# THE SPLIT LINE DOMINATION OF A JUMP GRAPH

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**ABSTRACT**

A dominating set D of a jump graph J(G)=(V, E) is a split line dominating set if the induced sub graph < E-D > is disconnected.

A split line dominating number $\surd'_s(J(G))$ of jump graph J(G) is the minimum cardinality of a split line dominating set. In this paper we study split line dominating sets and investigate the relation of $\surd'_s(J(G))$ with other known parameters of J(G).

## 1.INTRODUCTION

The jump graph considered here an finite, undirected without loop or multiple edge and have at least one component which is not complete or at least two components neither of which are isolated vertices, Unless otherwise stated, all graphs are assumed to have p vertices and q edges.

A set D of vertices in a jump graph J(G)= (V,E) is a dominating set of J(G) if every vertex in E-D is adjacent to some edge in D. The line domination number $\surd'(J(G))$ of J(G) is minimum cardinality of a line dominating set. A survey on $\surd'(J(G))$ can be found in[9]

And some recent results in[2]-[5]

The connectivity k(J(G)) is the minimum number of vertices whose removal results in a trivial or disconnected jump graph

The purpose of this paper is to combine the above two concepts as follows,

The line dominating set D of a jump graph J(G)=(V,E) is a split line dominating set if the induced sub graph <E-D> is disconnected. The split line domination number $\surd'_s(J(G))$ of jump graph J(G) is the minimum cardinality os a split dominating set.

A $\surd'$-set is a minimum line dominating set. A $\surd'_s$-set can be defined similarly, we note that $\surd'_s$-set exists if the graph is not complete and either it contains a non-split line dominating set, Further we also note that in a disconnected jump graph containing a $\surd'_s$-set is a $\surd'_s$-set. Thus, if for the rest of this paper we will assume that G is a non-complete connected jump graph. Any term not defined in this paper may be found in Harary [6]

## 2. RESULTS

We first obtain an upper bound for $\surd'_s(J(G))$,

**Theorem 1.** For any jump graph J(G)

$\surd'_s(J(G)) \leq \alpha_1(J(G))$ ................(1)

Where $\alpha_{1+}(J(G))$ is the edge covering number of jump graph J(G).

**Proof;** Let S be a maximum independent set of edges in J(G). Then S has at least two edges and every edge in S is adjacent to some edge in (E-S) This implies that (E-S) is a split line dominating set of J(G) Thus (1) holds.

We state without proof a straight forward result the characterize line dominating set of J(G) that are split line dominating sets.

**Theorem 2;** A line dominating set D of J(G) is a split line dominating set if and only if there exists two edges $e_1$ , $e_2$, ∈ E-D, such that every $e_1$-$e_2$ path contains a edge of D.

**Theorem 3;** For any jump graph J(G)

(i) $\sqrt{}'(J(G)) \leq \sqrt{}'_s(J(G))$….(2)

(ii) $K(J(G)) \leq \sqrt{}'_s(J(G))$….(3)

**Proof**; (2) and (3) follow from the definition of √'(J(G) and $\sqrt{}'_s(J(G))$.

Next we characterize split line dominating sets of J(G) which are minimal

**Theorem 4 ;** A split line dominating set D of J(G) is minimal if and only if for each edge e ∈ D one of the following is satisfied.

(i) There exists a vertex e∈ E-D such that N(e) ∩ D = {e}

(ii) e is isolated in <D>

(iii) < (E-D) ∪ {e} > is connected.

**Proof;** Suppose D is minimal and there exists a edge e ∈ D such that e does not satisfy any of the above conditions. Then by condition (i) and (ii), D'= D – {e} is a line dominating set of J(G) Also by (iii) <E-D> is disconnected. This implies that D' is a split line dominating set of J(G). a contradiction.\

**The converse is obvious.**

Now we obtain a another upper bound on $\sqrt{}'_s(J(G))$

**Theorem 5;** For any jump graph J(G)

$$\sqrt{}'_s(J(G)) \leq q.\ \frac{\Delta(J(G))}{(\Delta(J(G))+1)} \text{.....(4)}$$

Where Δ (J(G)) is the maximum degree of jump graph J(G)

**Proof;** Let D be a $\sqrt{}'_s$-set in J(G). Since D is minimal by theorem 4. It follows that for each vertex e ∈ D there exists a vertex

e ∈ E-D such that $e_1$ is adjacent to e. This implies that E-D is a dominating set of J(G). Then $\sqrt{}'(J(G)) \leq |E-D| \leq q- \sqrt{}'_s(J(G))$.

Hence (4) follows from the fact that

$$\sqrt{}'(J(G)) \geq \left|\frac{q}{\Delta(J(G))+1}\right|$$

In the next result we obtain a sufficient condition on J(G) such that $\sqrt{}'(J(G)) = \sqrt{}'_s(J(G))$

**Theorem 6;** For any jump graph J(G) with an end edge $\sqrt{}'(J(G)) = \sqrt{}'_s(J(G))$ ….(5)

Furthermore, there exists a $\sqrt{}'_s$-set of J(G) containing all edges adjacent to end -edges.

**Proof**: Let e be an end –edge of J(G).then there exists a cut-edge $e_1$ adjacent to e. Let D be a √'-set of J(G). suppose $e_1$ ∈ D. Then d is a $\sqrt{}'_s$-set of J(G). Suppose $e_1$ ∈ E-D Then e∈ D and hence D- {e} ∪ {$e_1$} is a $\sqrt{}'_s$-set of J(G). repeating this process for all such cut-edges adjacent to end-edges, we obtain a $\sqrt{}'_s$-set of J(G) containing all cut edges adjacent to end-edges.

We next consider the case where the diameter of J(G) is 2.

**Theorem 7;** If diam (J(G))=2 then

$$\sqrt{}'_s(J(G)) \leq \delta'(J(G))\text{........(6)}$$

Where δ(J(G)) is minimum degree of J(G).

**Proof;** Let e be a edge of minimum degree in J(G) since diam(J(G))=2,there exists a edge e is not adjacent to $e_1$. Hence it follows that N(e) is a split line dominating set of J(G) Thus (6) holds

The proof of the following proposition follow easily.

**Proposition 8;**

(i) For any cycle $J(C_p)$ with p ≥ 4 vertices $\sqrt{}'_s(J(C_p)) = \lceil q/3 \rceil$ …..(7) where $\lceil x \rceil$ is the least positive integer not less than x.

(ii) For any wheel $J(W_p)$ with p≥ 5 vertices $\sqrt{}'_s(J(w_p)) = 3$ …..(8)

(iii) For any complete bipartite jump graph $K_{m,n}$ with $2 \leq n \leq m$

$\surd'_s(J(K_{m,n}))=n$…..(9)

We obtain a sufficient condition for a cut edge to be in every $\surd'_s$-set of J(G).

**Theorem9 .** If J(G) have one cut edge e and at least two blocks $B_1$ and $B_2$ then e is in every $\surd'_s$-set of J(G).

**Proof**; Let D be a $\surd'_s$-set of J(G).Suppose $e \in E\text{-}D$.Then each $B_1$ and $B_2$ contributes at least one edge to D say $e_1$ and $e_2$ respectively. This implies that

$D' = D - \{e_1, e_2\} \cup \{e\}$ is a split line dominating set of J(G) a contradiction. Hence e is in every $\surd'_s$-set of J(G).

**Theorem 10**; Let v be a cut-edge of J(G).If there is a block H of J(G) such that e is the only cut-edge of H, and e is adjacent to all edges of H, then there is a $\surd'_s$-set of J(G) containing e,

**Proof**; If there exists at least two blocks in J(G) satisfying the given condition, then by Theorem 9. e is in every $\surd'_s$-set of J(G) and hence the result. Suppose there exists only one block H in J(G).satisfying the given condition. Let D be a $\surd'_s$-set of J(G). suppose $e \in E\text{-}D$ Then for some edge $e \in H$. $[e] \subset D$. This proves that $D' = D - \{e_1\} \cup \{e_2\}$ is a $\surd'_s$-set of J(G).

To prove our next result we make use of the following definitions from [7].

A line dominating set D of a jump graph is connected line domination set ,if the induced sub graph < D > is connected. The connected line domination number $\surd'_c(J(G))$ of J(G) is the minimum cardinality of a connected line dominating set.

**Theorem 11 ; If** $\surd'_s(J(G)) \leq \surd'_c(J(G))$, then for any $\surd'_s$-set D of J(G), E- D is a split line dominating set of J(G).

**Proof**; Since D is minimal, by Theorem 4 E-D is a line dominating set of J(G) and further it is split line dominating set since < D > is disconnected.

Finally we obtain a

Nordhus-Gaddum type result [8]

**Theorem 12** ; Let J(G) be a jump graph such that both J(G) and its complement $J(\bar{G})$are connected then

$\surd'_s(J(G)) + \surd'_s(J(\bar{G})) \leq q(q-3)$ …..(10)

**Proof;** By (1) $\surd'_s(J(G)) \leq \alpha_1(J(G))$ Since J(G) and $J(\bar{G})$ are connected

$\Delta'(J(G))$ , $\Delta'(J(\bar{G})) \leq q$ This implies that $\beta_1(J(G))$ $\beta_1(J(\bar{G}) \geq 2$. Hence

$\surd's(J(G) \leq q - 2 \leq 2(q-1) - q \leq 2q- q$

Similarly $\surd_s(J(\bar{G})) \leq 2\bar{q} -$ q Then

$\surd_s(J(G)) + \surd_s(J(\bar{G})) \leq 2(q + \bar{q}) - 2q \leq$

$q(q-1) - 2q \leq q(q-3)$

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