IMPLEMENTATION OF GEOMETRIC MEAN FOR HIGHER ORDER INTERVAL SYSTEMS

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Abstract : This note represents a method, for the reduction of the order of Interval system about a general shifting point 'a' .The selection of this shifting point 'a' done based upon the geometric mean of the real parts of the poles of four high order fixed systems obtained by Kharitonov's theorem. The denominator of the reduced model obtained by Least Square method while the numerator is obtained by matching the power series expansion of the original high order system with the reduced model .A numerical example illustrates its application.

Keywords: Least Squares Method, Large scale Interval system, order reduction, Geometric mean

I. Introduction

Analysis and implementation of large scale systems are both tedious and costly as a large scale linear control systems are often too complicated to be used in real problems. Hence, there is great demand in the control systems literature for realizing low-order simple linear models for the original large-scale linear control systems. Reducing a high order system into its lower order system is considered important in analysis, synthesis and simulation of practical systems. Numerical methods are available in the literature for order reduction of large scale systems. Recently it was shown in [1] that care has to be taken, if the system transfer function contains a pole of magnitude less than one, numerical problems can arise owing to a rapid increase in the magnitude of successive Time moments. This gives an ill conditioned set of linear equations to solve for the reduced denominator .To overcome this problem, it is sometimes possible to use a linear shift s→ (s+a) such that the pole of smallest magnitude has the modulus of approximately one, this tends to reduce the sensitivity of the method . However, the focus of the work so far appears to concentrate mainly on the basic idea of extending this technique for order reduction of fixed parameter systems. In this paper this method is extend for order reduction of high order Interval systems. Consequently, the method is more flexible than most other stability preserving methods and is simple to implement.

II. Reduction procedure

Given the nth order transfer function of a high order interval systems be represented as

$$H(s) = \frac{[a_0^-, a_0^+] + [a_1^-, a_1^+]s + \cdots + [a_{n-1}^-, a_{n-1}^+]s^{n-1}}{[b_0^-, b_0^+] + [b_1^-, b_1^+]s + \cdots + [b_n^-, b_n^+]s^n} \qquad \ldots (1)$$

Where $[a_i^-, a_i^+]$ for i = 0 to n-1 and $[b_i^-, b_i^+]$ for i = 0 to n are the interval parameters. Consider now the set δ(s) of real polynomials of degree 'n' of the form

$$\delta(s) = \delta_0 + \delta_1 s + \cdots + \delta_n s^n \qquad \ldots (2)$$

Where the coefficients lie within given ranges $\delta_0 \in [x_o, y_o]$, $\delta_1 \in [x_1, y_1]$, …. $\delta_n \in [x_n, y_n]$ .Write $\delta = [\delta_0, \delta_1, \ldots\ldots\ldots\ldots, \delta_n]$ using the Kharitonov's theorem the following four extreme polynomials are derived

$$K_1(s) = x_0 + x_1 s + y_2 s^2 + y_3 s^3 + \cdots$$

$$K_2(s) = x_0 + y_1 s + y_2 s^2 + x_3 s^3 + \cdots$$

$$K_3(s) = y_0 + x_1 s + x_2 s^2 + y_3 s^3 + \cdots$$

$$K_4(s) = y_0 + y_1 s + x_2 s^2 + x_3 s^3 + \cdots$$

From the above equations the numerator and the denominator polynomials are obtained.
Thus the four nth order system transfer functions are obtained each as

$$G_P(s) = \frac{A_{P0} + A_{P1}s + A_{P2}s^2 + \cdots + A_{Pn-1}s^{n-1}}{B_{P0} + B_{P1}s + B_{P2}s^2 + \cdots + B_{pn}s^n} \qquad \ldots (3)$$

Where p = 1, 2, 3, 4. and

n = order of the original system.

Substitute the $G_p(s)$ by $G_p(s+a)$ where the value of 'a' obtained by Geometric mean.

Let the nth order system transfer function of $G_p(s)$ can be written as:

$$G_p(s) = k\frac{\prod_{i=1}^{m}(s+Z_i)}{\prod_{i=1}^{n}(s+P_i)}$$

Where, P_i = poles of the system and Z_i = zeros of the system, $G_p(s)$. For this system, 'a' is given by the Geometric mean (G.M) of the magnitude of real parts of P_i $(|P_i|)$.

$$a=\prod_{i=1}^{n}(|P_i|)^{1/n}$$

The above equation gives value for the linear shift point 'a'. If $G_p(s+a)$ is expanded about s=0, then the Time moment proportional's, c_i are obtained by:

$$G_p(s+a) = \sum_{i=0}^{\infty} c_i s^i \quad \text{... (4)}$$

Similarly, if $G_p(s+a)$ is expanded about $s=\infty$, then the Markov parameters m_j are obtained by:

$$G_p(s+a) = \sum_{j=1}^{\infty} m_j s^{-j} \quad \text{… (5)}$$

The four reduced rth order models obtained as

$$R_p(s) = \frac{d_{p0}+d_{p1}s+d_{p2}s^2+\cdots+d_{pr-1}s^{r-1}}{e_{p0}+e_{p1}s+e_{p2}s^2+\cdots+e_{pr}s^r} \quad \text{… (6)}$$

Which retains 't' Time moments and 'm' Markov parameters, the coefficients e_{pk}, d_{pk} in (6) are derived from following set of equations

$$\begin{aligned}
d_{p0} &= e_{p0}c_0 \\
d_{p1} &= e_{p1}c_0 + e_{p0}c_1 \\
d_{p2} &= e_{p2}c_0 + e_{p1}c_1 + e_{p0}c_2 \\
&\;\;\vdots \\
d_{pr-1} &= e_{pr-1}c_0 + \cdots + e_{p0}c_{r-1} \\
0 &= e_{pr-1}c_1 + \cdots + e_{p0}c_r \\
0 &= e_{pr-1}c_2 + \cdots + e_{p0}c_{r+1} \\
&\;\;\vdots \\
0 &= e_{pr-1}c_t + \cdots + e_{p0}c_{r+t-1}
\end{aligned} \quad \text{… (7)}$$

And

$$\left.\begin{aligned}
d_{pr-1} &= m_1 \\
d_{pr-2} &= m_1 e_{pr-1} + m_2 \\
&\;\;\vdots \\
d_{pt} &= m_1 e_{pt+1} + m_2 e_{pt+2} + \cdots + m_{r-t}
\end{aligned}\right\} \text{… (8)}$$

Where the c_j and m_k are the Time moment proportional and Markov parameters of the system, such that j=(0,1,…..t-1) and k=(1,2,…..m) respectively. The denominator coefficients of the reduced model are obtained by substituting (8) in (7) and are given by the solution set.

$$\begin{bmatrix} c_{r+t-1} & c_{r+t-2} & \cdots & \cdots & \cdots & \cdots & c_t \\ c_{r+t-2} & c_{r+t-3} & \cdots & \cdots & \cdots & c_t & c_{t-1} \\ . & . & \cdots & \cdots & \cdots & . & . \\ c_{r-1} & c_{r-2} & \cdots & \cdots & \cdots & c_1 & c_0 \\ c_{r-2} & c_{r-3} & \cdots & \cdots & \cdots & c_0 & -m_1 \\ c_{r-3} & c_{r-4} & \cdots & \cdots & c_0 & -m_1 & -m_2 \\ . & . & & & . & . & . \\ . & . & & & . & . & . \\ . & . & \cdots & \cdots & . & . & . \\ c_t & c_{t-1} & \cdots & c_0 & -m_1 & \cdots & -m_{r-t-1} \end{bmatrix} \times \begin{bmatrix} e_{p0} \\ e_{p1} \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ e_{pr-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ m_1 \\ m_2 \\ m_3 \\ . \\ . \\ m_{r-t} \end{bmatrix} \quad \ldots (9)$$

or, the above equation can be represented as H e = m in matrix vector form and 'e' can be calculated from,

$$e = \left(H^T H\right)^{-1} H^T m \qquad \ldots (10)$$

are the coefficients of the reduced model denominator. If this estimate still does not yield a stable reduced denominator then H and m in (10) are extended by another row, which corresponds to using the next Markov parameter from the full system in Least Squares match. Once the reduced denominator obtained, formed by 'e', apply the inverse shift s→(s-a) to this reduced denominator. Later calculate the reduced numerator as before by matching proper number of Time moments of $G_p(s+a)$ to that of reduced model.

III. Illustrative Example

Consider the 6th order interval system given by its transfer function:

$$G(s) = \frac{[2\ 3]s^5 + [70\ 71]s^4 + [762\ 763]s^3 + [3610\ 3611]s^2 + [7700\ 7701]s^1 + [6000\ 6001]}{[1\ 2]s^6 + [41\ 42]s^5 + [571\ 572]s^4 + [3491\ 3492]s^3 + [10060\ 10061]s^2 + [13100\ 13101]s + [6000\ 6001]}$$

The four transfer functions obtained using the Kharitonov's theorem

$$G_1(s) = \frac{2s^5 + 70s^4 + 763s^3 + 3611s^2 + 7700s^1 + 6000}{2s^6 + 41s^5 + 571s^4 + 3492s^3 + 10061s^2 + 13100s + 6000}$$

$$G_2(s) = \frac{3s^5 + 70s^4 + 762s^3 + 3611s^2 + 7701s^1 + 6000}{2s^6 + 42s^5 + 571s^4 + 3491s^3 + 10061s^2 + 13101s + 6000}$$

$$G_3(s) = \frac{2s^5 + 71s^4 + 763s^3 + 3610s^2 + 7700s^1 + 6001}{s^6 + 41s^5 + 572s^4 + 3492s^3 + 10060s^2 + 13100s + 6001}$$

$$G_4(s) = \frac{3s^5 + 71s^4 + 762s^3 + 3610s^2 + 7701s^1 + 6001}{s^6 + 42s^5 + 572s^4 + 3491s^3 + 10060s^2 + 13101s + 6001}$$

The corresponding reduced models obtained using four Time moments

$$R_1(s) = \frac{2.367979s + 2.632555}{s^2 + 4.737273s + 2.632555} \text{ With GM=2.886004}$$

$$R_2(s) = \frac{2.1959623 + 6.528736}{s^2 + 8.071824s + 6.528736} \text{ With GM=2.94729}$$

$$R_3(s) = \frac{2.112907s + 10.068428}{s^2 + 11.17298s + 10.068428} \text{ With GM=4.2235}$$

$$R_4(s) = \frac{0.629336s + 30.981842}{s^2 + 28.508347s + 30.981842} \text{ With GM=4.225735}$$

Thus the transfer function of the reduced interval model obtained as

$$R(s) = \frac{[0.62936\ 2.367976]s + [2.632555\ 30.981842]}{[1\ 1]s^2 + [4.737273\ 28.508347]s + [2.632555\ 30.981842]}$$

The simulation results in figure1 and figure 2 , shows the accuracy of the step response when the reduced model compared with the original interval system.

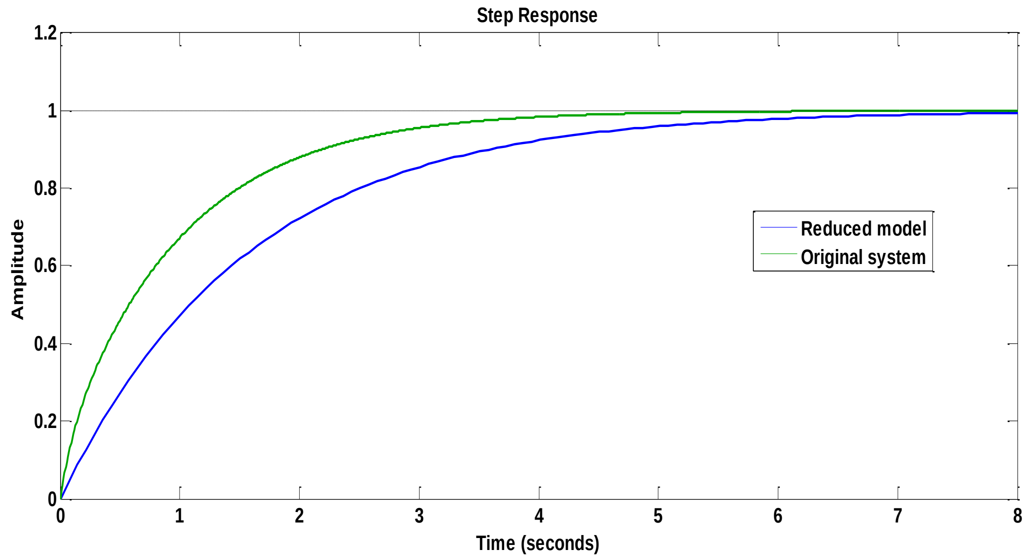


Fig 1: Lower Boundary Response

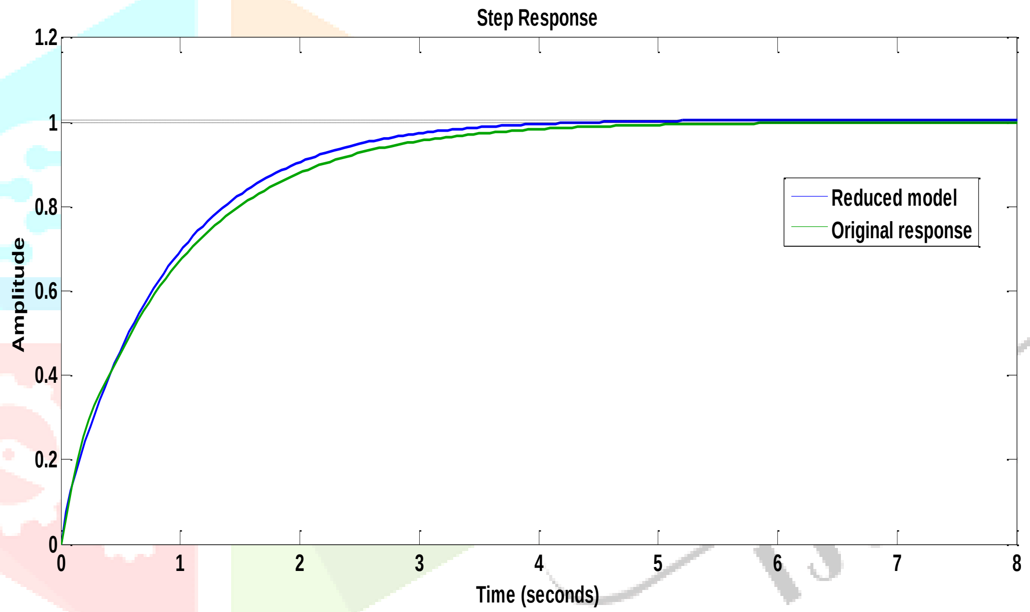
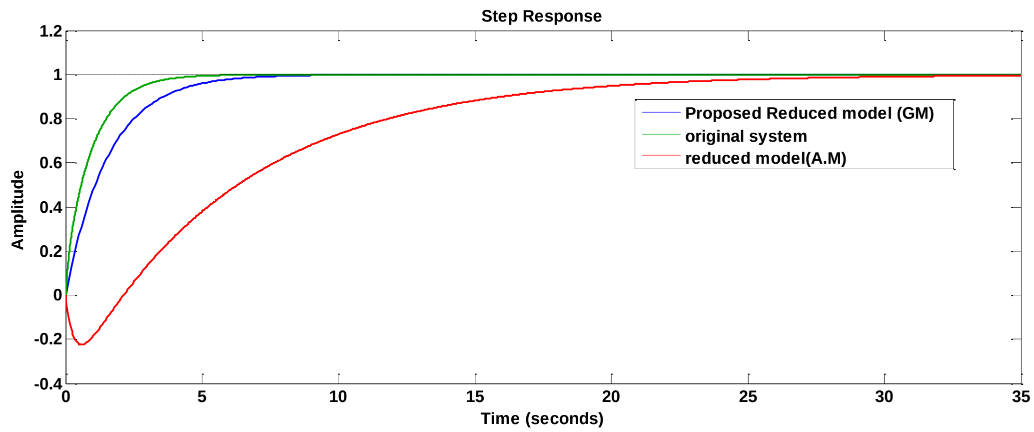
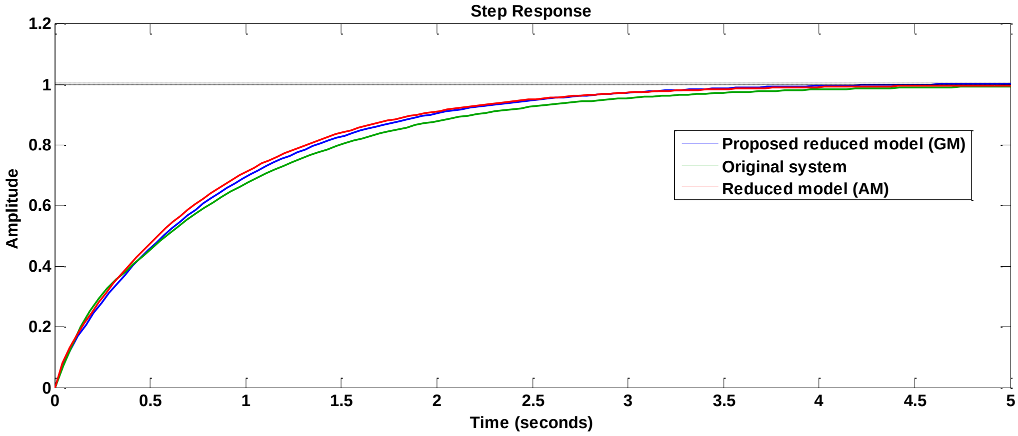


Fig 2: Upper boundary Response

IV. Comparisons





V. Conclusions:

A Novel method is suggested for the order reduction of high order Interval system based on improved Least Square moment matching method with a linear shift 'a', obtained by Geometric mean of real parts of the poles of original system. Stability is guaranteed in the reduced models for linear time invariant Interval systems. The proposed method is compared with Arithmetic mean and observed better responses.

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