrm{x}) = 0.3$

Then $\mu_A(\mathrm{x}) \min(\mu_A(x), \mu_B(x)) = 0.5 * min\{0.5, 0.2\}$

$= 0.5 * 0.2 = 0.1$

$\nu_A(\mathrm{x}) \max(\nu_A(x), \nu_B(x)) = 0.3 * max\{0.3, 0.3\}$

$= 0.3 * 0.3 = 0.09$

ie., $\mu_{B|A}(x) = 0.1$ & $\nu_{B|A}(x) = 0.09$

It can be interpreted as the degree that the object x belongs to the IFS $B|A$ is 0.1 and the degree that the object x does not belong to $B|A$ is 0.09.

IV. CONCLUSION

The membership and non-membership degree of the IFS $B|A$ depends on the variation of membership and non-membership degree of the IFS A.

V. ACKNOWLEDGEMENT

I would like to express my sincere gratitude to my Supervisor Dr. Souriar Sebastian (Department of Mathematics, St Albert's College, Ernakulam) who gave me the opportunity to do Research in this field. I also thank Rev. Fr. Dr James Philip for his valuable suggestions.

V. REFERENCES

[1]. George J. Klir and Bo Yuan, Fuzzy Sets and Fuzzy Logic, Theory and Applications, Prentice Hall PTR, New Jersey, (1995).
[2]. L. A. Zadeh, Fuzzy Sets, Information Control, 8(1965), 338 – 353.
[3]. Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems. 20(1986)87-96.
[4]. Atanassov, K., Two operators on intuitionistic fuzzy sets, Comptes Rendus de l'Academic Bulgare des Sciences 41(5)(1988)35-38.
[5]. P A Ejegwa , et al.,Intuitionistic fuzzy sets and its application in carrier determination via normalized Euclidean distance method, European Scientific Journal V10(15)(2014).