4-VARIABLE SIMPLE DISCRETE HEAT EQUATION

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ABSTRACT: We investigate the generalized partial difference equation operator and propose a model of it in discrete heat equation in this paper.The diffusion of heat is studied by the application of newton's law of cooling in dimension up to five and several solutions are postulated for the same. Through numerical simulations solutions are validated and applications are derived.

1.GENERALIZED DISCRETE HEAT EQUATION

Consider the temperature distribution of a very long rod

Assume that the rod is so long, that it can be laid on the set R of real numbers. Let $v(k_1, k_2)$ be the temperature at the real time (k_1) and real position (k_2) of a rod at time (k_1)

If the temperature $v(k_1, k_2 - l_2)$, $l_2 > 0$ is higher than $v(k_1, k_2)$ heat will flow from the point $k_2 - l_2$ to k_2

The amount of increase is $v(k_1 + l_1, k_2) - v(k_1, k_2)$ and

It is reasonable to postulate that the increase is proportional to the difference $v(k_1, k_2 - l_2) - v(k_1, k_2)$say

$$\alpha(v(k_1, k_2 - l_2) - v(k_1, k_2))$$

$v(k_1 + l_1, k_2) - v(k_1, k_2) = \alpha\big(v(k_1, k_2 - l_2) - v(k_1, k_2)\big), \alpha > 0$

(I,e)

$\underset{(l_1,0)}{\Delta} v(k_1, k_2) = \alpha \underset{(0,-l_2)}{\Delta} v(k_1, k_2)$

If $k_1 = k_2$ and $\alpha = \frac{l_1}{-l_2}$

Then $v(k_1, k_2) = (k_1 . k_2)$ is a solution of discrete heat equation

1.1: FORMATION OF 4-VARIABLE SIMPLE DHE

Consider the temperature distribution of a very long rod.Assume that the rod is so long, that it can be laid on the set R of real numbers. Let $v(k_1, k_2, k_3, k_4)$ be the temperature at the real time (k_1) and real position (k_2) of a rod at time (k_1)`If the temperature $v(k_1 - l_1, k_2 - l_2, k_3 - l_3, k_4)$, $l_2 > 0$ is higher than $v(k_1, k_2, k_3, k_4)$ heat will flow from the point $k_2 - l_2$ to k_2The amount of increase is $v(k_1 + l_1, k_2 + l_2, k_3 + l_3, k_4) - v(k_1, k_2, k_3, k_4)$ and It is reasonable to postulate that the increase is proportional to the difference $v(k_1 - l_1, k_2 - l_2, k_3 - l_3, k_4) - v(k_1, k_2, k_3, k_4)$say

$$\alpha(v(k_1 - l_1, k_2 - l_2, k_3 - l_3, k_4) - v(k_1, k_2, k_3, k_4))$$

$$v(k_1+l_1,k_2+l_2,k_3+l_3,k_4)-v(k_1,k_2,k_3,k_4)=\alpha\big(v(k_1-l_1,k_2-l_2,k_3-l_3,k_4)-v(k_1,k_2,k_3,k_4)\big),\ \alpha>0$$

(I,e)

$\underset{(l_1,l_2,l_3,0)}{\Delta} v(k_1,k_2,k_3,k_4)=\alpha \underset{(-l_1,-l_2,-l_3,0)}{\Delta} v(k_1,k_2,k_3,k_4)$

If $\alpha=\frac{2^{l_1+l_2+l_3-1}}{2^{-l_1-l_2-l_3-1}}$ discrete a heat equation Equation (4.1) is a model of simple heat equation

wherek_1,k_2,k_3,k_4 are variable and l_1,l_2,l_3,l_4 are parameters

TO FIND SECOND TYPE SOLUTION

$$v(k_1,k_2,k_3,k_4)=\frac{1}{1-\alpha}v(k_1+l_1,k_2+l_2,k_3+l_3,k_4)-\frac{\alpha}{1-\alpha}v(k_1-l_1,k_2-l_2,k_3-l_3,k_4)$$

Replace $k_1\ by\ k_1+l_1$ and $k_2\ by\ k_2+l_2$ and $k_3\ by\ k_3+l_3$

$$v(k_1+l_1,k_2+l_2,k_3+l_3,k_4)=\frac{1}{1-\alpha}v(k_1+2l_1,k_2+2l_2,k_3+2l_3,k_4)-\frac{\alpha}{1-\alpha}v(k_1,k_2,k_3,k_4)$$

Substitute Eqn (5.2), we have

$$v(k_1,k_2,k_3,k_4)=\frac{1}{(1-\alpha)^2}v(k_1+2l_1,k_2+2l_2,k_3+2l_3,k_4)-\frac{\alpha}{(1-\alpha)^2}v(k_1,k_2,k_3,k_4)-\frac{\alpha}{1-\alpha}v(k_1-l_1,k_2-l_2,k_3-l_3,k_4)$$

In general

$$v(k_1,k_2,k_3,k_4)=\frac{1}{(1-\alpha)^m}v(k_1+ml_1,k_2+ml_2,k_3+ml_3,k_4)-\sum_{r=1}^{m}\frac{\alpha}{(1-\alpha)^r}v(k_1+(r-2)l_1,k_2+(r-2)l_2,k_3+(r-2)l_3,k_4)$$

EXAMPLE 1.2.1

$$v(k_1,k_2,k_3,k_4)=\frac{1}{1-\alpha}v(k_1+l_1,k_2+l_2,k_3+l_3,k_4)-\frac{\alpha}{1-\alpha}v(k_1-l_1,k_2-l_2,k_3-l_3,k_4)$$

Taking $k_1=2,k_2=4,k_3=6,k_4=8$ and $l_1=3,l_2=5,l_3=7$

$$\alpha=\frac{2^{l_1+l_2+l_3}-1}{2^{-l_1-l_2-l_3}-1}=\frac{2^{3+5+7}-1}{2^{-3-5-7}-1}=-32768$$

If $v(k_1,k_2,k_3,k_4)=2^{k_1+k_2+k_3+k_4}$

$$2^{k_1+k_2+k_3+k_4}=\frac{1}{1-\alpha}(2^{k_1+l_1+k_2+l_2+k_3+l_3+k_4})-\frac{\alpha}{1-\alpha}2^{k_1-l_1+k_2-l_2+k_3-l_3+k_4}$$

$$2^{2+4+6+8}=\frac{1}{1+32768}2^{2+3+4+5+6+7+8}+\frac{32768}{1+32768}2^{2-3+4-5+6-7+8}$$

$$1048576 = 1048576$$

TO FIND THIRD TYPE SOLUTION

$v(k_1,k_2,k_3,k_4) = \frac{1}{1-\alpha} v(k_1+l_1,k_2+l_2,k_3+l_3,k_4) - \frac{\alpha}{1-\alpha} v(k_1-l_1,k_2-l_2,k_3-l_3,k_4)$(5.3)

Replace k_1 *by* $k_1 - l_1$ *and* k_2 *by* $k_2 - l_2$ *and* k_3 *by* $k_3 - l_3$ in (5.3)

$$v(k_1-l_1,k_2-l_2,k_3-l_3,k_4) = \frac{1}{1-\alpha} v(k_1,k_2,k_3,k_4) - \frac{\alpha}{1-\alpha} v(k_1-2l_1,k_2-2l_2,k_3-2l_3,k_4)$$

$$v(k_1,k_2,k_3,k_4) = \frac{1}{1-\alpha} v(k_1+l_1,k_2+l_2,k_3+l_3,k_4) - \frac{\alpha}{(1-\alpha)^2} v(k_1,k_2,k_3,k_4) + \frac{\alpha^2}{(1-\alpha)^2} v(k_1-2l_1,k_2-2l_2,k_3-2l_3,k_4)$$

In general

$$v(k_1,k_2,k_3,k_4) = \frac{1}{1-\alpha} v(k_1+l_1,k_2+l_2,k_3+l_3,k_4) - \frac{\alpha}{(1-\alpha)^m} v(k_1+(m-2)l_1,k_2+(m-2)l_2,k_3+(m-2)l_3,k_4) + \sum_{r=1}^{m} \frac{\alpha^2}{(1-\alpha)^r} v(k_1+(r-4)l_1,k_2+(r-4)l_2,k_3+(r-4)l_3,k_4)$$

EXAMPLE 1.2.4

$$v(k_1,k_2,k_3,k_4) = \frac{1}{1-\alpha} v(k_1+l_1,k_2+l_2,k_3+l_3,k_4) - \frac{\alpha}{(1-\alpha)^2} v(k_1,k_2,k_3,k_4) + \frac{\alpha^2}{(1-\alpha)^2} v(k_1-2l_1,k_2-2l_2,k_3-2l_3,k_4)$$

Taking $k_1 = 2, k_2 = 4, k_3 = 6, k_4 = 8$ and $l_1 = 3, l_2 = 5, l_3 = 7$

$$\alpha = \frac{2^{l_1+l_2+l_3} - 1}{2^{-l_1-l_2-l_3} - 1} = \frac{2^{3+5+7} - 1}{2^{-3-5-7} - 1} = -32768$$

If $v(k_1,k_2,k_3,k_4) = 2^{k_1+k_2+k_3+k_4}$

$$2^{2+4+6+8} = \frac{1}{1+32768} 2^{2+3+4+5+6+7+8} + \frac{32768}{(1+32768)^2} 2^{2+4+6+8} + \frac{(32768)^2}{(1+32768)^2} 2^{2-6+4-10+6-14+8}$$

$$1048576 = 1048576$$

CONCLUSION

In this project, we derived several types of solution of discrete heat equation for one, two and three dimensional system

Here we obtained several results and the theorems by introducing partial difference operator with and without variable coefficient

All the theorem are nearly derived and suitable examples are provide and valuate the finding verify the results

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