PRE-DFT COMBINING CODED SIMO OFDM SYSTEM IN MULTI-OBJECTIVE OPTIMIZATION

[1] B.Ramu, [2]K.Victor

Abstract: For coded SIMO-OFDM systems, pre- DFT combining was shown to provide a good trade-off between error-rate performance and processing complexity. Max-sum SNR and max-min SNR are two reasonable ways for obtaining these combining weights. In this paper multi attribute augmentation is employed to further reveal the suitability and limitation of these two criteria. The results show that neither max-sum SNR nor max-min SNR is universally good. For better error-rate performance, the means for weight calculation should be adapted according to the capability of the error-correcting code used, and multi attribute augmentation can help in the determination.

I.INTRODUCTION

ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM) combined with multiple receive antennas, namely, single-input multiple-output (SIMO) OFDM, has recently been investigated for use in wireless communication systems. It can provide high spectrum efficiency and high data rate for information transmission. On one hand, OFDM divides the entire channel into many parallel sub channels which increases the symbol duration and therefore reduces the inter-symbol interference (ISI) caused by multipath propagation. Besides, since the subcarriers are orthogonal to each other, OFDM can utilize the spectrum very efficiently. It is known that subcarrier-based maximum ratio combining (MRC) performs the best for coded SIMO-OFDM systems; however, it requires high processing complexity. Pre-discrete Fourier transform (DFT) combining was then developed, in which only one DFT block is necessary at the receiver. It was previously shown to provide a good trade-off between error-rate performance and processing complexity. In this letter, we employ multi-objective optimization to reveal the suitability and limitation of two previously-proposed criteria for obtaining the pre-DFT combining weights, i.e., maximization of the sum of subcarrier signal-to-noise ratio (SNR) values (called max-sum SNR hereafter) and maximization of the minimum subcarrier SNR value (called max-min SNR hereafter) . Our results show that neither max-sum SNR nor max-min SNR is universally good. Furthermore, for better error rate performance, the means for weight calculation should be adapted according to the capability of the error-correcting code used, and multi-objective optimization can help in the determination. Monte Carlo simulations are finally provided to verify the correctness of these sayings. Throughout the letter, we use boldface letters, boldface letters with over bar, lower-case letters, and upper-case letters to denote vectors, matrices, time-domain signals, and frequency domain signals, respectively. Besides $(\cdot)^T$, $(\cdot)^H$ trace(·), rank(·), and diag{·} are used to represent the matrix transpose, matrix Hermitical, matrix trace, matrix rank calculation, and diagonal matrix with its main diagonal being the included vector, respectively

Pre-DFT Combining SIMO-OFDM System:

We consider an SIMO-OFDM system with M receive antennas. Define an N×1 signal vector.

$$\mathbf{S}(k) = [S(kN)\ S(kN+1)\ \cdots\ S(kN+N-1)]^T$$

as the k^{th} OFDM data block to be transmitted, where N is the number of subcarriers. This data block is first modulated by the inverse DFT (IDFT). With matrix representation, we can write the output of the IDFT as

$$\mathbf{s}(k) = [s(kN)\ s(kN+1)\ \cdots\ s(kN+N-1)]^T = \bar{\mathbf{F}}^H\mathbf{S}(k)\ \bar{\mathbf{F}}$$

Where is an N × DFT matrix with elements

$[\bar{\mathbf{F}}]_{p,q} =$ $(1/\sqrt{N})\exp(-j2\pi pq/N)$ For $p, q = 0, 1, \cdots, N-1$ and $j = \sqrt{-1}$. A cyclic prefix (CP) is inserted afterwards and its length (Lcp) is chosen to be longer than the maximum length of the multipath fading channel (L). Also define an N × 1vector $\mathbf{h}_m = [h_m(0)\ h_m(1)\ \cdots\ h_m(L-1)\ 0\ \cdots\ 0]^T$

Where $h_m(l)$ represents the l^{th} channel coefficient for the m^{th} receive antenna, with l = 0, 1, · · · , L − 1 and m = 0, 1, · · · ,M − 1. Collecting all channel vectors from the M different receive antennas, we construct an N × M channel matrix

$\bar{\mathbf{h}} = [\mathbf{h}_0\ \mathbf{h}_1\ \cdots\ \mathbf{h}_{M-1}]$ And its frequency response as

$$\bar{\mathbf{H}} = [\mathbf{H}_0\ \mathbf{H}_1\ \cdots\ \mathbf{H}_{M-1}] = \bar{\mathbf{F}}\bar{\mathbf{h}} \qquad (1)$$

with $\mathbf{H}_m = \bar{\mathbf{F}}\mathbf{h}_m$. In an ordinary OFDM signal reception process, after CP removal and DFT demodulation, the resultant N × 1 signal vector from the mth receive antenna, denoted by $\mathbf{R}_m(k)$ can be shown to be

$$\mathbf{R}_m(k) = \text{diag}\{\mathbf{S}(k)\}\mathbf{H}_m + \mathbf{N}_m(k) \qquad (2)$$

Where $\mathbf{N}_m(k)$ is an N × 1 complex Gaussian noise vector with zero mean and equal variance for each element.

For the considered SIMO scenario, we can collect the M received

Signal vectors and form an $N \times M$ received signal matrix as

$$\bar{\mathbf{R}}(k) = [\mathbf{R}_0(k)\ \mathbf{R}_1(k)\ \cdots\ \mathbf{R}_{M-1}(k)]. \qquad (3)$$

Let $\mathbf{w} = [w_0\ \ w_1\ \ \cdots\ \ w_{M-1}]^T$ be an $M \times 1$ weight vector. With (1)-(3), the pre-DFT combining operation and the resultant $N \times 1$ signal vector can be expressed as

$$\mathbf{Y}(k) = \bar{\mathbf{R}}(k)\mathbf{w} = \mathrm{diag}\{\mathbf{S}(k)\}\bar{\mathbf{H}}\mathbf{w} + \bar{\mathbf{N}}(k)\mathbf{w} \qquad (4)$$

With $\bar{\mathbf{N}}(k) = [\mathbf{N}_0(k)\ \mathbf{N}_1(k)\ \cdots\ \mathbf{N}_{M-1}(k)]$.

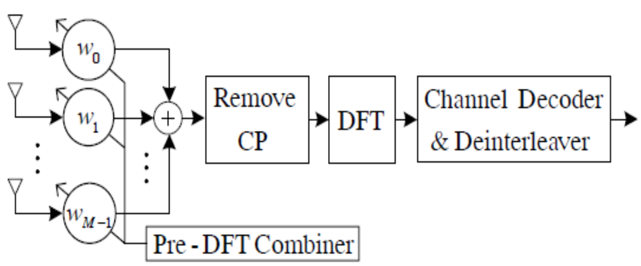

Fig. Block diagram of OFDM diversity receiver with pre-DFT combining.

Block diagram of a simplified OFDM receiver performing pre- DFT combining. W was calculated based on max-sum SNR. For that case, the optimum w can be shown to be the solution of the following optimization problem:

$$\max_{\mathbf{w}}\ \mathbf{w}^H\bar{\mathbf{H}}^H\bar{\mathbf{H}}\mathbf{w},\ \ \text{subject to}\ \ \mathbf{w}^H\mathbf{w} = 1 \qquad (5)$$

Where $\mathbf{w}^H\bar{\mathbf{H}}^H\bar{\mathbf{H}}\mathbf{w}$ indicates the sum of the signal power in all N subcarriers. As an alternative, pre-DFT combining based on max-min SNR. Define a $1 \times M$ vector γ_n as the n^{th} row of the channel matrix $\bar{H}$ given in (1), with $n = 0, 1, \cdots, N-1$ For that approach, the optimization of w can be described as

$$\max_{\mathbf{w}}\ \min_{n}\ |\gamma_n\mathbf{w}|^2,\ \ \text{subject to}\ \ \mathbf{w}^H\mathbf{w} = 1 \qquad (6)$$

In which $|\gamma_n\mathbf{w}|^2$ indicates the signal power of the nth subcarrier after pre-DFT combining.

It is understood that while max-sum SNR tends to help the good, max-min SNR tends to help the bad. Both criteria are reasonable for obtaining the pre-DFT combining weights. Nevertheless, two questions are naturally raised:

1) Is one of the two criteria strictly superior to the other?

2) Can we further improve the error-rate performance with pre-DFT combining? We try to answer these questions through the use of multi-objective optimization in the following.

Multi-objective Optimization For PRE-DFT Combining: Although max-sum SNR and max-min SNR are both practical, they are normally in conflict with each other, i.e., an improvement in one leads to deterioration in the other, which will be shown later in this section. This motivates the use of multi-objective optimization for gaining further insight into the two criteria. Formally, a multi-objective optimization problem for the case at hand can be stated as follows:

$$\max_{\mathbf{w}}\ \mathbf{g}(\mathbf{w}) = \begin{bmatrix} g_1(\mathbf{w}) \\ g_2(\mathbf{w}) \end{bmatrix},\ \ \text{subject to}\ \ \mathbf{w}^H\mathbf{w} = 1 \qquad (7)$$

With
$g_1(\mathbf{w}) = \min_n\ |\gamma_n\mathbf{w}|^2$ and $g_2(\mathbf{w}) = \left(\mathbf{w}^H\bar{\mathbf{H}}^H\bar{\mathbf{H}}\mathbf{w}\right)/N,$

in which $g2(\mathrm{w})$ is normalized for convenience during numerical calculation. With (7), we can generally look for some good trade-offs, rather than a single solution of either max-sum SNR or max-min SNR. For this problem, a solution is optimal if there exists no other solution that gives enhanced performance with regard to both $g1(\mathrm{w})$ and $g2(\mathrm{w})$ - Pareto optimizers. The set of Pareto optimizers is called the Pareto front . However, there is no systematic manner to find the Pareto front in (7). Instead, we use a simple and popular way, i.e., the weighted-sum method, to approach to the solution set. This essentially converts the multi-objective optimization problem\ into a single objective problem. Mathematically speaking, the objective function in this circumstance is changed to be a linear combination of the two objectives as

$$\max_{\mathbf{w}}\ \lambda g_1(\mathbf{w}) + (1-\lambda)g_2(\mathbf{w}),\ \ \text{subject to}\ \ \mathbf{w}^H\mathbf{w} = 1\ (8)$$

Where $\lambda \in [0, 1]$ is a parameter determining the relative importance between max-sum SNR and max-min SNR? Solving (8) yields the solution that gives the best compromise for a typical λ. Next, we show that (8) can be efficiently evaluated via convex optimization techniques. Without loss of generality, we can recast the optimization problem in (8) to be

$$\max_{\mathbf{W}}\ \lambda\left[\min_{n}\ \mathrm{trace}(\mathbf{\Gamma}_n\mathbf{W})\right] + (1-\lambda)\left[\mathrm{trace}(\mathbf{Q}\mathbf{W})\right]$$

Subject to

$$\mathrm{trace}(\mathbf{W}) = 1,\ \mathrm{rank}(\mathbf{W}) = 1,\ \mathbf{W} \succeq \mathbf{0} \qquad (9)$$

With $\mathbf{\Gamma}_n = \gamma_n^H\gamma_n$ and $\mathbf{Q} = \bar{\mathbf{H}}^H\bar{\mathbf{H}}$. In (9),

$\mathbf{W}$ is an $M \times M$ matrix to be determined and the inequality $\mathbf{W} \succeq 0$ means that $\mathbf{W}$ is symmetric positive semi definite. Instead of solving the above nondeterministic polynomial-time hard (NP-hard) problem directly, we seek an approximation of the solution. By dropping the no convex rank-one constraint, this weighted sum objective function can be relaxed to

$$\max_{\mathbf{W}} \ \lambda \left[\min_{n} \ \mathrm{trace}(\mathbf{\Gamma}_n \mathbf{W})\right] + (1-\lambda)\left[\mathrm{trace}(\mathbf{Q}\mathbf{W})\right]$$

Subject to

$$\mathrm{trace}(\mathbf{W}) = 1, \ \mathbf{W} \succeq \mathbf{0}. \qquad (10)$$

Let $z1$ and $z2$ be two scalars. The relaxation is equivalent to

$$\max_{\mathbf{W}} \ \lambda z_1 + (1-\lambda) z_2,$$

Subject to $\mathrm{trace}(\mathbf{\Gamma}_n \mathbf{W}) \geq z_1, \ \mathrm{trace}(\mathbf{Q}\mathbf{W}) \geq z_2$

$$\mathrm{trace}(\mathbf{W}) = 1, \ \mathbf{W} \succeq \mathbf{0} \qquad (11)$$

Which becomes convex? It is not difficult to see that (11) can be categorized to be a semi definite programming problem. The optimal choice of W, i.e., Wopt, can be obtained systematically using the efficient interior point method, and then a randomization step is used to produce an approximated solution to (7). In general, the complexity from weight calculation can be ignored as compared with the complexity saving from the reduction of DFT components. An example of a typical Pareto front solved via (11) is illustrated in Fig. 2. To obtain the entire approximation set, the search is repeated with various values of λ. We clearly see the trade-off between max-sum SNR and max- in SNR. Besides, the weighted-sum method along with the convex formulation can efficiently approach the Pareto front, as expected.

EXPERMENTAL RESULTS:

A comparison of the bit-error-rate (BER) performance with different pre-DFT combining is made by Monte Carlo simulations carried out regarding a 1 × 2 coded OFDM System. Quadrature phase-shift keying (QPSK) is used for modulation. Besides,

$$N = 64, \ L_{\mathrm{cp}} = 16, \text{ and } L = 2$$

(Independently generated with the Rayleigh distribution) are set. Convolution codes with different error-correcting capabilities (different minimum free distance $d$free) are used for error protection. At the receiver, the Viterbi algorithm with hard decision is employed for decoding. Figs. 3 and 4 present the corresponding BER performance. From these figures, we have the following observations: For the case of higher error-correcting capability (Fig. 3), max-sum SNR Performs slightly better than max-min SNR. Note that maxsum SNR generally focuses on the good and ignores the bad. With the relatively large amount of error protection, the low subcarrier SNR values may be compensated. Together with the “boosted” high-SNR subcarriers, max-sum SNR provides better BER performance in this case. On the contrary, for the case of lower error-correcting capability (Fig. 4), max-min SNR outperforms max-sum SNR, especially in the high SNR region. The small amount of error protection makes each subcarrier equally essential. Max- in SNR usually does a good job in balancing the subcarrier SNR values, and thus gives better ER performance. Moreover, it is interesting to note that in either Fig. 3 or Fig. 4, the weighted-sum method which successfully captures the advantages of both max-sum SNR and max-min SNR is superior to these two previously-proposed criteria. By varying λ, there exists some cases in which a lower BER can be achieved. That is to say, multi-objective optimization can be employed to form some better pre-DFT combining weights over the pure max-sum SNR and max-min SNR. By means of exhaustive simulations, we find that the effect of max-min SNR is more substantial than that of maxsum SNR in most circumstances. Thus, as a rule of thumb, λ should be set close to 1 for better BER performance.

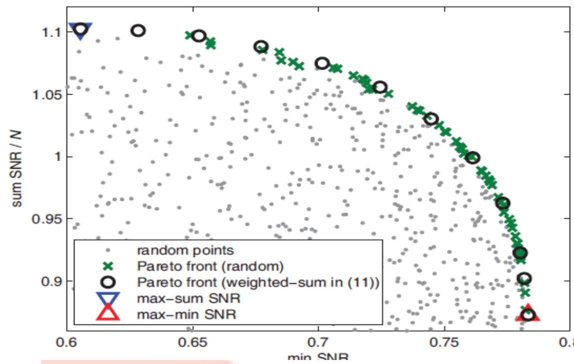

Fig. Pareto front for max-sum SNR and max-min SNR with SNR=15 dB, M = 2, N = 64, Lcp = 16, L = 2, and λ = [0 : 0.1 : 0.8 0.9 : 0.05 : 1].

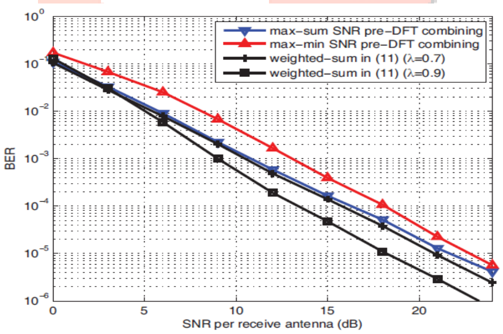

Fig. BER for 1/2-rate convolution-coded SIMO-OFDM

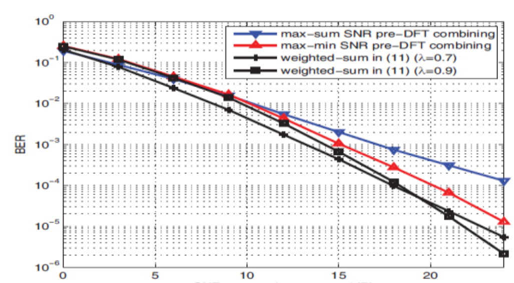

Fig. BER for 3/4-rate convolution-coded SIMO-OFDM with generator sequence

CONCLUSION: This letter has discussed and compared the error-rate performance for coded SIMO-OFDM systems with different pre-DFT combining. Our results show that multi-objective optimization can be used to determine some better pre-DFT combining weights, which are generally superior to both maxsum SNR and max-min SNR for achieving a lower BER. To this end, finding a more exact relation between the weighted sum parameter λ and the BER is surely interesting, and can Serve as our future area of research. single chip VLSI implementation," in *the Mobile*

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