COMMON FIXED POINT THEOREM ON FUZZY METRIC SPACE USING COMPATIBLE MAPPINGS OF TYPE (R)

B.VIJAYABASKER REDDY[1], V.SRINIVAS[2]
[1] Department of Mathematics, Sreenidhi Institute of Science and Technology, Ghatkesar, Hyderabad, India-501 301.
[2] Department of Mathematics, University college of Science Saifabad, Osmania University Hyderabad, India.

Abstract:
The purpose of this paper is to prove a common fixed point theorem for four self maps in a fuzzy metric space using the concept compatible mappings of type (R). Finally we give an example to our main result.

Keywords: Fixed point, Fuzzy metric space and Compatible mappings of Type (R).
AMS (2000) Mathematics Classification: 54H25, 47H10

1. Introduction

In 1965,the concept of fuzzy set was introduced by Zadeh [1] which laid the foundation of fuzzy mathematics.George and Veeramani[3] modified the notation of fuzzy metric space introduced by Kramosil and Mechalek[2]. Grabic [4] obtained the Banach contraction principle in fuzzy version. In 2004 R Singh etal[14] initiated concept of compatible mappings of Type (R).

Definitions and Preliminaries

Definition 1.1: A binary operation * :$[0,1]\times[0,1] \to[0,1]$ is called continuous t-norm if * satisfies the following conditions:

(i) * is commutative and associative
(ii) * is continuous
(iii)$a*1=a$ for all $a\in[0,1]$
(iv)$a*b \le c*d$ whenever $a \le c$ and $b\le d$ for all $a, b,c,d \in[0,1]$

Definition 1.2: A 3-tuple (X, M,*) is said to be fuzzy metric space if X is an arbitrary set,* is continuous t-norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions for all $x,y,z\in X$, $s,t>0$

(FM-1) $M(x,y,0)=0$
(FM-2) $M(x,y,t)=1$ for all $t>0$ if and only if $x=y$
(FM-3) $M(x,y,t)= M(y,x,t)$
(FM-4) $M(x,y,t) * M(y,z,s) \le M(x,z,t+s)$
(FM-5) $M(x,y,.) : [0,\infty) \to[0,1]$ is left continuous
(FM-6) $\lim_{t\to\infty} M(x,y,t)=1$ for all x,y in X

the function M(x, y,t) denote the degree of nearness between x and y with respect to ‘t’.

Example 1.3 (Induced fuzzy metric space**):** Let (X,d) be a metric space defined $a*b=\min\{a,b\}$ for all $x,y\in X$ and $t>0$,

$$M(x,y,t)=\frac{t}{t+d(x,y)} \qquad ----(a)$$

Then (X, M,*) is a fuzzy metric space. We call this fuzzy metric *M* induced by metric *d* the standard fuzzy metric. From the above example every metric induces a fuzzy metric but there exist no metric on X satisfying (a).

Definition 1.4: Let (X, M,*) be a fuzzy metric space then a sequence $<x_n>$ in X is said to be convergent to a point $x\in X$, if

$$\lim_{n\to\infty} M(x_n,x,t)=1 \quad for\ all\ t>0.$$

Definition 1.5: A sequence $<x_n>$ in X is called a Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ *for all* $t > 0$ *and* $p > 0$.

Definition 1.6: A fuzzy metric space (X, M,*) is said to be complete if every Cauchy sequence is convergent to a point in X.

Lemma 1.7 : For all $x,y\in X$, M(x,y,.) is non decreasing.
Lemma 1.8: let (X,M,*)be a fuzzy metric space if there exists $k\in(0,1)$ such that $M(x,y,kt)\geq M(x,y,t)$ then x=y.

Proposition 1.9: In the fuzzy metric space (X, M,*) if $a*a\geq a$ for all $a\in[0,1]$ then $a*b= \min\{a,b\}$.

In 2004,Rohan et al. introduced the concept of compatible mappings of Type (R) in metric space as follows:

Definition 1.10: Let f and g be two mappings of a metric space (X,d) into itself. Then f and g are called compatible of type (R) if $\lim_{n\to\infty} d(fgx_n,gfx_n)=0$ and $\lim_{n\to\infty} d(ffx_n,ggx_n)=0$ whenever $<x_n>$ is a sequence in X such that $\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gx_n= z$ for some $z\in X$.
Now Compatible mapping of Type (R) in fuzzy metric space as follows

Definition 1.11: Let S and T be two mappings of a fuzzy metric space (X,M,*) into itself. Then S and T are called compatible of type (R) if $\lim_{n\to\infty} M(STx_n,TSx_n,t)=1$ and $\lim_{n\to\infty} M(SSx_n,TTx_n,t)=0$ whenever $<x_n>$ is a sequence in X such that $\lim_{n\to\infty} Sx_n=\lim_{n\to\infty} Tx_n= z$ for some $z\in X$.
Proposition1.12: Let S and T be compatible mappings of type (R) of a complete fuzzy metric space (X,M,*) into itself If Sz=Tz for some $t\in X$ then STz=SSz=TTz=TSz

Proposition1.13: Let S and T be mappings from a complete fuzzy metric space into itself. If a pair (S,T) is compatible is compatible type (R) on X and if $\lim_{n\to\infty} Sx_n=\lim_{n\to\infty} Tx_n= z$ for some $z\in X$ then we have

(i) $\lim_{n\to\infty} M(TSx_n,Sz,t)=1$ if S is continuous
(ii) $\lim_{n\to\infty} M(STx_n,Tz,t)=1$ if T is continuous
(iii)STz=TSz if S and T are continuous at z.

Proof: (i) Suppose that S is continuous at z. Since $\lim_{n\to\infty} Sx_n=\lim_{n\to\infty} Tx_n= z$ for some $z\in X$, we have $\lim_{n\to\infty} SSx_n=\lim_{n\to\infty} STx_n= Sz$ .Since S and T are compatible of type (R), then we have $\lim_{n\to\infty} M(TSx_n,Sz,t)=1$ and $\lim_{n\to\infty} M(SSx_n,TTx_n,t)=1$.Therefore $\lim_{n\to\infty} TSx_n= Sz$
(ii) Similar arguments as in (i)
(iii)Suppose S and T are continuous at z and $<x_n>$ is a sequence in X defined $x_n =z$ (n=1,2,..) for some $z\in X$.
Since $Sx_n\to z$ $Tx_n\to z$ as $n\to\infty$ and S is continuous at z, by (i) $\lim_{n\to\infty} M(TSx_n,Sz,t)=1$.
On the other hand ,T is also continuous at z , $\lim_{n\to\infty} M(TSx_n,Tz,t)=1$.Thus,we have Sz=Tz by the uniqueness of limit and also by proposition 1.12 , STz=TSz.
This completes the proof.

Main results:

2.1 Theorem: Let A, B, S and T are self maps of a complete Fuzzy metric space (X, M,*) satisfying the conditions

2.1.1 $B(X) \subseteq S(X)$ and $A(X) \subseteq T(X)$

2.1.2 $$[M(Ax,By,kt)]^2 * M(Ax,By,kt)M(Ty,Sx,kt) \ge \{k_1[M(By,Sx,2kt)*M(Ax,Ty,2kt)] + k_2[M(Ax,Sx,kt)*M(By,Ty,kt)]\}M(Ty,Sx,t)$$

Where for all x,y in X and $k_1,k_2 \ge 0$, $k_1+k_2 \ge 1$

2.1.3 one of the mappings A,B,S and T is continuous

2.1.4 the pairs (T, B) and (S,A) are compatible of type(R)

then A, B,S and T have a unique common fixed point z in X.

2.1.5 **Lemma:** Let A,B,S and T be self mappings from a complete fuzzy metric space (X,M,*) into itself satisfying the conditions (2.1.1)and (2.1.2). Then the sequence $\{y_n\}$ defined by $y_{2n}=Bx_{2n}= Sx_{2n+1}$ and $y_{2n+1}=Ax_{2n+1} = Tx_{2n+2}$ for $n \ge 0$ relative to four self maps is a Cauchy sequence in X.

Proof of the Lemma: Let x_0 be any arbitrary point of X, $B(X) \subseteq S(X)$ and $A(X) \subseteq T(X)$ and there exists $x_1,x_2 \in X$ such that $Bx_0=Sx_1$ and $Ax_1=Tx_2$.
Inductively we construct a sequence $<x_n>$ and $<y_n>$ in X such that $y_{2n}=Bx_{2n}= Sx_{2n+1}$ and $y_{2n+1}=Ax_{2n+1} = Tx_{2n+2}$ for $n \ge 0$.

By taking $x= x_{2n}$, $y= x_{2n+1}$ in 2.1.1

$$[M(Ax_{2n+1},Bx_{2n},kt)]^2 * M(Ax_{2n+1},Bx_{2n},kt)M(Tx_{2n},Sx_{2n+1},kt) \ge \{k_1[M(Bx_{2n},Sx_{2n+1},2kt)*M(Ax_{2n+1},Tx_{2n},2kt)] + k_2[M(Ax_{2n+1},Sx_{2n+1},kt)*M(Bx_{2n},Tx_{2n},kt)]\}M(Tx_{2n},Sx_{2n+1},t)$$

$$[M(y_{2n+1},y_{2n},kt)]^2 * M(y_{2n+1},y_{2n},kt)M(y_{2n-1},y_{2n},kt) \ge \{k_1[M(y_{2n},y_{2n},2kt))*M(y_{2n+1},y_{2n-1},2kt)] + k_2[M(y_{2n+1},y_{2n},kt)*M(y_{2n},y_{2n-1},kt)]\}M(y_{2n-1},y_{2n},t)$$

$$[M(y_{2n+1},y_{2n},kt)]\{M(y_{2n+1},y_{2n},kt)*M(y_{2n-1},y_{2n},kt)\} \ge \{k_1[M(y_{2n+1},y_{2n-1},2kt)] + k_2[M(y_{2n+1},y_{2n},kt)*M(y_{2n},y_{2n-1},kt)]\}M(y_{2n-1},y_{2n},t)$$

$$[M(y_{2n},y_{2n+1},kt)]\{M(y_{2n+1},y_{2n-1},2kt)\} \ge \{k_1[M(y_{2n+1},y_{2n-1},2kt)] + k_2[M(y_{2n-1},y_{2n+1},2kt)]\}M(y_{2n},y_{2n-1},t)$$

$$[M(y_{2n},y_{2n+1},kt)][M(y_{2n+1},y_{2n-1},2kt)] \ge \{k_1+k_2\}[M(y_{2n+1},y_{2n-1},2kt)]M(y_{2n},y_{2n-1},t)$$

$$[M(y_{2n},y_{2n+1},kt)] \ge \{k_1+k_2\}\ M(y_{2n-1},y_{2n},t)$$

$$[M(y_{2n},y_{2n+1},kt)] \ge M(y_{2n-1},y_{2n},t)$$

$$[M(y_n,y_{n+1},kt)] \ge M(y_{n-1},y_n,t)$$

$$M(y_n,y_{n+1},t) \ge M(y_{n-1},y_n,t/k) \ge M(y_{n-1},y_n,t/k^2) \ge M(y_{n-1},y_n,t/k^3) \ldots\ldots \ge M(y_{n-1},y_n,t/k^n)$$

This implies $M(y_n,y_{n+1},t) \to 1$ as $n \to \infty$

For each $\varepsilon > 0$ and $t>0$ we can choose $n_0 \in N$ such that $M(y_n,y_{n+1},t) > 1-\varepsilon$

For $m,n \in N$ suppose $m \ge n$

$$M(y_n,y_m,t) \ge \left[M(y_n,y_{n+1},t/(m-n))*M(y_{n+1},y_{n+2},t/(m-n))*\ldots*M(y_{m-1},y_m,t/(m-n))\right]$$
$$\ge (1-\varepsilon)*(1-\varepsilon)*\ldots..(1-\varepsilon)$$
$$\ge (1-\varepsilon)$$

This shows that the sequence $\{y_n\}$ is a cauchy sequence in X and, it converges to a limit, say $z \in X$.
Consequently ,the sub sequences $\{Bx_{2n}\}$,$\{Sx_{2n+1}\}$,$\{Ax_{2n+1}\}$,$\{Tx_{2n+2}\}$ of sequence $\{y_n\}$ also converges to z.

Proof of main Theorem:

Now suppose that T is continuous: .
Since the pair (T,B) is compatible of type (R) by preposition 1.13, TTx_{2n}, BTx_{2n} converges to Tz as $n \to \infty$.

We claim Tz=z .Putting y=Tx_{2n}, x=x_{2n+1} in inequality 2.1.2

$$[M(Ax_{2n+1},BTx_{2n},kt)]^2 * M(Ax_{2n+1},BTx_{2n},kt)M(TTx_{2n},Sx_{2n+1},kt) \ge \{k_1[M(BTx_{2n},Sx_{2n+1},2kt)*M(Ax_{2n+1},TTx_{2n},2kt)] + k_2[M(Ax_{2n+1},Sx_{2n+1},kt)*M(BTx_{2n},TTx_{2n},kt)]\}M(TTx_{2n},Sx_{2n+1},t)$$

$$[M(z,Tz,kt)]^2 * M(z,Tz,kt)M(Tz,z,kt) \ge \{k_1[M(Tz,z,2kt)*M(z,Tz,2kt)] + k_2[M(z,z,kt)*M(Tz,Tz,kt)]\}M(Tz,z,t)$$

$$[M(z,Tz,kt)]^2 * [M(z,Tz,kt)]^2 \ge \{k_1[M(Tz,z,2kt)] + k_2[1]\}M(Tz,z,t)$$

$$[M(z,Tz,kt)] \ge \{k_1[M(Tz,z,2kt)] + k_2[1]\}$$

$$[M(z,Tz,kt)] \ge \frac{k_2}{1-k_1}$$

$$[M(z,Tz,kt)] \ge 1$$

therefore Tz=z.

next we claim that Bz=z.

Putting x=x_{2n+1} , y=z in inequality 2.1.2

$$[M(Ax_{2n+1},Bz,kt)]^2 * M(Ax_{2n+1},Bz,kt)M(Tz,Sx_{2n+1},kt) \ge \{k_1[M(Bz,Sx_{2n+1},2kt)*M(Ax_{2n+1},Tz,2kt)] + k_2[M(Ax_{2n+1},Sx_{2n+1},kt)*M(Bz,Tz,kt)]\}M(Tz,Sx_{2n+1},t)$$

$$[M(z,Bz,kt)]^2 * M(z,Bz,kt)M(z,z,kt) \ge \{k_1[M(Bz,z,2kt)*M(z,z,2kt)] + k_2[M(z,z,kt)*M(Bz,z,kt)]\}M(z,z,t)$$

$$[M(z,Bz,kt)]^2 * [M(z,Bz,kt)] \ge \{k_1[M(Bz,z,2kt)] + k_2[M(Bz,z,kt)]\}M(z,z,t)$$

$$[M(z,Bz,kt)]^2 \ge k_1 + k_2[M(Tz,z,2kt)]$$

$$[M(z,Bz,kt)] \ge k_1 + k_2$$

$$[M(z,Bz,kt)] \ge 1$$

Therefore Bz=z

Since from the condition B(X) $\subseteq$ S(X), there exists a point u $\in$ X such that z=Su=Bz.

Put x=u , y=z in inequality 2.1.2

$$[M(Au,Bz,kt)]^2 * M(Au,Bz,kt)M(Tz,Su,kt) \ge \{k_1[M(Bz,Su,2kt)*M(Au,Tz,2kt)] + k_2[M(Au,Su,kt)*M(Bz,Tz,kt)]\}M(Tz,Su,t)$$

$$[M(Au,z,kt)]^2 * M(Au,z,kt)M(z,z,kt) \ge \{k_1[M(z,z,2kt)*M(Au,z,2kt)] + k_2[M(Au,z,kt)*M(z,z,kt)]\}M(z,z,t)$$

$$[M(Au,z,kt)]^2 \ge \{k_1[M(Au,z,2kt)] + k_2[M(Au,z,kt)]\}M(z,z,t)$$

$$[M(Au,z,kt)] \ge k_1 + k_2$$

$$[M(Au,z,kt)] \ge 1$$

Therefore Au=z

Since (S,A) is compatible type (R) and Su=Au=z,by Proposition SAu=ASu and hence Sz=SAu=ASu=Az

To prove Az = z Put x = z, y = z

$$[M(Az,Bz,kt)]^2 * M(Az,Bz,kt)M(Tz,Sz,kt) \ge \{k_1[M(Bz,Sz,2kt)*M(Az,Tz,2kt)] + k_2[M(Az,Sz,kt)*M(Bz,Tz,kt)]\}M(Tz,Sz,t)$$

$$[M(Az,z,kt)]^2 * M(Az,z,kt)M(z,Az,kt) \ge \{k_1[M(z,Az,2kt)*M(Az,z,2kt)] + k_2[M(Az,Az,kt)*M(z,z,kt)]\}M(z,Az,t)$$

$$[M(Az,z,kt)]^2 \ge \{k_1[M(Az,z,2kt)] + k_2[1]\}M(z,Az,t)$$

$$\left[M(Az,z,kt)\right] \geq \frac{k_2}{1-k_1}$$

$$\left[M(Az,z,kt)\right] \geq 1$$

Az=z implies Sz=z

Hence z=Az=Bz=Sz=Tz gives z is common fixed point of A,B,S and T.
Similarly we can prove when S is continuous.
Suppose B is continuous. Since the pair (T,B) is compatible of type R.
By preposition 1.13, BBx_{2n},TBx_{2n} converges to Bz as $n\to\infty$.
We claim that z=Bz
Putting $x=x_{2n+1}$, $y=Bx_{2n}$ in inequality 2.1.2

$$\begin{aligned}\left[M(Ax_{2n+1},BBx_{2n},kt)\right]^2 * M(Ax_{2n+1},BBx_{2n},kt)M(TBx_{2n},Sx_{2n+1},kt) \geq \{k_1\left[M(BBx_{2n},Sx_{2n+1},2kt) * M(Ax_{2n+1},TBx_{2n},2kt)\right]\\ +k_2\left[M(Ax_{2n+1},Sx_{2n+1},kt) * M(BBx_{2n},TBx_{2n},kt)\right]\}M(TBx_{2n},Sx_{2n+1},t)\end{aligned}$$

$$\begin{aligned}\left[M(z,Bz,kt)\right]^2 * M(z,Bz,kt)M(Bz,z,kt) \geq \{k_1\left[M(Bz,z,2kt) * M(z,Bz,2kt)\right]\\ +k_2\left[M(z,z,kt) * M(Bz,Bz,kt)\right]\}M(Bz,z,t)\end{aligned}$$

$$\left[M(z,Bz,kt)\right]^2 * \left[M(z,Bz,kt)\right]^2 \geq \{k_1\left[M(Bz,z,2kt)\right] + k_2\left[1\right]\}M(Bz,z,t)$$

$$\left[M(z,Bz,kt)\right] \geq \{k_1\left[M(Bz,z,2kt)\right] + k_2\left[1\right]\}$$

$$\left[M(z,Bz,kt)\right] \geq \frac{k_2}{1-k_1}$$

$$\left[M(z,Bz,kt)\right] \geq 1$$

This implies Bz=z

From the condition B(X) $\subseteq$ S(X) implies there exists $v \in X$ such that z=Bz= Sv.
We claim that z=Av
Putting x=v , $y=Bx_{2n}$ in inequality 2.1.2

$$\begin{aligned}\left[M(Av,BBx_{2n},kt)\right]^2 * M(Av,BBx_{2n},kt)M(TBx_{2n},Sv,kt) \geq \{k_1\left[M(BBx_{2n},Sv,2kt) * M(Av,TBx_{2n},2kt)\right]\\ +k_2\left[M(Av,Sv,kt) * M(BBx_{2n},TBx_{2n},kt)\right]\}M(TBx_{2n},Sv,t)\end{aligned}$$

$$\begin{aligned}\left[M(Av,Bz,kt)\right]^2 * M(Av,Bz,kt)M(Bz,Bz,kt) \geq \{k_1\left[M(Bz,Bz,2kt) * M(Av,Bz,2kt)\right]\\ +k_2\left[M(Av,Bz,kt) * M(Bz,Bz,kt)\right]\}M(Bz,Bz,t)\end{aligned}$$

$$\left[M(Av,z,kt)\right]^2 * M(Av,z,kt) \geq \{k_1\left[M(Av,z,2kt)\right] + k_2\left[M(Av,z,kt)\right]\}M(z,z,t)$$

$$\left[M(Av,z,kt)\right] \geq \{k_1 + k_2\}$$

$$\left[M(Av,z,kt)\right] \geq 1$$

Implies Av=z
Since the pair (S,A) is compatible of type (R) and Sv=Av=z.By preposition 1.13, SAv=ASv and hence Sz=SAv=ASv=Az.

We claim that Az=z
Putting x=z , $y=x_{2n}$ in inequality 2.1.2

$$[M(Az,Bx_{2n},kt)]^2 * M(Az,Bx_{2n},kt)M(Tx_{2n},Sz,kt) \geq \{k_1[M(Bx_{2n},Sz,2kt)*M(Az,Tx_{2n},2kt)] + k_2[M(Az,Sz,kt)*M(Bx_{2n},Tx_{2n},kt)]\}M(Tx_{2n},Sz,t)$$

$$[M(Az,z,kt)]^2 * M(Az,z,kt)M(z,Az,kt) \geq \{k_1[M(z,Az,2kt)*M(Az,z,2kt)] + k_2[M(Az,Sz,kt)*M(z,z,kt)]\}M(z,Az,t)$$

$$[M(Az,z,kt)]^2 \geq \{k_1[M(z,Az,2kt)] + k_2[1]\}M(z,Az,t)$$

$$[M(Az,z,kt)] \geq \{k_1[M(z,Az,kt)] + k_2[1]\}$$

$$[M(Az,z,kt)] \geq \frac{k_2}{1-k_1}$$

$$[M(Az,z,kt)] \geq 1$$

Implies Az=z

Since the condition A(X) $\subseteq$T(X) implies there exists w$\in$X such that z=Az=Tw

We claim that z=Bw
Putting x=z,y=w in inequality 2.1.2

$$[M(Az,Bw,kt)]^2 * M(Az,Bw,kt)M(Tw,Sz,kt) \geq \{k_1[M(Bw,Sz,2kt)*M(Az,Tw,2kt)] + k_2[M(Az,Sz,kt)*M(Bw,Tw,kt)]\}M(Tw,Sz,t)$$

$$[M(z,Bw,kt)]^2 * M(z,Bw,kt)M(Bw,z,kt) \geq \{k_1[M(Bw,z,2kt)*M(z,Bw,2kt)] + k_2[M(z,z,kt)*M(Bw,Bw,kt)]\}M(Bw,z,t)$$

$$[M(z,Bw,kt)]^2 \geq \{k_1[M(Bw,z,2kt)] + k_2[1]\}M(Bw,z,t)$$

$$[M(z,Bw,kt)] \geq \frac{k_2}{1-k_1}$$

$$[M(z,Bw,kt)] \geq 1$$

Implies Bw=z
Since the pair (B,T) is compatible type (R) and Bw=Tw=z
By preposition1.13, TBw=BTw and hence Tz=TBw=BTw=Bz
Therefore z=Az=Bz=Sz=Tz which gives z is common fixed point of A,B,S and T.
Similarly we can complete the proof when A is continuous.
Uniqueness completes the proof.

2.2 Example: Let X= [0,1], $M(x,y,t) = \dfrac{t}{t+d(x,y)}$ where d(x,y)=$|x-y|$

$$Ax = Bx = \begin{cases} \frac{1}{6} & if\ 0 \leq x < \frac{1}{8} \\ \frac{1}{8} & if\ \frac{1}{8} \leq x \leq 1 \end{cases} \qquad Sx = Tx = \begin{cases} \frac{1}{4} - x & if\ 0 \leq x < \frac{1}{8} \\ \frac{1}{8} & if\ \frac{1}{8} \leq x \leq 1 \end{cases}$$

Then A(X) =B(X)=$\left\{\frac{1}{8},\frac{1}{6}\right\}$ while S(X) =T(X) =$\left\{\frac{1}{8} \cup \left[\frac{1}{8},\frac{1}{4}\right]\right\}$ so that the conditions A(X) $\subseteq$T(X) and B(X) $\subseteq$S(X) are satisfied . For this, take a sequence x_n=$\left(\frac{1}{8}+\frac{1}{n}\right)$ for n≥1.

From the example given above, satisfies all the conditions of Theorem 2.1
Clearly 1/8 is the unique common fixed point of A, B, S and T.

REFERENCES:

[1] Zadeh L A,Fuzzy Sets,Information and Control,8(1965),338-353.

[2] Kramosil and J. Machalek, Fuzzy metric and statistical metric spaces, Kybernetika 11, (1975), 336-344

[3] George A, Veeramani P, On some results in fuzzy metric spaces, Fuzzy Sets and Systems, 64 (1994), 395-399.

[4] Grabiec M, Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems 27, (1988), 385-389.

[5] Sharma S, Common fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems 127(2002), 345-352.

[6] Cho Y J, Pathak H K, Kang S M & Jung J S, Common fixed point of compatible maps of type β on fuzzy metric spaces, Fuzzy *sets and Systems*, 93 (1998), 99-111.

[7] Jungck G, Compatible mappings and common fixed points (2) Internet. *.J.Math & Math. Sci.* (1988), 285-288.

[8] Sisodia K S, Rathore M S, & Deepak Singh , A common fixed point theorem in fuzzy metric spaces, Vol.5, 2011, No.17, 819-826.

[9] Kutukcu S, Sharma S & Tokgoz H A, A fixed point theorem in fuzzy metric spaces, *Int. J.Math. Analysis*, 1(18) (2007), 861-872.

[10] Mishra S N, Sharma N & Singh S L, Common fixed points of mappings on fuzzy metric spaces, *Internet .J.Math &Math .Sci*,17(1994), 253-258.

[11] Pant R P & Jha K, A Remark on common fixed points of four mappings in a fuzzy metric space, J.Fuzzy *Math*.12 (2) (2004),433-437.

[12] Srinivas V, B.V.B Reddy & Umamaheshwar Rao R, A focus on common fixed point theorem using weakly compatible mappings *KUSET* ,Vol.2, No 3, 2012,60-65.

[13] Koireng M and Yumnam Rohen, Common fixed point theorems of compatible mappings of type(P) of fuzzy metric spaces, *Int. Journal of Math. Analysis*, vol.6, No.4, 2012, 181-188.

[14] Y Rohan ,M.R.Singh And Shambhu , Common Fixed Point Of Compatible Mapping Of Type (R) In Banach spaces,*Proc.Of ath.Soci.Bhu*,20, 2004, 77-87.

[15] Shin Min Kang,Praveen Kumar and Sanjay Kumar,& Bu Young Lee, Common Fixed Points for Compatible Mappings of Types in Multiplicative Metric Spaces, *International Journal of Mathematical Analysis*,Vol.9 no.36,(2015),1755-1767.

[16] Dinesh Singh Yadav ,Shailendra Singh Thakur and Samajh Singh Thakur, Common fixed point theorem for compatible mappings of type (R) in fuzzy metric spaces, *Journal of Nonlinear Analysis and its Applications*,No2.2015.111-114.