A Review on the Algorithms used for Constructing Experimental Designs in Conjoint Analysis

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***Abstract*—For the purpose of measuring the consumer preferences for products or services, the technique used is the conjoint analysis. The conjoint analysis is one of the methods of finding the possible reaction of the consumer on a particular product or a particular service. One of the basic problems in performing conjoint analysis is how to generate experimental designs. The purpose of an experimental design is to give a rough overall idea as to the shape of the experimental response surface, while only requiring a relatively small number of runs. These designs are expected to be orthogonal and balanced in an ideal case. In practice, though, it is hard to construct optimal designs and thus constructing of near optimal and efficient designs is carried out. In this paper, review on the basic criteria of the design efficiency and some algorithms will be discussed which can be used for its construction.**

Keywords—Conjoint analysis, experimental design, efficiency, optimality criteria, algorithms

I. Introduction

Lately, the importance of preference analysis techniques and its usage has grown rapidly. The importance of preference analysis techniques could be understood by its wide spread use for the purpose of new product development and manufacturing and also in the diverse areas like marketing, financial services etc. In marketing research, for example, preference measuring techniques may provide an answer to questions as to which product will be successful or which attributes of a product drive the purchase decision and may thus serve as a valuable aid for managerial decision. One method that has become particularly popular in this context is Conjoint analysis.

Conjoint analysis is decompositional method [1] which assumes that product/services can be "break-down" into their attributive components and which implies study of join effects of variety products' attributes on their preference.

In Conjoint analysis, respondents have to evaluate a set of alternatives that are represented by factorial combinations of the levels of certain attributes. In traditional Conjoint approach, the alternatives have to be rank ordered or rated on some graded scale. It is assumed that these preference judgments are based on the overall utility values of the considered profile's levels. These unknown parameters are than estimated from the data. If the data consists of ranking techniques from linear programming, nonmetric versions of ANOVA can be used. Variants of conjoint analysis that use rating scales are referred as metric conjoint analysis. Here, the utility values are usually estimated by least squares procedures. Because of the metric response format and the linear relationship between preference judgments and attributes it is especially this last type of conjoint analysis to which techniques from optimal design theory can be readily applied.

The quality of statistical analysis heavily depends on the alternatives presented in the experimental design. An experimental design is a plan for running an experiment. Experiments are performed to study the effects of the factor levels on the dependent variable. The factors of an experimental design are variables that have two or more fixed values or levels of the factors. In Conjoint analysis, the factors are the attributes of the hypothetical products or services, and the response is preference or choice.
Using all combinations of attribute levels, i.e. a full factorial design, the number of evaluations required from every respondent soon becomes prohibitively large when the number of attributes and/or levels increases. To deal with this problem, the application of formal experimental designs was suggested. Many of the researchers have proposed the use of orthogonal arrays, incomplete block designs and fractional factorial designs of different resolutions to reduce the number of evaluations to be performed. In this reduction process it is especially important the goodness of the reduced designs. This goodness is named as efficiency.

There are several ways to quantify the relative efficiency of experimental designs. The choice of measure will determine which types of experimental designs are favored as well as the algorithms for choosing efficient designs.

The paper is organized as follows. In Section 2 we study some of the fundamental concepts in Conjoin experimental design including standard factorial designs, as well as fractional factorial designs, orthogonal arrays and nonorthogonal designs. Design terminology introduces and design efficiency explains. Section 3 reviews the basic optimality criteria as measure of the design efficiency. There are many algorithms for constructing efficient experimental designs. Some standard algorithms are studied in Section 4. In Section 5 we give conclusions and further research directions.

II. STUDYING EXPERIMENTAL DESIGN IN CONJOINT ANALYSIS

The design of experiments is a fundamental part of Conjoint analysis. Experimental designs are used to construct the hypothetical products or services. A simple experimental design is the full-factorial design, which consists of all possible combinations of the levels of the factors. These combinations in Conjoint analysis are referred as profiles or concepts. For example, with five factors, two at two levels and three at three levels (denoted as $2^2 3^3$), there are 108 possible combinations. In a full factorial design, all main effects, two-way interactions, and higher-order interactions are estimable and uncorrelated. The problem with a full-factorial design is that, for more practical situations, it is too cost-prohibitive and tedious to have subjects rate all possible combinations. For this reason, researchers often use fractional-factorial designs, which have fewer runs than full-factorial designs. The basic difficulty is how to construct such fractional-factorial design which can provide quality data. In order to obtain valuable and reliable data, two basic principles must be taken into account: orthogonality and balance.

A design is *orthogonal* if all effects can be estimated independently of all of the other effects, and it is *balanced* when each level occurs equally often within each factor, which means the intercept is orthogonal to each effect. In ideal case experimental design is orthogonal and balanced, hence optimal [8]. This is case for full-factorial designs.

A special type of fractional-factorial design is the *orthogonal array*, in which all estimable effects are uncorrelated. Orthogonal arrays are categorized by their resolution [8]. The resolution identifies which effects, possibly including interactions, are estimable. Higher resolutions require larger designs. Orthogonal arrays come in specific numbers of runs (e.g., 16, 18, 20, 24, 27, 28) for specific numbers of factors with specific numbers of levels. Resolution III orthogonal arrays are frequently used in marketing research. The term .orthogonal array,. as it is sometimes used in practice, is imprecise. It correctly refers to designs that are both orthogonal and balanced, and hence optimal. It is also imprecisely used to refer to designs that are orthogonal but not balanced, and hence potentially non-optimal. Imbalance is a generalized form of non-orthogonality, which increases the variances of the parameter estimates.

Orthogonal designs are available for only a relatively small number of very specific problems. They may not be available from follow reasons [7]:

- when there are many attributes in the survey,
- when the number of attribute levels is different for most of factors,
- when some combinations of factor levels are infeasible,
- when a nonstandard number of runs (factor level combinations or hypothetical products) is desired or when the number of runs must be limited,
- when some factor levels combinations are unrealistic, such as of the best product at the lowest price, or
- when a nonstandard model is being used, such as a model with interactions.

When an orthogonal design is not available, non-orthogonal designs must be used. The measure of experimental design's quality refers as "efficiency". In efficient experimental designs variance and covariance of parameters which estimates are minimal. Some orthogonal designs are not always more efficient than other orthogonal or non-orthogonal designs.

There are a number of techniques for constructing such efficient designs. Two basic are manual, which is typically used in surveys with small number of attributes and levels, and computerized search which is based on approximate algorithms.

Before a design is used, it must be coded [4]. One standard coding is the binary or dummy variable or (1, 0) coding. Another standard coding is effects or deviations from means or (1, 0,-1) coding. However, for evaluating design efficiency, an orthogonal coding is most appropriate. This is because standard non-orthogonal coding such as effects or binary is generally correlated, even for orthogonal designs.

III. OPTIMALITY CRITERIA

Efficiencies are measures of design goodness. An optimality criterion is a single number that summarizes how good a design is, and it is maximized or minimized by an optimal design. In order to generate an efficient design, specifically methodology was developed. Efficient designs can be efficient for one criterion and less efficient for another one. There are some standard criteria for measuring efficiency of experimental design in Conjoint analysis [8]. Two general types are: *information-based* criteria and *distance-based* criteria.

Consider the linear model where consumers provide utility scores, y_j, for each profile:

$$y_j = \alpha + \beta_1 x_{1j} + \beta_2 x_{2j} + \dots + \beta_m x_{mj} + \varepsilon_j \qquad (1)$$

for $j = \{1,\dots,n\}$, where x_{ij} are independent variables. In matrix notation (1) can be written as $y = \alpha + X\beta + e$. Let **X** is the orthogonally coded design matrix of independent variables. The information-based criteria such as D- and A-optimality are both related to the information matrix **X'X** for the design. This matrix is important because it is proportional to the inverse of the variance-covariance matrix for the least-squares estimates of the linear parameters of the model. Roughly, a good design should "minimize" the variance $(\mathbf{X'X})^{-1}$, which is the same as "maximizing" the information **X'X**. D- and A-efficiency are different ways of saying how large (**X'X**) or $(\mathbf{X'X})^{-1}$ are.

For the distance-based criteria, the candidates are viewed as comprising a point cloud in *p*-dimensional Euclidean space, where *p* is the number of parameters in the model. The goal is to choose a subset of this cloud that "covers" the whole cloud as uniformly as possible or that is as broadly "spread" as possible.

D-optimality is based on the determinant of the information matrix for the design, which is the same as the reciprocal of the determinant of the variance-covariance matrix for the least-squares estimates of the linear parameters of the model.

$$(X'X) = 1/|(X'X)^{-1}| \qquad (2)$$

The determinant is thus a general measure of the size of $(X'X)^{-1}$. D-optimality is the most common criterion for computer-generated optimal designs.

The D-optimality criterion has the following characteristics:

- D-optimality is the most computationally efficient criterion to optimize for the low-rank update algorithms, since each update depends only on the variance of prediction for the current design.
- $(X'X)$ is inversely proportional to the size of a confidence ellipsoid for the least squares estimates of the linear parameters of the model.
- $(X'X)^{1/p}$ is equal to the geometric mean of the eigenvalues of $X'X$ where *p* is a number of parameters in the model (number of columns in coded matrix **X**)
- The D-optimal design is invariant to non-singular coding of the design matrix. A-optimality is based on the sum of the variances of the estimated parameters for the model, which is the same as the sum of the diagonal elements, or trace, of $(X'X)^{-1}$.

For both criteria, if a balanced and orthogonal design exists, then it has optimum efficiency; conversely, the more efficient a design is, the more it tends toward balance and orthogonality. Assuming an orthogonally coded X:

- A design is balanced and orthogonal when $(X'X)^{-1}$ is diagonal.
- A design is orthogonal when the sub-matrix of $(X'X)^{-1}$, excluding the row and column for the intercept, is diagonal; there may be off-diagonal non-zeros for the intercept.
- A design is balanced when all off-diagonal elements in the intercept row and column are zero.

- As efficiency increases, the absolute values of the diagonal elements get smaller.

For appropriate coded matrix X , measures of efficiency can be scaled to be in interval 0 to 100. For Helmert's coded data (matrix) it is more appropriate to use A optimality criterion:

$$A - \text{eff} = 100 * \frac{1}{N_D . tr(X'X)^{-1} / p} \quad (3)$$

When data are coded by Chakravarty's procedure, it is more appropriate to use Doptimality criterion:

$$D - \text{eff} = 100 * \frac{1}{N_D \mid X'X)^{-1} \mid^{1/p}} \quad (4)$$

In the equations (3) and (4), p is number of parameters in model. The total number of parameters to be estimated is given by the formula: total *number of levels - number of attributes* + 1. N_D is number of runs (profiles) in fractional factorial design specified by the user. It is suggested, when possible, including between two to three times the number of runs as parameters estimated. However, design efficiency is not the only reason for including two to three times as many runs as parameters to be estimated. All real-world respondents answer conjoint questions with some degree of error, so those observations beyond the minimum required to permit utility estimation are needed to refine and stabilize utility estimates.

These optimality criteria measure the goodness of the design relative to hypothetical orthogonal designs that may be far from possible, so they are not useful as absolute measures of design efficiency. Instead, they should be used relatively, to compare one design to another for the same situation. Efficiencies that are not near 100 may be perfectly satisfactory.

IV. Standard Algorithms

As mentioned above, finding exact optimal designs is hard. Finding exact optimal designs in general requires solving a large nonlinear mixed integer programming problem, as the number of feasible designs explodes rapidly as the number of factors and levels increases. But we live in the real world, and we don't.t need the absolute best design, only one that's good enough. This is where approximation algorithms come in.

One of most simple algorithms for generating information-efficient designs is Dykstra's sequential search method [7]. The method starts with an empty design and adds candidate points so that the chosen efficiency criterion is maximized at each step. This algorithm is fast, but it is not very reliable in finding a globally optimal design. Also, it always finds the same design.

A typical approximation algorithm seeks to locate a good solution by the following sequential process [11]:
1. Choose initial feasible solution (random/greedy)
2. Modify solution slightly (random/greedy)
3. Repeat 2. Until finished, then report best solution seen

Random methods modify the current solution in some random way, and this change is accepted or rejected via some decision routine. Event worse solutions may be accepted under certain decision routines. Simulated annealing is an example of a random approximation algorithm. Greedy methods modify the current solution in a way that improves the score; as they.re seeking to improve the score for each iteration of the process they're frequently referred to as hill climbing algorithms.

One large class of pure greedy algorithms for generating efficient designs are the exchange algorithms. Exchange algorithms hill climb by adding new design points and removing existing design points to improve the objective. There are both Rank-1 and Rank-2 exchange algorithms, and these classifications are based on how the algorithm changes the points in the current candidate design matrix [11]:
Rank-1: Choose points to add and delete sequentially (Wynn, DETMAX)
Rank-2: Choose points to add and delete simultaneously (Fedorov, modified Fedorov, *k*exchange, *kl*-exchange)

The Mitchell and Miller (1970) simple exchange algorithm is a slower than Dykstra's but more reliable method. It improves the initial design by adding a candidate point and then deleting one of the design points, stopping when the chosen criterion ceases to improve. The DETMAX algorithm of Mitchell (1974) generalizes the simple exchange method. Instead of following each addition of a point by a deletion, the algorithm makes excursions in which the size of the design may vary. These algorithms add and delete points one at a time.

The next two algorithms add and delete points simultaneously, and for this reason, are usually more reliable for finding the truly optimal design; but because each step involves a search over all possible pairs of candidate and design points, they generally run much more slowly. The Fedorov (1972) algorithm simultaneously adds one candidate point and deletes one design point. Cook and Nachtsheim (1980) define a modified Fedorov algorithm that finds the best candidate point to switch with each design point. The resulting procedure is generally as efficient as the simple Fedorov algorithm in finding the optimal design, but it is up to twice as fast.

The *k*-exchange algorithm modifies the current candidate design via the process:

1. Examine k least critical points only.
2. Least critical: x with smallest $v(x)$, where $v(x)= f'(x) .D.f(x)$), x is a point in p dimensional design space, where the total number of factors is p, $f(x)$ is the corresponding row of our design matrix **X**, and $f'(x)$ is corresponding column.
3. Among these k, find the best exchange to make.

Some researchers have proposed nonstandard algorithms and criteria for constructing efficient experimental design [7]. Steckel, DeSarbo, and Mahajan (SDM) (1991) proposed using computer-generated experimental designs for conjoint analysis to exclude unacceptable combinations from the design. They considered a nonstandard measure of design goodness based on the determinant of the (m-factor × m-factor) correlation matrix (|**R**|) instead of the customary determinant of the (p-parameter × p-parameter) variance matrix $(\mathbf{X'X})^{-1}$. The SDM approach represents each factor by a single column rather than as a set of coded indicator variables. Designs generated using nonstandard criteria will not generally be efficient in terms of standard criteria like A-efficiency and D-efficiency, so the parameter estimates will have larger variances.

V. Concusion

Conjoint analysis has been widely used method for measuring customer preferences since the 1970s. This method is based on idea that customers' decisions depend on all tangible and intangible product features. One of the fundamental steps in performing Conjoint analysis is construction of experimental designs. These designs are expected to be orthogonal and balanced in an ideal case. In practice, though, it is hard to construct optimal designs and thus constructing of near optimal and efficient designs is carried out. Efficient designs are typically nonorthogonal; however they are efficient in the sense that the variances and covariances of the parameter estimates are minimized. There are several ways to quantify the relative efficiency of experimental designs. The choice of measure will determine which types of experimental designs are favoured as well as the algorithms for choosing efficient designs. In this paper we have presented some standard optimality criteria for measuring design efficiency, as well as some widely used algorithms for constructing such efficient designs. These algorithms are typically approximate and can be random or greedy, sequential or simultaneous.

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