Heat and Mass transfer on the MHD flow of polar fluid over a vertical plate in the presence of radiation in slip flow regime

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Abstract: Unsteady two-dimensional MHD free convective heat and mass transfer flow of polar fluid past a vertical porous plate immersed in a porous medium with time dependent suction in presence of radiation with viscous dissipation has been considered by employing perturbation technique. Numerical evaluation of the analytical results is performed and the effect of different involved parameters such as rotational parameter, velocity slip parameter, temperature jump parameter, Prandtl number, radiation parameter, Schmidt number on velocity, temperature, skin friction and concentration profiles are plotted and discussed in this paper. We notice that when Radiation parameter decreases Nusselt number decreases on the other hand decrease in temperature jump parameter tends the Nusselt number to rise.

Keywords: Viscous dissipation, MHD, Polar fluid, Radiation, Mass transfer, Porous medium.

1. Introduction:

The problem of fluid flow in an electromagnetic field has been studied for its importance in geophysics, metallurgy, aerodynamics and extrusion of plastic sheets and other engineering processes such as in petroleum engineering, chemical engineering, composite or ceramic engineering and heat exchangers. Many investigations dealing with heat and mass transfer over a vertical porous plate with variable suction, heat absorption/generation have been reported involving heat transfer occurring frequently in the environment. Many practical diffusive operations involve the molecular diffusion of a species in the presences of a chemical reaction within or at the boundary. Chemical reactions can be codified as either heterogeneous or homogeneous processes.

Neild and Bejan [4] discussed study of heat and mass transfer with chemical reactions is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Bakr [1] investigated free convection in MHD Micro polar fluid with radiation and chemical reaction effects. Some other related works can also be found in the paper by Srinivasacharya and Upender [5] and Damesh et. Al [15]. Chamka et al. [2] have reported unsteady MHD natural convection from a heated vertical porous plate in micro polar fluid with joule heating, chemical reaction and radiation effects. Ram Reddy et al. [3] analyzed numerical Solution for mixed convection in a Darcy porous medium saturated with a micro polar fluid under convective Boundary Condition using Spectral Quasi-Linearization method. Nield and Bejan [4] have done a comparative study on various fields of free convection flows and also its application. The effect of thermal radiation on MHD flow and heat transfer problem has become more important industrially. At high operating temperatures radiation effect can be quite significant, many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer became very important. Such flows have been studied by Abo-eldohad and Ghonaim [6] and Rahman [13]. Prabir Kumar et al. [8] investigated MHD micro polar fluid with thermal radiation and thermal diffusion in a rotating frame. Das [9] analyzed the effect of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micro polar fluid in a rotating frame of reference. The radiation effects on boundary layer flow with and without applying a magnetic field under different situations has been studied by Mahmoud [11]. Samad and Rahman [12] have analyzed thermal radiation interaction with unsteady MHD flow past a vertical porous plate immersed in porous medium. Ram and Jain [14] presented the result of a study on hydro-magnetic Ekman layer on convective heat generating fluid in slip flow regime.

Siva Reddy Sheri and Srinivasa Raju [16] have studied soret effect on unsteady MHD free convective flow past a semi-infinite vertical plate in the presence of viscous dissipation.

In view of the above studies, magnetic parameter (M), permeability parameter (K), slip parameter h_1, temperature jump parameter h_2, rotational parameter $\alpha_{1,}$ couple stress parameter $\beta_{1,}$ and thermal and mass Grashof numbers (G_r and G_c resp.) on the unsteady free convective magneto polar flow with variable suction velocity and jump in temperature in a slip flow regime. The effects, on velocity (u), angular velocity (ω), temperature (θ), concentration (C), skin friction (C_f) and rate of heat transfer (N_u), of all the parameter are shown graphically.

2. Formulation of the problem:

Consider the problem of an unsteady two-dimensional, MHD free convective, heat and mass transfer flow with radiation of a polar fluid through a porous medium over a vertical plate with slip boundary condition for velocity field and jump for temperature field. A transfer magnetic field of strength is applied. The plate is moving in its own plane with velocity $U_0(1+\epsilon e^{-nt})$. The permeability of the porous medium is considered to be constant and $V = -V_0(1 + A \in e^{-nt})$, under these conditions and using the Boussinesq's approximation, governing equations of the flow are given by:

Continuity equation:

$$\frac{\partial V}{\partial y} = 0 \qquad \ldots (2.1)$$

Linear momentum:

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta'(C - C_\infty) + (\nu + \nu_r)\frac{\partial^2 u}{\partial y^2} + 2\nu_r\frac{\partial \omega}{\partial y} - \frac{\nu}{K}u - \sigma\frac{{\beta_0}^2}{\rho}u \qquad \ldots (2.2)$$

Angular momentum:

$$\frac{\partial \omega}{\partial t} + V\frac{\partial \omega}{\partial y} = \frac{\gamma}{I}\left(\frac{\partial^2 \omega}{\partial y^2}\right) \qquad \ldots (2.3)$$

Energy equation:

$$\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \qquad \ldots (2.4)$$

Concentration equation:

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + D_l\frac{\partial^2 T}{\partial y^2} \qquad \ldots (2.5)$$

Where u, ω, T and C are velocity, angular velocity, temperature and concentration of the fluid particles. g is acceleration due to gravity, β is coefficient of volumetric expansion, β ′ is coefficient of species concentration expansion, $\rho, \nu, \nu_r k, c_p, \sigma, D_m$, K are density, kinematic viscosity, rotational ,kinematic viscosity, thermal conductivity, specific heat at constant pressure, electrical conductivity, mass diffusivity and permeability of the porous medium respectively. I is a scalar constant equal to moment of inertia of unit mass and $\gamma = c_a + c_d$, where c_a and c_d are coefficient of couple stress viscosities.

The initials and boundary conditions are as follows:

$$y = 0: u = U_0(1+\epsilon e^{-nt}) + L_1\frac{\partial u}{\partial y}, \omega = \frac{-1}{2}\frac{\partial u}{\partial y}, T = T_\omega + \xi\frac{\partial T}{\partial y}, C = C_\omega$$

$$y \to \infty: u \to 0, \omega \to 0, T \to T_\infty, C \to C_\infty \qquad \ldots (2.6)$$

The local radiant for the case of an optically thin gray gas is expressed by:

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma^*\left(T_\infty{}^4 - T^4\right) \qquad \ldots(2.7)$$

We assume that the temperature difference within the flow is sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting the higher order, thus:-

$$T^4 \cong 4T_\infty{}^3 - 3T_\infty{}^4 \qquad \ldots(2.8)$$

By using equations (2.8) in (2.7) we obtain:

$$\frac{\partial q_r}{\partial y} = 16a^*\sigma^*T_\infty^3(T - T_\infty) \qquad \ldots(2.9)$$

Where σ^* is Stefan-Boltzmann constant and a^* is absorption coefficient. On introducing the following non-dimensional quantities:

$$y^* = \frac{V_0 y}{\nu}, t^* = \frac{V_0{}^2}{\nu}, u^* = \frac{u}{V_0}, n^* = \frac{n\nu}{V_0{}^2}, \omega^* = \frac{\nu\omega}{V_0 U_0}, \theta = \frac{T-T_\infty}{T_\omega - T_\infty}, C^* = \frac{C-C_\infty}{C_\omega - C_\infty}$$

Equations (2.2) to (2.5), substituting (2.9) in (2.4), in non-dimensional form after dropping the asterisk are:

$$\frac{\partial u}{\partial t} = (1 + A\epsilon e^{-nt})\frac{\partial u}{\partial y} = \theta G_r + CG_c + (1 + \alpha_1)\frac{\partial^2 u}{\partial y^2} + 2\alpha_1\frac{\partial \omega}{\partial y} - \left[M^2 + \frac{1}{K}\right]u \qquad \ldots(2.10)$$

$$\frac{\partial \omega}{\partial t} - (1 + A\epsilon e^{-nt})\frac{\partial w}{\partial y} = \frac{1}{\beta_1}\frac{\partial^2 \omega}{\partial y^2} \qquad \ldots(2.11)$$

$$\frac{\partial \theta}{\partial t} - (1 + A\epsilon e^{-nt})\frac{\partial \theta}{\partial y} = \frac{1}{p_r}\frac{\partial^2 \theta}{\partial y^2} - \frac{R}{p_r}\theta \qquad \ldots(2.12)$$

$$\frac{\partial C}{\partial t} - (1 + A\epsilon e^{-nt})\frac{\partial C}{\partial y} = \frac{1}{s_c}\frac{\partial^2 C}{\partial y^2} + S_o\frac{\partial^2 T}{\partial y^2} \qquad \ldots(2.13)$$

With corresponding boundary conditions as:

$$y = 0: u = (1 + A\epsilon e^{-nt}) + h_1\frac{\partial u}{\partial y}, \omega = \frac{-1}{2}\frac{\partial u}{\partial y}, \theta = 1 + h_2\frac{\partial \theta}{\partial y}, C = 1$$

$$y \to \infty,\ u \to 0, \omega \to 0, \theta \to 0, C \to 0 \qquad \ldots(2.14)$$

Where

$K^* = K\frac{V_0{}^2}{\nu^2}$ (Permeability parameter), $M^2 = \frac{\sigma\beta^2\nu}{\rho V_0{}^2}$ (Magnetic parameter),

$h_1 = \frac{L_1 V_0}{\nu}$ (Velocity slip parameter), $h_2 = \frac{\xi V_0}{\nu}$ (Temperature jump parameter),

$G_r = \frac{\nu g\beta(T_\omega - T_\infty)}{V_0{}^2 U_0}$ (Thermal Grashof number), $G_C = \frac{\nu g\beta'(C_\omega - C_\infty)}{V_0{}^2 U_0}$ (Mass Grashof number),

$\alpha_1 = \frac{V_r}{\nu}$ (Rotational viscosity parameter),

$\beta_1 = \frac{I\nu}{\gamma}$ (Couple stress parameter) , $S_C = \nu / D_m$ (Schmidt number),

$P_r = \frac{\mu c_p}{k}$ (Prandtl number), $R = \frac{16a^*\sigma^*\nu^2 T_\infty{}^3}{V_0{}^2 k}$ (Radiation parameter),

$L_1 = \left(\left(\frac{2-m_1}{m_1}\right)L\right)$, m_1 being the Maxwell's reflex ion coefficient and L the free path.

$\xi = \left(\frac{2-a}{a}\right)\left(\frac{1.996\eta}{(\eta+1)}\right)\frac{L}{P_r}$, here a is the thermal accommodation coefficient and η is the gas constant and L the free path.

3. Solution of the problem:

ϵ is a small quantity, we reduce the system of partial differential equations to ordinary differential equations by assuming :

$$f(y,t) = f_0(y) + \epsilon e^{-nt} f_1(y) + o(\epsilon^2) + \cdots \qquad \text{... (3.1)}$$

Where f stands for u, ω, θ, and C.

Substituting equation (3.1) in equations (2.10) to (2.13) and equating the like terms, neglecting the coefficient of $o(\epsilon^2)$and higher orders, we get:

$$u_0'' + \frac{1}{(1+\alpha_1)}u_0' - \frac{\left(M^2+\frac{1}{K}\right)}{(1+\alpha_1)}u_0 = \frac{1}{(1+\alpha_1)}(-\theta_0 G_r - C_0 G_c - 2\alpha_1\omega_0') \qquad \text{... (3.2)}$$

$$u_1'' + \frac{1}{(1+\alpha_1)}u_1' - \frac{\left(M^2+\frac{1}{K}-n\right)}{(1+\alpha_1)}u_1 = \frac{1}{(1+\alpha_1)}(-A_0 u_0' - \theta_1 G_r - C_1 G_C - 2\alpha_1\omega_1') \qquad \text{... (3.3)}$$

$$\omega_0'' + \beta_1\omega_0' = 0 \qquad \text{... (3.4)}$$

$$\omega_1'' + \beta_1\omega_1' + n\beta_1\omega_1 = -A\beta_1\omega_0' \qquad \text{... (3.5)}$$

$$\theta_0'' + P_r\theta_0' - R\theta_0 = 0 \qquad \text{... (3.6)}$$

$$\theta_1'' + P_r\theta_1' - (R - nP_r)\theta_1 = -AP_r\theta_0' \qquad \text{... (3.7)}$$

$$C_0'' + S_c C_0' = -S_o\theta_0''$$

$$C_1'' - S_c C_1' + nS_c C_1 = -AS_c C_0' - S_c S_o \theta_1'' \qquad \text{... (3.8)}$$

Here primes denote differentiation with respect to y.

The corresponding boundary conditions can be written as:

$$y = 0: u_0 = 1 + h_1 u_0', \omega_0 = -\frac{1}{2}u_0',\ \theta_0 = 1 + h_2\theta_0', C_0 = 1,, u_1 = 1 + h_1 u_1',$$

$$\omega_1 = -\frac{1}{2}u_1', \theta_1 = h_2\theta_1', C_1 = 0$$

$$y \to \infty: u_0 \to 0,\ \omega_0 \to 0,\ \theta_0 \to 0, C_0 \to 0, u_1 \to 0,\ \omega_1 \to 0,\ \theta_1 \to 0,\ C_1 \to 0 \qquad \text{... (3.9)}$$

Solving equations (3.2) to (3.8) with satisfying boundary conditions (3.9), and substituting back in (3.1), we get:

$$u = \{m_7 e^{x_8 y} + b_1 e^{x_3 y} + b_2 e^{-S_c y} + b_3 e^{-\beta_1 y}\} + \epsilon e^{-nt}\{m_8 e^{x_{11} y} + b_4 e^{x_8 y} + b_{14} e^{x_3 y} + b_{15} e^{-S_c y} + b_{16} e^{-\beta_1 y} + b_8 e^{x_5 y} + b_{10} e^{x_7 y} + b_{12} e^{x_1 y}\} \qquad \text{...(3.10)}$$

$$\omega = m_1 e^{-\beta_1 y} + \epsilon e^{-nt}\{m_2 e^{x_1 y} + b_{17} e^{-\beta_1 y}\} \qquad \text{... (3.11)}$$

$$\theta = m_3 e^{x_3 y} + \epsilon e^{-nt}\{m_4 e^{x_5 y} + b_{18} e^{x_3 y}\} \qquad \text{... (3.12)}$$

$$C = m_5 e^{-S_c y} + \epsilon e^{-nt}\{m_6 e^{z_7 y} + b_{19} e^{-S_c y}\} \qquad \text{... (3.13)}$$

4. Skin Friction:

Knowing the velocity field, the non dimensional skin friction (C_f) at the plate is given by:

$$C_f = \frac{\tau W}{\rho U_0 V_0}$$

$$C_f = (1+\alpha_1)\left[(m_7 x_9 + b_1 x_3 - b_2 S_c - b_3\beta_1) + \epsilon e^{-nt}(m_8 x_{11} + b_4 x_9 + b_{14} x_3 - b_{15} S_c - b_{16}\beta_1 + b_8 x_5 + b_{10} x_7 + b_{12} x_1\right] \qquad \text{... (4.1)}$$

5. Nusselt Number:

Another important physical parameter of interest viz. Nusselt number in dimensionless form is:

$$N_u = \left(\frac{\partial\theta}{\partial y}\right)_{y=0}$$

$$N_u = -[m_3 x_3 + \epsilon e^{-nt}(m_4 x_4 + b_{18} x_3)] \quad \ldots(5.1)$$

Where,

$X_1, X_2 = \frac{-\beta_1 \mp \sqrt{\beta_1^2 - 4n\beta_1}}{2}$, $X_3, X_4 = \frac{-P_r \mp \sqrt{P_r^2 + 4R}}{2}$,

$X_5, X_6 = \frac{-P_r \mp \sqrt{P_r^2 + 4(R - nP_r)}}{2}$, $X_7, X_8 = \frac{-S_c \mp \sqrt{S_c^2 + 4nS_c}}{2}$,

$X_9, X_{10} = \frac{-1 \mp \sqrt{1 + \left(M^2 + \frac{1}{K}\right)(1+\alpha_1)}}{2(1+\alpha_1)}$, $X_{11}, X_{12} = \frac{-1 \mp \sqrt{1 + 4\left(M^2 + \frac{1}{K} - n\right)(1+\alpha_1)}}{2(1+\alpha_1)}$,

$b_1 = \frac{-m_3 G_r}{(1+\alpha_1)(x_3 - x_8)(x_3 - x_{10})}$, $b_2 = \frac{-m_5 G_c}{(1+\alpha_1)(S_c + x_8)(S_c + x_{10})}$,

$b_3 = \frac{2\alpha_1 m_1 \beta_1}{(1+\alpha_1)(\beta_1 + x_8)(\beta_1 + x_{10})}$, $b_4 = \frac{-A m_7 x_8}{(1+\alpha_1)(x_8 - x_{11})(x_8 - x_{12})}$,

$b_5 = \frac{-A b_1 x_3}{(1+\alpha_1)(x_3 - x_{11})(x_3 - x_{12})}$, $b_6 = \frac{-A b_2 S_c}{(1+\alpha_1)(S_c + x_{11})(S_c + x_{12})}$,

$b_7 = \frac{A b_3 \beta_1}{(1+\alpha_1)(\beta_1 + x_{11})(\beta_1 + x_{12})}$, $b_8 = \frac{-m_4 G_r}{(1+\alpha_1)(x_5 - x_{11})(x_5 - x_{12})}$,

$b_9 = \frac{A P m_3 x_3 G_r}{(1+\alpha_1)(x_3 - x_5)(x_3 - x_6)(x_3 - x_{11})(x_3 - x_{12})}$, $b_{10} = \frac{-m_6 G_c}{(1+\alpha_1)(x_7 - x_{11})(x_7 - x_{12})}$,

$b_{11} = \frac{-A m_5 G_c S_c^2}{(1+\alpha_1)(S_c + x_7)(S_c + x_8)(S_c + x_{11})(S_c + x_{12})}$, $b_{12} = \frac{-2\alpha_1 m_2 x_1}{(1+\alpha_1)(x_1 - x_{11})(x_1 - x_{12})}$,

$b_{13} = \frac{2A m_1 \alpha_1 \beta_1^3}{(1+\alpha_1)(\beta_1 + x_1)(\beta_1 + x_2)(\beta_1 + x_{11})(\beta_1 + x_{12})}$, $b_{14} = b_5 + b_9$,

$b_{15} = b_6 + b_{11}$, $b_{16} = b_7 + b_{11}$,

$b_{17} = \frac{A m_1 \beta_1^2}{(1+\alpha_1)(\beta_1 + x_1)(\beta_1 + x_2)}$, $b_{18} = \frac{-A P_r m_3 x_3}{(x_3 - x_5)(x_3 - x_6)}$,

$b_{19} = \frac{A m_1 S_c^2}{(S_c + x_7)(S_c + x_8)}$,

$m_1 = -\frac{1}{2}[m_7 x_9 - b_1 x_3 - b_2 S_c - b_3 \beta_1]$,

$m_2 = -b_{17} - \frac{1}{2}[m_8 x_{11} + b_4 + x_9 + b_{14} x_3 - b_{15} S_c - b_{16} \beta_1 + b_8 x_5 + b_{10} x_7 + b_{12} x_1]$,

$m_3 = \frac{1}{(1 - h_2 x_3)}$, $m_4 = \frac{b_{18}(1 - h_2 x_3)}{(h_2 x_5 - 1)}$, $m_5 = 1$,

$m_6 = -b_{19}$,

$m_7 = \frac{(1 - b_1)(1 - x_3 h_1) - b_2(1 + h_1 S_c) - b_3(1 + h_1 \beta_1)}{(1 - h_1 x_{11})}$,

6. Results and Discussion:

In order to understand the physical importance of the flow and to find the effects of different parameters, calculations have been carried out for velocity, angular velocity, temperature, concentration, skin friction and the rate of heat transfer, for different values of the permeability parameter (K), the magnetic parameter (M), the thermal Grashof number (G_r), the mass Grashof number (G_c), the velocity slip parameter (h_1), the temperature jump parameter (h_2), the rotational viscosity parameter (α_1), the couple stress parameter (β_1), the Prandtl number (P_r), the schmidt number (S_c). Result is also shown for particular cases of no slip $(h_1 = 0)$ and for no jump in temperature $(h_2 = 0)$. $\in = o.1$, $n = 0.1$ are considered to be fixed.

In figures 1 and 2, the velocity distribution is plotted against y for air $(P_r = 0.71), (S_c = 0.22)$ and for water $(P_r = 7, S_c = 0.61)$ fixing $t = 1$, $A = 0.5$ and R =0.2. For both air and water, we observe that on decreasing k, (G_r) and (G_c) velocity decreases where as on decreasing (M) and (h_2) velocity increases. It is specially observed that on decreasing (h_1) velocity decreases near the plate but rises up as we move away from the plate for both the basic fluids. Results differ for (α_1) and(β_1). For air on decreasing (α_1) velocity rises near the plate but drops as we move away and on increasing (β_1) velocity drops where as for water on decreasing (α_1) velocity drops and on increasing (β_1) velocity rises near the plate but then drops. Results are specially observed for the case of no slip $(h_1 = 0)$ and no jump in temperature $(h_2 = 0)$ for both the basic fluids air and water.

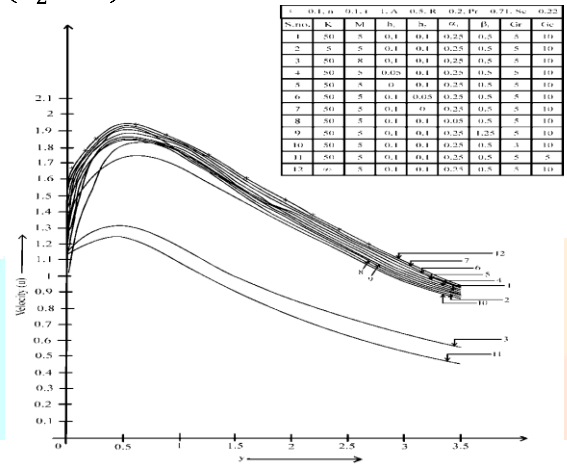
Figure 1: Velocity profiles in air plotted against y for different values of K, M, h_1, h_2, α_1, β_1, Gr and Gc.

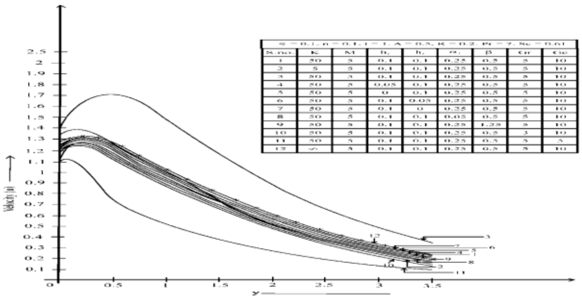
Figure 2: Velocity profiles in water plotted against y for different values of K, M, h_1, h_2, α_1, β_1, Gr and Gc.

Angular velocity distribution is plotted against y for air $(P_r = 0.71), (S_c = 0.22)$ and $(P_r = 7, S_c = 0.61)$ water in figures 3 and 4, fixing t=1, A=0.5 and R=0.2. It is observed that on decreasing k, G_r and (G_c) velocity increases whereas on decreasing M, h_1, h_2, α_1 and β_1 velocity decreases. Cases for no slip $(h_1 = 0)$ and no jump in temperature $(h_2 = 0)$ are also observed for both the basic fluids.

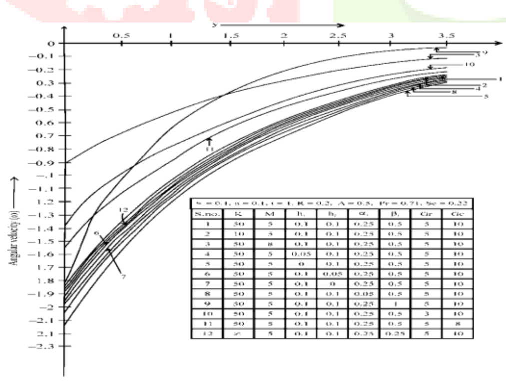
Figure 3: Angular velocity profiles in air plotted against y for different values of K, M, h_1, h_2, α_1, β_1, Gr and Gc.

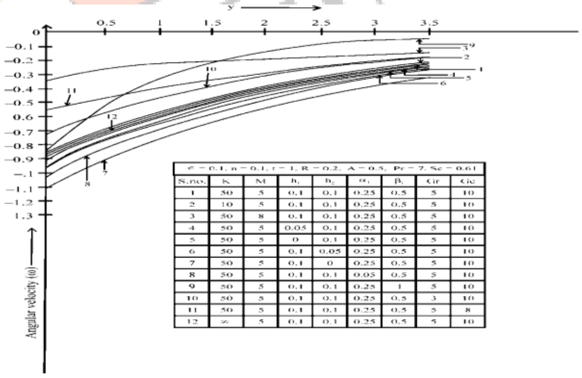
Figure 4: Angular velocity profiles in water plotted against y for different values of K, M, h_1, h_2, α_1, β_1, Gr and Gc.

In figure 5, temperature profiles are plotted against y for both air $(P_r = 0.71), (S_c = 0.22)$ and water $(P_r = 7, S_c = 0.61)$ fixing t=1. We observe that as R, A and h_2 decrease, temperature increases. Also for negative of radiation (absorption) temperature rises. Concentration profiles are plotted against y in figure 6, fixing t=1. We notice that increasing value of Schmidt number decreases the concentration of the fluid.

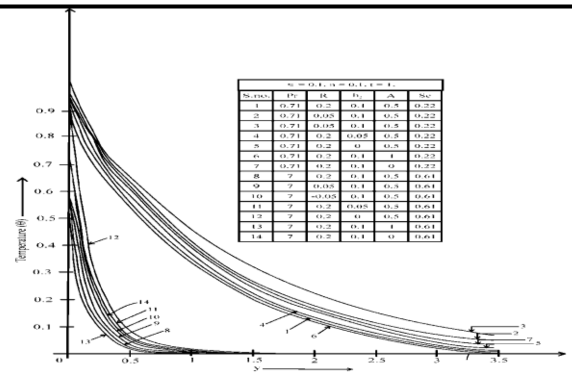

Figure 5: Temperature profiles plotted against y of different values of Pr, R, h_2, A and Sc.

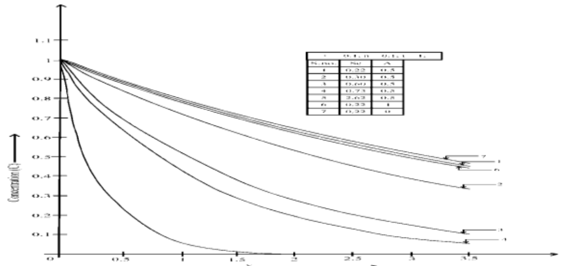

Figure 6: Concentration profiles plotted against y for different values Sc and A.

Here, we may say that concentration is highest for hydrogen ($S_c = 0.22$) but least for Propyl benzene ($S_c = 2.62$) . Also, we notice that increasing the value of A, in figures 7 and 8, skin friction is plotted against K for both the basic fluids air ($P_r = 0.71$), ($S_c = 0.22$) and water ($P_r = 7, S_c = 0.61$) respectively. We notice that for both air and water, on decreasing M and h_1 skin friction increases where as on decreasing G_c and R skin friction drops. Results differ for h_1, α_1, β_1 and G_r. For air, if h_2 decreases skin friction increases on the other hand decreasing α_1, β_1 and G_r drops the skin friction. For water, decrease in h_2 tends the skin friction to drop where as decrease in $\alpha_1, \beta_1$and G_r increases the skin friction. Also for negative of radiation (absorption) skin friction drops further.

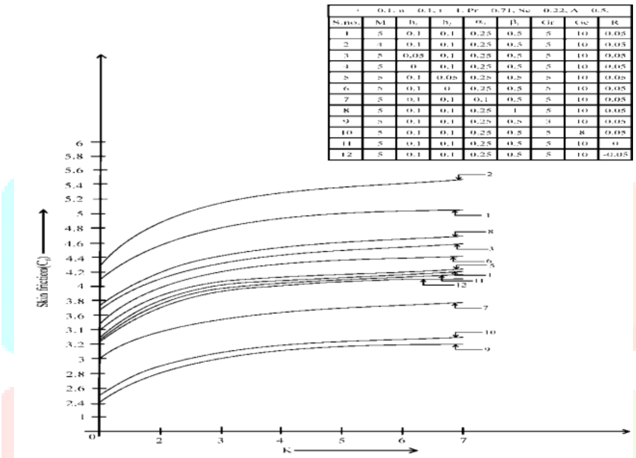

Figure 7: Skin friction in air plotted against K for different values of M, h_1, h_2, α_1, β_1, Gr, Gc and R.

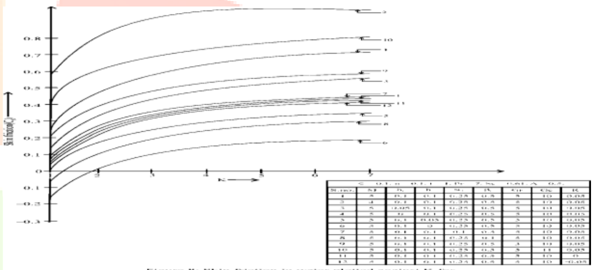

Figure 8: Skin friction in water plotted against K for different values of M, h_1, h_2, α_1, β_1, Gr, Gc and R.

Nusselt number is plotted against t for both basic fluids air($P_r = 0.71$), ($S_c = 0.22$) and water($P_r = 7, S_c = 0.61$) in figure 9. We notice that when radiation parameter (R) decreases Nusselt number decreases on the other hand decrease in (h_2) tends the Nusselt number to rise. Also for negative of radiation (absorption) Nusselt number falls further.

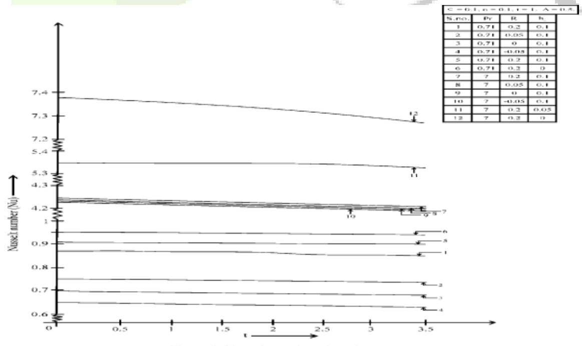

Figure 9: Nusselt number plotted against t for different values of Pr, R and h_2.

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