DETERIORATING ITEMS WITH STOCK-DEPENDENT

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ABSTRACT

In this paper we discuss a partial backlogging inventory model for deteriorating items with stock-dependent demand rate, along with the effects of inflation and time value of money. We also establish theorems to find the minimum total relevant cost and optimal order quantity. We have established an inventory model for deteriorating items with stock-dependant, time-dependent and permissible delay in payments. The optimal order quantity and minimum present value of total relevant cost are also derived. Numerical examples are used to illustrate the theorem and results.

1.INTRODUCTION

Deterioration is defined as decay, change, damage, spoilage or obsolescence that results in decreasing usefulness from its original purpose. Some kinds of inventory products (e.g., vegetables, fruit, milk, and others) are subject to deterioration. Ghare and Schrader (1963) [9] first established an economic order quantity model having a constant rate of deterioration and constant rate of demand over a finite planning horizon. Covert and Philip (1973) [7] extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution. Dave and Patel (1981) [8] discussed an inventory model for deteriorating items with time-proportional demand when shortages were not allowed. Mandal and Phaujdar (1989) [13] however, assumed a production inventory model for deteriorating items with uniform rate of production and stock dependent demand.

In some fashionable products, some customers would like to wait for backlogging during the shortage period. But the willingness is diminishing with the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Chang and Dye (1999) [3] developed an inventory model in which the demand rate is a time-continuous function and items deteriorate at a constant rate with partial backlogging rate which is the reciprocal of a linear function of the waiting time. Papachristos and Skouri (2000) [15] developed an EOQ inventory model with time-dependent partial backlogging.

Shortages in inventory were allowed and partially backlogged with waiting time dependent backlogging rate. Optimal ordering policy for deteriorating items with partial backlogging was formulated by Ouyang et al. (2006) [14] when delay in payment was permissible. An inventory lot-size model for deteriorating items with partial backlogging was formulated by Chern et al. (2008) [4]. Arya and Shakya (2010) [2] investigate the effect of life time on an inventory model for decaying items with and without shortages.

Today, inflation has become a permanent feature of the economy. Many researchers have shown the inflationary effect on inventory policy. Hou (2006) [10] established an inventory model with stock-dependent consumption rate simultaneously considered the inflation and time value of money when shortages are allowed over a finite planning horizon.

Both in deterministic and probabilistic inventory models of classical type, it is observed that payment is made to the supplier for goods just after getting the consignment. But actually nowadays a supplier grants some credit period to the retailer to increase the demand. An EOQ model for inventory control in the presence of trade credit is presented by Chung and Huang (2005) [5]. The optimal replenishment policy for EOQ models under permissible delay in payments is also discussed by Chung and Hwang (2003) [6]. In recent times to make the real inventory systems more practical and realistic, Aggarwal and Jaggi (1995) [1] extended the model with a constant deterioration rate. Hwang and Shinn (1997) [11] determined lot-sizing policy for exponential demand when delay in payment is permissible. After that Jamal et al. (1997) [12] developed further following the lines of Aggarwal and Jaggi's (1995) [1] model to take into consideration for shortage and make it more practical and acceptable in real situation.

2. MODELLING ASSUMPTIONS

The following assumptions are used in this paper.

(1) Only a single-product item is considered during the planning horizon H.

(2) Replenishment rate is infinite and lead time is zero.

(3) A constant fraction of the on-hand inventory deteriorates per unit of time and there is no repair or replacement of the deteriorated inventory.

(4) Shortage are allowed and backlogged partially. The backlogging rate is a decreasing function of the waiting time. Let the backlogging rate be $B(T'-t)=e^{-\delta(T-t)}$, where $\delta \geq 0$, and $T'-t$ is the waiting time up to the next replenishment.

(5) A Discounted Cash Flow (DCF) approach is used to consider the various costs at various times.

3. MATHEMATICAL MODEL AND SOLUTION

In this section we discuss mathematical model and solution optimization.

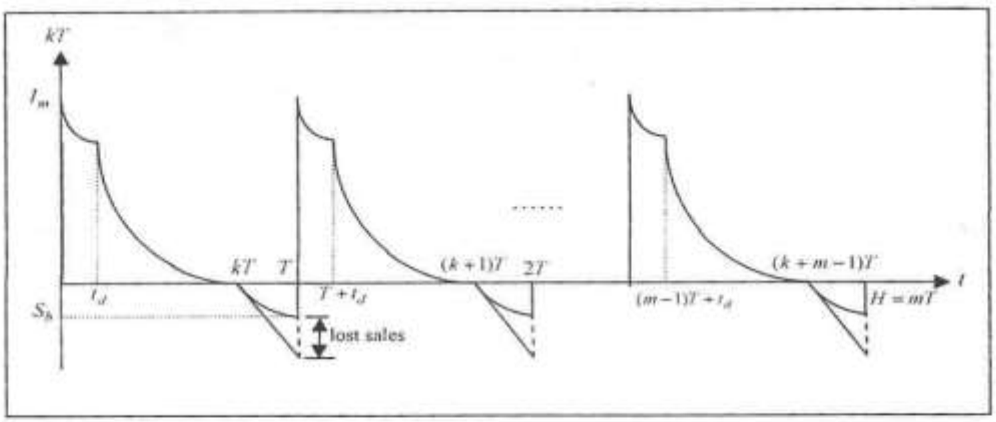

Figure The graphical representation of inventory model

The inventory model is shown in Fig. The planning horizon H is divided into m equal parts of length $T'=H/m$. The j the replenishment is made at time $jT'(j=0,1,2,\ldots,m)$. The maximum inventory level for each cycle is I_m. During the time interval $[jT', jT'+t_d](j=0,1,2,\ldots,m-1)$ the product has no deterioration, the inventory level is decreasing due to demand only. During the time interval $[jT'+t_d, jT'+kT'](j=0,1,2,\ldots,m-1)$, the inventory level gradually reduces to zero owing to deterioration and demand. And shortage happens during the time interval $[jT'+kT',(j+1)T'](j=0,1,2,\ldots,m-1)$. The quantity received at $jT'(j=1,2,3,\ldots,m-1)$ is used partly to meet the accumulated backorders in the previous cycle from time $(k+j-1)T'$ to jT', where $k(t_d/T' \leq k \leq 1)$ is the ratio of no-shortage period to scheduling period T' in each cycle. The last extra replenishment at time H is needed to replenish shortages generated in the last cycle. The objective of the inventory problem here is to determine the replenishment number m and the ratio k in order to minimize the total relevant cost.

In the first replenishment cycle, owing to stock-dependent consumption rate only, the inventory level at time t during the time interval $[0, t_d]$ is governed by the following differential equation:

$$\frac{dI_1(t)}{dt} = -[\alpha + \beta I_1(t)] \qquad 0 \le t \le t_d \tag{1}$$

with the boundary condition $I_1(0) = I_{m.}$ The solution of Eq. (1) can be represented by

$$I_1(t) = e^{-\beta t} I_m - \frac{\alpha}{\beta}(1 - e^{-\beta t}] \quad 0 \le t \le t_d \tag{2}$$

Owing to stock-dependent consumption rate and deterioration, the inventory level at time t during the time interval $[t_{d,} kT']$ is governed by the following differential equation:

$$\frac{dI_2(t)}{dt} = \theta I_2 - [\alpha + \beta I_2(t)] \qquad t_d \le t \le kT' \tag{3}$$

with the boundary condition $I_2(kT') = 0$. The solution of Eq. (3) can be represented by

$$I_2(t) = \frac{\alpha}{\theta + \beta}\left[e^{(\theta+\beta)(kT'-t)} - 1\right] \quad t_d \le t \le kT' \tag{4}$$

Because $I_1(t_d) = I_2(t_d)$, the maximum inventory level I_m is

$$I_m = \frac{\alpha}{\theta + \beta}\left[e^{(\theta+\beta)(kT'-t_d)} - 1\right]e^{\beta t_d} + \frac{\alpha}{\beta}(e^{\beta t_d} - 1) \tag{5}$$

Hence, $I_1(t)$ in Eq. (2.2) can be represented as

$$I_1(t) = \frac{\alpha}{\theta + \beta}[e^{(\theta+\beta)(kT'-t_d)} - 1]e^{-\beta(t-t_d)} + \frac{\alpha}{\beta}[e^{-\beta(t-t_d)} - 1] \tag{6}$$

Since the backlogging rate is a decreasing function of the waiting time, we let the backlogging rate be $B(T'-t) = e^{-\delta(T'-t)}$, the shortage level at time t during the time interval $[kT', T']$ is governed by the following differential equation:

$$\frac{dI_3(t)}{dt} = \alpha e^{-\delta(T'-t)} \qquad kT' \le t \le T' \tag{7}$$

with the boundary condition $I_3(kT') = 0$. The solution of Eq.(7) can be represented by

$$I_3(t) = \frac{\alpha}{\delta}\left[e^{-\delta(T'-t)} - e^{-\delta(I-k)T'}\right] \quad kT' \le t \le T' \tag{8}$$

And the amount of lost sale at time t during the time interval $[kT', T']$ is

$$L(t) = \alpha\int_{kT'}^{t}[1-e^{-\delta(T'-\tau)}]d\tau = \alpha\left\{t - kT' - \frac{1}{\delta}[e^{-\delta(T'-t)} - e^{-\delta(t-k)T'}]\right\} kT' \le t \le T' \quad (9)$$

Let S_b be the maximum shortage quantity per cycle.

$$S_b = I_3(T') = \frac{\alpha}{\delta}[1-e^{-\delta(T'-kT')}] \quad (10)$$

Replenishment is made at time $jT'(j = 0,1,2,...m)$, the maximum inventory level for each cycle is I_m. The last replenishment at time mT' is just to satisfy the backorders generated in the last cycle. There are $m+1$ replenishments in the entire time horizon H. The total relevant inventory cost involves following five factors.

(a) Ordering cost: The present value of the ordering cost in the entire time horizon H is

$$T'C_o = c_o\sum_{j=0}^{m} e^{-RjT'} = c_0\frac{e^{RH/m} - e^{-RH}}{e^{RH/m} - 1} \quad (11)$$

(b) Purchasing cost: The present value of the purchasing cost in the entire time horizon H is

$$T'C_p = \sum_{j=0}^{m-1} c_p I_m e^{-RjT'} + \sum_{j=1}^{m} c_p S_b e^{-RjT'}$$

$$= c_p\alpha\left\{\frac{1}{\theta+\beta}[e^{(\theta+\beta)(kT'-t_d)} - 1]e^{\beta t_d} + \frac{1}{\beta}(e^{\beta t_d} - 1)\right\} \quad (12)$$

$$\times\frac{1-e^{-RH}}{1-e^{-RH/m}} + \frac{c_p\alpha}{\delta}[1-e^{-\delta(1-k)H/m}]\frac{1-e^{-RH}}{e^{RH/m}-1}$$

(c) Holding cost: The present value of the holding cost in the entire time horizon H is

$$T'C_h = \sum_{j=0}^{m-1} c_h\left[\int_0^{t_d} e^{-Rt}I_1(t)dt + \int_{t_d}^{kT'} e^{-Rt}I_2(t)dt\right]e^{-RjT'} \quad (13)$$

(d) Shortage cost: The present value of the shortage cost in the entire time horizon H is

$$T'C_s = \sum_{j=0}^{m-1} c_s\left[\int_{KT'}^{T'} e^{-Rt}I_3(t)(dt)\right]e^{-RjT'}$$

$$= \frac{c_s\alpha}{R}\left[\frac{e^{(R-\delta)(1-k)H/m} - 1}{R-\delta} + \frac{e^{-\delta(1-k)H/m} - 1}{\delta}\right]\frac{1-e^{-RH}}{e^{RH/m} - 1} \quad (14)$$

(e) Lost sale cost: The present value of the lost sale cost in the entire time horizon H is

$$T'C_L = \sum_{j=0}^{m-1}\left[c_L \int_{KT'}^{T'} e^{-Rt}\alpha[1-e^{-\delta(T'-t)}]dt\right]e^{-RjT'}$$

$$= c_L\alpha[\frac{e^{R(1-k)H/m}-1}{R} + \frac{1-e^{(R-\delta)(1-k)H/m}}{R-\delta}]\frac{1-e^{-RH}}{e^{RH/m}-1} \qquad (15)$$

Hence, the present value of the total relevant inventory cost in the entire time horizon H is

$$T'C(m,k) = T'C_o + T'C_p + T'C_h + T'C_s + T'C_L \qquad (16)$$

Let

$$U = \frac{e^{RT'}-e^{-RH}}{e^{RT'}-1} \quad V = \frac{1-e^{-RH}}{1-e^{-RT'}} \quad W = \frac{1-e^{-RH}}{e^{RT'}-1} \quad T' = \frac{H}{m}$$

We substitute Eqs. (11)-(15) into Eq. (16) and obtain

$$T'C(m,k) = c_o U + c_p\alpha\left\{\frac{1}{\theta+\beta}[e^{(\theta+\beta)(kT'-t_d)}-1]e^{\beta t_d} + \frac{1}{\beta}(e^{\beta t_d}-1)\right\}V$$

$$+c_h\alpha\left\{\frac{[e^{(\theta+\beta)(kT'-t_d)}-1](e^{\beta td}-e^{-Rt_d})}{(\theta+\beta)(R+\beta)} + \frac{1}{\beta}(\frac{e^{\beta t_d}-e^{-Rt_d}}{R+\beta} + \frac{e^{-Rt_d}-1}{R})\right.$$

$$\left.+\frac{1}{\theta+\beta}[\frac{e^{-(\theta+\beta+R)t_d+(\theta+\beta)kT'}-e^{-RkT'}}{(\theta+\beta+R)} + \frac{e^{-RkT'}-e^{-Rt_d}}{R}]\right\}V \qquad (17)$$

$$+\alpha\left[(\frac{c_s}{R}-c_L)\frac{e^{(R-\delta)(1-k)T'}-1}{R-\delta} + (c_p - \frac{c_s}{R})\frac{1-e^{-\delta(1-k)T'}}{\delta} + c_L\frac{e^{R(1-k)T'}-1}{R}\right]W$$

There are two variables in the present value of the total inventory cost $T'C(m,k)$. One is the replenishment number m which is a discrete variable, the other is the ratio k, where $kT' \le t \le T'$, which is a continuous variable. For a fixed value of m, the condition for $TC(m,k)$ to be minimized is $dT'C(m,k)/dk = 0$. Consequently, we obtain

$$c_p e^{(\theta+\beta)kT'-\theta t_d} + c_h\left[\frac{e^{(\theta+\beta)(kT'-t_d)}(e^{\beta t_d}-e^{-Rt_d})}{R+\beta} + \frac{e^{(\theta+\beta)(kT'-t_d)-Rt_d}-e^{-RkT'})}{\theta+\beta+R}\right]$$

$$-\left[(c_p - \frac{c_s}{R})e^{-\delta(1-k)T'} + (\frac{c_s}{R}-c_{L)}e^{(R-\delta)(1-k)T'} + c_L e^{R(1-k)T'}\right]e^{-RT'} = 0 \qquad (18)$$

Theorem 1

(a) If

$$c_p e^{\beta t_d} + c_h \frac{e^{\beta t_d} - e^{-Rt_d}}{R+\beta} <$$

$$\left[(c_p - \frac{c_s}{R}) e^{-\delta(T'-t_d)} + (\frac{c_s}{R} - c_L) e^{(R-\delta)(T'-t_d)} + c_L e^{R(T'-t_d)} \right] e^{-RT'} \quad (19)$$

there exists a unique solution k^*, where $t_d < k^* T' < T'$, such that $T'C(m,k^*)$ is the minimum value of k when m is given.

(b) If

$$c_p e^{\beta t_d} + c_h \frac{e^{\beta t_d} - e^{-Rt_d}}{R+\beta} >$$

$$\left[(c_p - \frac{c_s}{R}) e^{-\delta(T'-t_d)} + (\frac{c_s}{R} - c_L) e^{(R-\delta)(T'-t_d)} + c_L e^{R(T'-t_d)} \right] e^{-RT'} \quad (20)$$

$T'C(m, mt_d / H)$ is the minimum value when m is given.

Proof:

Part (a)

$$dT'C(m,k)/dk = \alpha T' \{ c_\rho e^{(\theta+\beta)kT' - \theta t_d}$$
$$+ c_h \frac{e^{(\theta+\beta)(kT'-t_d)}(e^{\beta t_d} - e^{-Rt_d})}{R+\beta} + c_h \frac{[e^{(\theta+\beta+R)(kT'-t_d)} - 1] e^{-RkT'}}{\theta+\beta+R}$$
$$-[(c_p - \frac{c_s}{R}) e^{-\delta(1-k)T'} + (\frac{c_s}{R} - c_L) e^{(R-\delta)(1-k)T'} + c_L e^{R(1-k)T'}] e^{-RT'} \} V$$

$$d^2 T'C(m,k)/dk^2 = \alpha T'^2 \{ c_\rho [(\theta+\beta) e^{(\theta+\beta)kT' - \theta t_d} - \delta e^{-\delta(1-k)T' - RT'}]$$
$$+ c_h (\theta+\beta) \left[\frac{e^{(\theta+\beta)(kT'-t_d)}(e^{\beta t_d} - e^{-Rt_d})}{R+\beta} + \frac{e^{(\theta+\beta)(kT'-t_d) - Rt_d} + \mathrm{Re}^{-RkT'}}{\theta+\beta+R} \right]$$
$$+ \left[[\frac{c_s}{R} - c_L](R-\delta) e^{(R-\delta)(1-k)T'} + \frac{c_s}{R} \delta e^{-\delta(1-k)} T' + c_L \mathrm{Re}^{R(1-k)T'} \right] e^{-RT'} \} V > 0$$

Clearly, $dT'C$ (m,k)/dk is a strictly increasingly function of k. Besides,

$$\frac{dT'C(m,1)}{dk} = \alpha T' \{ c_\rho [e^{(\theta+\beta)T' - \theta t_d} - e^{-RT'}] + c_h [\frac{e^{(\theta+\beta)(T'-t_d)}(e^{\beta t_d} - e^{-Rt_d})}{R+\beta}$$

$$+\frac{e^{(\theta+\beta+R)(T'-t_d)}-1}{\theta+\beta+R}e^{-RT'}]\}V>0$$

$$\frac{dT'C(m,t_d/T')}{dk}=\alpha T'\{c_p e^{\beta t_d}+c_h\frac{e^{\beta t_d}-e^{-Rt_d}}{R+\beta}$$

$$-\left[(c_p-\frac{c_s}{R})e^{-\delta(T'-t_d)}+(\frac{c_s}{R}-c_L)e^{(R-\delta)(T'-t_d)}+c_L e^{R(T'-t_d)}\right]e^{-RT'}\}V$$

if

$$c_p e^{\beta t_d}+c_h\frac{e^{\beta t_d}-e^{-Rt_d}}{R+\beta}<\left[(c_p-\frac{c_s}{R})e^{-\delta(T'-t_d)}+(\frac{c_s}{R}-c_L)e^{(R-\delta)(T'-t_d)}+c_L e^{R(T'-t_d)}\right]e^{-RT'}$$

then $dT'C(m,t_d/T')/dk<0$. From the Intermediate Value Theorem, there will exist a unique solution k^* that satisfies $dT'C(m,k^*)/dk=0$, where $t_d<k^*T'<T'$. Because $d^2T'C(m,k)/dk^2>0, T'C(m,k)$ is a convex function of k for a fixed value m. Hence, $T'C(m,k^*)$ is the minimum value of k when m is given.

Part (b).

If

$$c_p e^{\beta t_d}+c_h\frac{e^{\beta t_d}-e^{-Rt_d}}{R+\beta}>\left[(c_p-\frac{c_s}{R})e^{-\delta(T'-t_d)}+(\frac{c_s}{R}-c_L)e^{(R-\delta)(T'-t_d)}+c_L e^{R(T'-t_d)}\right]e^{-RT'},$$

owing to $dT'C(m,1)/dk>0$ and $d^2T'C(m,k)/dk^2>0$, $T'C(m,k)$ is a strictly increasing function of k in the interval $mt_d/H\le k\le 1$ when m is given. Consequently, the minimum value of $T'C(m,k)$ will happen at $k=mt_d/H$ when m is given.

From theorem 1, we can use Newton-Raphson method to find the optimal value k^* when the replenishment number m is given. However, since the high-power expression of the exponential function in $T'C(m,k)$, it is difficult to show analytic solution of m such that it makes $T'C(m,k)$ minimized. Following the optimal solution procedure proposed by Montgomery (1982), we let (m^*,k^*) denote the optimal solution to $T'C(m,k)$ and let $(m,k^*(m))$ denote the optimal solution to $T'C(m,k)$ when m is given. If m^* is the smallest integer such that $T'C(m^*,k^*,(m^*))$ less than each value of $T'C(m,k(m))$ in the interval $m^*+1\le m\le m^*+10$. Then we take $(m^*,k^*(m^*))$ as the optimal solution to $T'C(m,k(m))$. And we can obtain the maximum inventory level I_m as

$$I_m = \frac{\alpha}{\theta+\beta}[e^{(\theta+\beta)(\frac{K^* H}{m^*}-t_d)} - 1]e^{\beta t_d} + \frac{\alpha}{\beta}(e^{\beta t_d} - 1) \qquad (21)$$

Also the optimal order quantity Q^* is

$$Q^* = I_m + s_b$$

$$= \frac{\alpha}{\theta+\beta}[e^{(\theta+\beta)(\frac{k^* H}{m^*}-t_d)} - 1]e^{\beta t_d} + \frac{\alpha}{\beta}(e^{\beta t_d} - 1) + \frac{\alpha}{\delta}[1 - e^{-\delta(1-k^*)\frac{H}{m^*}}] \qquad (22)$$

Theorem 2 $T'C^*$ is the minimum total relevant cost of $T'C(m,k)$.*If* $c_\rho < c_s / R < c_L$, then $T'C^*$ is a strictly increasing function of δ *for* $\delta \geq 0$.

Proof

Now, we consider the relation between variable δ and the minimum total relevant cost $T'C^*$

$$\frac{dT'C^*}{d\delta} = \{(\frac{c_s}{R} - c_L)[\frac{[1-(1-k^*)T'^*(R-\delta)]e^{(R-\delta)(1-k^*)T'^*} - 1}{(R-\delta)^2}]$$

$$+(c_p - \frac{c_s}{R})\frac{[1+(1-k^*)T'^*\delta]e^{-\delta(1-k^*)T'^*} - 1}{\delta^2}\}\alpha W^*$$

Let $x = (1-k)T'$, where $0 \leq x \leq T' - t_d$, and

$$f(x) = (\frac{c_s}{R} - c_L)\frac{[1-x(R-\delta)]e^{(R-\delta)x} - 1}{(R-\delta)^2} + (c_p - \frac{c_s}{R})\frac{(1+x\delta)e^{-\delta x} - 1}{\delta^2}$$

$$\frac{df(x)}{dx} = -\left[(\frac{c_s}{R} - c_L)e^{Rx} + (c_p - \frac{c_s}{R})\right]xe^{-\delta x}$$

If $c_p < c_s / R < c_L$, it must be $df(x)/dx > 0$ for $0 \leq x \leq T' - t_d$. Hence, $f(x)$ is a strictly increasing function of x for $0 \leq x \leq T' - t_d$. Because $f(0) = 0$, we can sure $fx) > 0$ for $0 \leq x \leq T' - t_d$. This implies $dT'C^* / d\delta > 0$, we say $T'C^*$ is a strictly increasing function of δ for $\delta \geq 0$.

Theorem 3 $T'C^*$ is the minimum total relevant cost of $T'C(m,k)$ and Q* is the optimal order quantity. If $t_d < k^* T'^* < T'^*$, i.e., it satisfies Theorem 2.1(a), then $T'C^*$ and Q* are strictly decreasing functions of t_d for $t_d \geq 0$.

Proof

Considering $T'C^*$ and Q* derivative to t_d respectively.

$$\frac{dT'C^*}{dt_d} = -\frac{\alpha\theta}{\theta+\beta}[e^{(\theta+\beta)(k^* T'^* - t_d)} - 1][c_p e^{\beta t_d} + \frac{c_h(e^{\beta t_d - e^{-Rt_d}})}{R+\beta}]V^*$$

and

$$\frac{dQ^*}{dt_d} = -\frac{\alpha\theta}{\theta+\beta}[e^{(\theta+\beta)(k^*T'^*-t_d)} - 1]e^{\beta t_d}$$

The conditions for $dT'C^*/dt_d > 0$ and $dQ^*/dt_d > 0$ are $e^{(\theta+\beta)(k^*T'^*-t_d)} - 1 < 0$, i.e., $k^*T'^* < t_d$. It violates the definition $t_d \le k^*T'^* \le T'^*$. Hence, $dT'C^*/dt_d$ and dQ^*/dt_d would be smaller than zero in the interval $t_d < k^*T'^* < T'^*$. It also satisfies Theorem. $1(a)$ which judges whether $k^*T'^*$ exists between t_d and T^* or not. In this situation, $T'C^*$ and Q^* are strictly decreasing functions of t_d for $t_d \ge 0$.

Theorem 4 Q^* is the optimal order quantity. Q^* is a strictly decreasing function of δ $for\, \delta \ge 0$.

Proof:

Now, we consider the relation between variable δ and optimal order quantity Q^*.

$$\frac{dQ^*}{d\delta} = \alpha\frac{[\delta(1-k^*)T'^* + 1]e^{-\delta(1-k^*)T'^*} - 1}{\delta^2}$$

Let $\lambda = \delta(1-k)T'$, where $0 \le \lambda \le \delta(T' - t_d)$, and $g(\lambda) = \alpha[(\lambda+1)e^{-\lambda} - 1]/\delta^2$. Because $dg(\lambda)/d\lambda = -\alpha e^{-\lambda}/\delta^2 < 0$, $g(\lambda)$ is a strictly decreasing function of λ for $0 \le \lambda \le \delta(T' - t_d)$. Because $g(0) = 0$, we can sure $g(\lambda) < 0$ for $0 \le \lambda \le \delta(T' - t_d)$. This implies $dQ^*/d\delta < 0$, we can say Q^* is a strictly decreasing function of δ for $\delta \ge 0$.

4.CONCLUSIONS:

We have presented the mathematical models under two different cases are also discussed which can help the decision makes to determine the total average inventory cast. To the purchaser to earn more by inventing the resource otherwise from the sale-proceed of the inventory, which results in the lower cost

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