Laminar free convection flow of an incompressible, electrically conducting viscous fluid past an infinite vertical porous plate

Hoshiyar Singh[1], Sonam Yadav[2*] and Yogesh Khandelwal
Department of Mathematics, Jaipur National University, Jaipur (302017), Rajasthan, India

Abstract: A steady laminar free convective two-dimensional flow of a viscous incompressible electrically conducting over an infinite vertical plate embedded in the porous medium with heat and mass transfer is analyzed by taking into account the effect of viscous dissipation. The dimensionless governing equations for this investigation are solved by using perturbation technique. Numerical evaluation of the analytical results is performed and graphical results for velocity, temperature and concentration profiles with the boundary layer are discussed. We observe that skin friction (τ) is lower for water (Pr=7, Sc=0.61) than for air (Pr=0.71, Sc=0.22).

Keywords: MHD , Magnetic Field, Radiation , Porous Medium and Free Convection.

1. Introduction

The problem of free convection flow has drawn attention of the researchers due to its application in engineering and technology. To make heat transfer at the surface more effective, it is very needful to study the free convection flow through a non-homogeneous porous medium and to estimate its effect on heat and mass transfer. The study of flow through porous medium finds application in geophysics, agricultural engineering and technology. Further the free convection flow in enclosures has become increasingly important in engineering applications in recent years due to fact growth of technology, effecting cooling of electronic equations ranges from individual transistors to main frame computers and so on. Several authors including Gupta and Sharma[7], Singh et. al.[17], Reddy et.al. [14] has studied MHD flow through porous medium in slip flow regime. A Soret effect due to natural convection between heated inclined plates with magnetic field was studied by Raju et al.[11].In their study Reddy et al.[12-13], considered effect of thermal diffusion on heat and mass transfer flow problems in different geometries. Ravikumar et al. [16] investigated, heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in presence of temperature dependent heat source. MHD transient free convection and chemically reactive flow past a porous vertical plate with radiation and temperature gradient dependent heat source in slip flow regime was investigated by Rao et al. [15].

Non-Darcy fully developed mixed convection in a porous medium channel with heat generation/absorption and hydro magnetic effects were considered by Chamkha [1]. Non-similar solutions for heat and mass transfer by hydro magnetic mixed convection flow over a plate in porous media with surface suction or injection and effects of heat generation/absorption and thermophoresison hydromagnetic flow with heat and mass transfer over a flat surface were considered by Chamkha[2-3]. Magetohydrodynamicsis attracting the attention of the many authors due to its applications in geophysics it is applied to study the stellar and solar structures, inter stellar matter, radio propagation through the ionosphere etc.

In engineering in MHD pumps, MHD bearings etc. at high temperatures attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic and alter heat transfer and friction characteristic. Since some fluid scan also emit and absorb thermal radiation, it is of interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. This is of interest because heat transfer by thermal radiation is becoming of greater importance when we are concerned with space applications and higher operating temperatures. The effects of transversely magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al. [18].

Again, Soundalgekar and Takhar [19] studied the effect of radiation on the natural convection flow of a gas pasta semi-infinite plate using the Cogly- Vincentine-Gillesequilibrium model. For the same gas Takhar et al.[20] investigated the effect of radiation on the MHD free convection flow past a semi-infinite vertical plate. Later, Hossain et al.[8] studied the effect of radiation on free convection from a porous vertical plate. Muthucumarswamy and Kumar [10] examined the thermal radiation effects on moving infinite vertical plate in presence of variable temperature and mass diffusion. An analytical solution for unsteady free convection in porous media has been studied by Magyari et al. [9]. Simultaneous radiative and convective heat transfer in a variable porosity medium was studied by Nagaraju et al. [4]. An analytical solution for hydromagnetic natural convection flow of a particulate suspension through a channel with heat generation or absorption effects was studied by Subaie and Chamkha [5]. Recently Patil et al. [6] studied double diffusive mixed convection flow over a moving vertical plate in the presence of internal heat generation and chemical reaction. Motivated by above cited work, in this chapter we have considered an unsteady MHD free convection flow of a viscous fluid past a vertical porous plate embedded with porous medium in the presence of thermal diffusion. In obtaining the solution, the terms regarding radiation effect and temperature gradient dependent heat source are taken into account in energy equation. The permeability of the porous medium and the suction velocity are considered to be exponentially decreasing function of time.

2. Formulation of the problem

We consider two dimensional steady, laminar free convection flow of an incompressible, electrically conducting viscous fluid past an infinite vertical porous plate. The flow is considered in presence of the radiation effect.

In Cartesian coordinate system, we take x-axis along the surface in the upward direction and y-axis is taken normal to it.

Under the above stated assumptions the governing equations of continuity, momentum and energy are:

Equation of Continuity

$$\frac{\partial v}{\partial y} = 0 \qquad \ldots(2.1)$$

Equation of Momentum

$$v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \nu\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\nu u}{K} - \frac{\sigma B_0^2 u}{\rho} \qquad \ldots(2.2)$$

Equation of Energy

$$v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial y^2}\right) - \frac{1}{\rho C_p}\frac{dq_r}{dy} \qquad \ldots(2.3)$$

Equation of Concentration

$$v\frac{\partial c}{\partial y} = D_m\frac{\partial^2 c}{\partial y^2} + D_l\frac{\partial^2 T}{\partial y^2} \qquad \ldots(2.4)$$

The permeability of the porous medium is assumed to be the form

$$K = K_0 \qquad \ldots(2.5)$$

The radiative heat flux q_r is given by (Cogley et. al [16])

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)\, I \qquad \ldots(2.6)$$

where I is the absorption coefficient and

$$I = \int_0^{\infty} K_{\lambda n} \frac{\partial e_{b\lambda}}{\partial T} d\lambda,$$

$K_{\lambda n}$ is the radiation absorption coefficient at the wall, $e_{b\lambda}$ is Plank's function and λ is frequency.

The boundary conditions are:

$$\left.\begin{array}{ll} u = U_0 + L_1 \frac{\partial u}{\partial y}, \ \frac{\partial T}{\partial y} = -\frac{q}{k}, \ \frac{\partial C}{\partial y} = -\frac{m}{D} & \text{at } \ y = 0 \\ u = 0, \ \ T \to T_\infty, \ \ \ C \to C_\infty & \text{at } y \to \infty \end{array}\right\} \qquad \ldots(2.7)$$

where q and m are uniform heat and concentration flux at the plate respectively and $L_1 = \left(\frac{2 - m_1}{m_1}\right) L, L$ being mean free path and m_1 the Maxwell's reflection coefficient.

Integration of equation (2.1) for variable suction gives

$$v = -v_0 \qquad \ldots(2.8)$$

where B is a real positive constant and $\in$ is small such that $\in$ B << 1.

Now introducing the following non-dimensional quantities

$$y^* = \frac{y \nu_0}{\nu}, \ , C = \frac{D v_0 (C - C_\infty)}{\nu m},$$

$$u^* = \frac{u}{v_0}, \ \theta = \frac{k v_0 (T - T_\infty)}{\nu q}, \ M^2 = \frac{\sigma B_0^2 \nu}{\rho v_0^2},$$

$$Gr = \frac{\nu^2 g \beta q}{k v_0^4}, \ \ Gc = \frac{g \beta^* \nu^2 m}{D v_0^4}, \ K_0^* = \frac{K_0 v_0^2}{\nu^2},$$

$$Pr = \frac{\mu C_p}{k}, \ S = \frac{4 \nu I}{\rho C_p v_0^2}, \ Sc = \frac{\nu}{D}, \ h_1 = \frac{L_1 v_0}{\nu}, \alpha = \frac{U_0}{v_0}, S_0 = \frac{D l V_0 D q}{k \nu m},$$

Equation (2.2), (2.3) and (2.4) reduce to the following form after dropping the asterisks over them:

$$\frac{\partial u}{\partial y} = -Gr\,\theta - Gc\,C - \frac{\partial^2 u}{\partial y^2} + \left(M^2 + \frac{1}{K}\right) u \qquad \ldots(2.9)$$

$$\frac{\partial \theta}{\partial y} = -\frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + S\theta \qquad \ldots(2.10)$$

$$\frac{\partial C}{\partial y} = -\frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - S_0 \frac{\partial^2 T}{\partial y^2} \qquad \ldots(2.11)$$

with corresponding boundary conditions:

$$\left.\begin{array}{ll} u = \alpha + h_1 \dfrac{\partial u}{\partial y}, \dfrac{\partial \theta}{\partial y} = -1, \dfrac{\partial C}{\partial y} = -1 & \text{at } y = 0 \\ u = 0, \ \theta \to 0, \ C \to 0 & \text{at } y \to \infty \end{array}\right\} \qquad \ldots(2.12)$$

3 . Solution of the problem

ϵ is a small quantity, we reduce the system of partial differential equations to ordinary differential equations by assuming:

$$f(y) = f_o(y) + \varepsilon f_1(y) \qquad \ldots(3.1)$$

where f stands for u, θ and C. With the help of equations (3.1), the equations (2.9) to (2.11) reduce to the following differential equations on equating like powers of $\in$ (neglecting $0(\in^2)$)

$$u_0'' + u_0' - \left(M^2 + \frac{1}{K}\right) u_0 = -\, Gr\, \theta_0 - Gc\, C_0 \qquad \ldots(3.2)$$

$$u_1'' + u_1' - \left(M^2 + \frac{1}{K}\right) u_1 = -Gr\, \theta_1 - Gc\, C_1 \qquad \ldots(3.3) \ \frac{1}{Pr}\theta_0'' + \theta_0' - S\,\theta_0 = 0$$

$$\ldots(3.4)\ \frac{1}{Pr}\theta_1'' + \theta_1' - S\,\theta_1 = 0$$

$\ldots(3.5)$

$$\frac{1}{S_0} C_0'' + C_0' + S_0 \theta''_0 = 0 \qquad \ldots(3.6)$$

$$\frac{1}{S_c} C_1'' + C_1' + S_0 \theta''_1 = 0 \qquad \ldots(3.7)$$

with corresponding boundary conditions:

$$\left.\begin{array}{ll} u_0 = \alpha + h_1 u_0', u_1 = \alpha + h_1 u_1', \theta_0' = -1, \theta_1' = 0, C_0' = -1, C_1' = 0 & \text{at } y = 0 \\ u_0 \to 0, u_1 \to 0, \ \theta_0 \to 0, \theta_1 \to 0, C_0 \to 0, C_1 \to 0 & \text{at } y \to \infty \end{array}\right\} \ldots(3.8)$$

Equations (3.2) to (3.7) are second order linear differential equations with constant coefficients, the solutions of which are:

$$u = C_1 e^{A_4 y} + C_2 e^{-A_5 y} + (R_2 + R_3) e^{-A_2 y} + R_4 e^{-S_c y} \qquad \ldots(3.9)$$

$$\theta = R_1 e^{-A_2 y} \qquad \ldots(3.10)$$

$$C = A_3 e^{-A_2 y} + m_1 e^{-S_c y} \qquad \ldots(3.11)$$

where

$$A_1 = \frac{1}{2}\left[-Pr + \sqrt{Pr^2 + 4\,Pr\,S}\right], \ N^2 = M^2 + \frac{1}{K}$$

$$A_2 = -\frac{1}{2}\left[\Pr + \sqrt{\Pr^2 + 4\Pr S}\right], \quad A_3 = -\frac{S_0 S_c}{A_2 + S_c}$$

$$A_4 = \frac{-1+\sqrt{1+4N^2}}{2}, \quad A_5 = -\frac{1}{2}\left[1+\sqrt{1+4N^2}\right]$$

$$m_1 = \frac{1+A_2A_3}{S_c}, \quad R_1 = \frac{1}{A_2}$$

$$R_2 = \frac{-G_r R_1}{A_2^2 + A_2 - N^2}, \quad R_3 = \frac{-G_c A_3}{A_2^2 + A_2 - N^2}, \quad R_4 = \frac{-G_c m_1}{S_c^2 + S_c - N^2}$$

3. Skin Friction

With the help of velocity and temperature profiles, the expression for the skin friction (τ) is given

$$\tau = \left(\frac{\partial u}{\partial y}\right) = C_1A_4 - C_2A_5 - A_2(R_2 + R_3) - S_cR_4$$

4.Nusselt Number

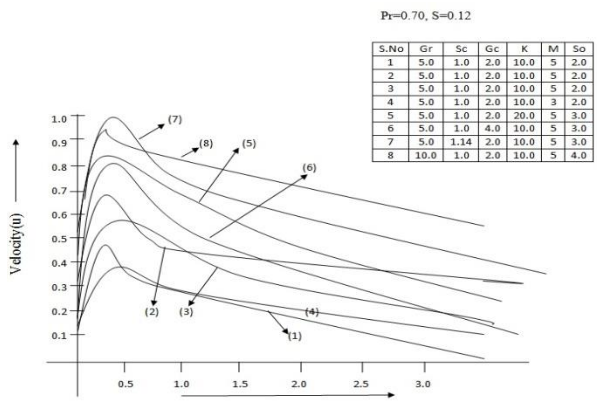
The expression for the Nusselt number (Nu) is

$$Nu = \left(\frac{1}{\theta_0}\right) = A_2 e^{A_2 y}$$

5. Discussion and Conclusion

In order to get physical insight of the problem, calculations have been made for velocity and temperature profiles, species concentration function, skin friction and Nusselt number for different values of the parameters entered into the problem viz. K_0 (Permeability parameter), M (Magnetic parameter), Gr (Grashoff number), Pr (Prandtl number), Sc (Schmidth number), S (Radiation parameter) and Gc (Grashoff number for mass transfer), So(soret number).

In figure 1, the velocity distribution is plotted against y for fixed values Pr = 0.70 (air as a fluid), and S=0.12. We observe that velocity increases with the increase in K_0, So and Gr but decreases with increase in M, Gc and Sc . It is also being observed that the velocity increases rapidly near the plate and then decreases slowly far away from the plate.



S.No	Gr	Sc	Gc	K	M	So
1	5.0	1.0	2.0	10.0	5	2.0
2	5.0	1.0	2.0	10.0	5	2.0
3	5.0	1.0	2.0	10.0	5	2.0
4	5.0	1.0	2.0	10.0	3	2.0
5	5.0	1.0	2.0	20.0	5	3.0
6	5.0	1.0	4.0	10.0	5	3.0
7	5.0	1.14	2.0	10.0	5	3.0
8	10.0	1.0	2.0	10.0	5	4.0

Figure 1: Velocity profiles plotted against y for different values of k,M,Gr,Sc,Gc,So.

Temperature distribution, which depends on Pr and S, is shown in figure 2. It is clear from the figure 2, that the temperature increases with the increases of Pr andS. For the case of the heat sink (S < 0) the temperature of the fluid is less as compared to heat source (S > 0).

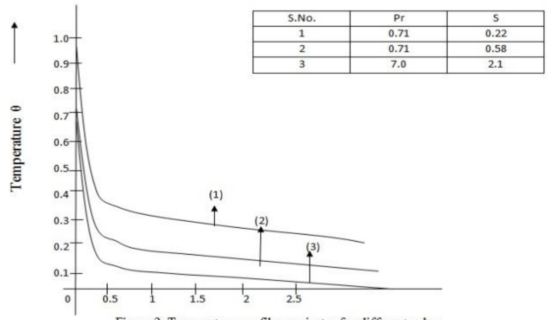


Figure 2: Temperature profiles against y for different value of Pr and S.

The concentration profile (C) is shown in figure 3 against y for different values of Sc and So. It is concluded that increase in Sc decreases species concentration. It is also observed that for (So = 4.0) taking Sc = 2.52 species concentration further decreases.

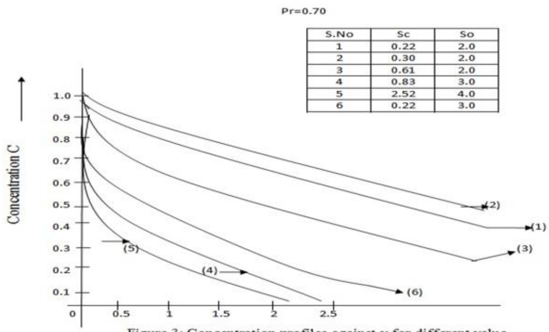


Figure 3: Concentration profiles against y for different value of Sc and So.

We observe from figure 5 that important parameter viz. skin friction (τ) increases when K_0, S, Gc and Gr are increased but decreases with increases in Sc, Pr and M . Another important parameter viz. the Nusselt number (Nu) at the plate is plotted against Prandtl number(Pr) in figure 4. It follows that Nusselt number decreases when Pr increases.

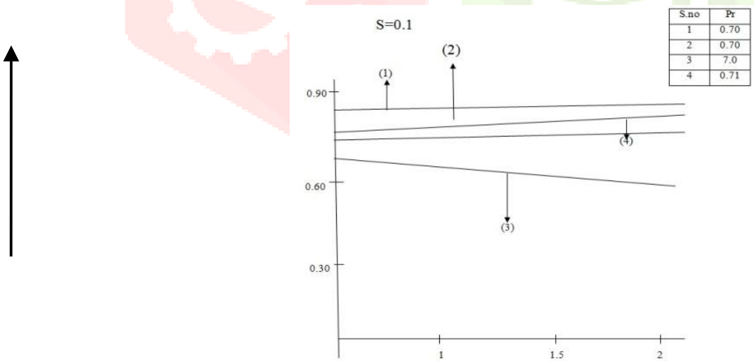


Figure 4: Variation of Nusselt number

τ

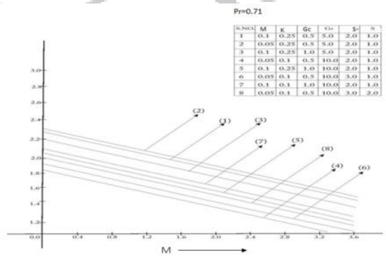


Fig 5. Skin Fraction for different values of Sc, K, M, Gr, Gc and S

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