A MATHEMATICAL STUDY OF TWO SPECIES COMMENSALISM MODEL HOMOTOPY ANALYSIS METHOD APPROACH

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Abstract:

In this present paper we discussed two species commensalism model. Here first species (x) is commensal and the second species (y) is host. Commensalism is Ecological model interaction between two organisms. One organism benefits from other without harmed by the organism. Here we governed two non linear differential equations with natural resources and the model is represented by coupled non linear ordinary differential equations. The analytical solution of the model was identified by using Homotopy Analysis method technique. The solutions are supported by plotting h-curves using Mat Lab.

Keywords: Commensal, Host, Embedding parameter, Deformation equation, h-curves.

1. Introduction:

In this chapter a two species Commensalism model with limited resources for both the species was taken up for analytic study. The model is represented by coupled non-linear ordinary differential equations. The series solution of the non-linear system is approximated by Homotopy Analysis Method.

Symbioses are a broad class of interactions among organisms commensalism involves one organism is benefited by another without any positive or negative benefit for itself.

2. Mathematical Model

The governing equations of the system are as follows

$$\frac{dx}{dt} = a_1 x(t) - \alpha_{11} x^2(t) + \alpha_{12} x(t) y(t) \qquad (2.1)$$

$$\frac{dy}{dt} = a_2 y(t) - \alpha_{22} y^2(t)$$

2.2. Solutions as polynomials of the model (2.1) by HAM

Consider the nonlinear differential equation (2.1) with initial conditions x_0 and y_0 .The solutions x(t),y(t) can be expressed by following set of base functions in the form

$$x(t) = \sum_{m=1}^{+\infty} a_m t^m,\ y(t) = \sum_{m=1}^{+\infty} b_m t^m \qquad (2.2.1)$$

Where a_m, b_m are coefficients to be determined.

Choose the linear operator and non-linear operators are denoted as follows.

$$L_1[x(t;p)] = \frac{dx(t;p)}{dt}, L_1[y(t;p)] = \frac{dy(t;p)}{dt} \qquad (2.2.2)$$

$$L_1[x(t,p)] = \frac{dx(t;p)}{dt} - a_1 x(t;p) + \alpha_{11} x^2(t;p) - \alpha_{12} x(t;p) y(t;p) \qquad (2.2.3)$$

$$L_1[y(t,p)] = \frac{dy(t;p)}{dt} - a_2 y(t;p) + \alpha_{22} y^2(t;p) \qquad (2.2.4)$$

The zero order deformation equation can be constructed using the above definition.

$$(1-p)L_1[x(t;p) - x_0(t)] = ph_1 N[x,y], \qquad (2.2.5)$$

$$(1-p)L_2[y(t;p) - y_0(t)] = ph_2 N[x,y],$$

When p=0 and p=1, from the zero-deformation equations one has,

$$x(t;0) = x_0(t) \quad x(t;1) = x(t)$$

$$y(t;0) = y_0(t) \quad y(t;1) = y(t) \qquad (2.2.6)$$

And expanding x(t;p) and y(t;p) in Taylors series, with respect to embedding parameter p, one obtains

$$x(t;p) = x_0(t) + \sum_{m=1}^{+\infty} x_m(t) p^m \qquad (2.2.7)$$

$$y(t;p) = y_0(t) + \sum_{m=1}^{+\infty} y_m(t) p^m$$

$$x_m(t) = \frac{1}{m!} \frac{d^m x(t;p)}{dp^m}\bigg|_{p=0} \qquad (2.2.8)$$

$$y_m(t) = \frac{1}{m!} \frac{d^m y(t;p)}{dp^m}\bigg|_{p=0}$$

$$p = 1 \begin{cases} x_m(t) = x_0(t) + \sum_{m=1}^{+\infty} x_m(t) \\ y_m(t) = y_0(t) + \sum_{m=1}^{+\infty} y_m(t) \end{cases} \qquad (2.2.9)$$

Define the vector

$$\vec{x}_m = [x_0(t), x_1(t), \ldots. x_m(t)] \qquad (2.2.10)$$

$$\vec{y}_m = [y_0(t), y_1(t), \ldots. y_m(t)]$$

And apply the procedure stated before. The following mth-order deformation Equations will be achieved.

$$L_1[x_m(t) - \chi_m x_{m-1}(t)] = \bar{h}_1 H_1(t) R_{1m}(\vec{x}_{m-1}, \vec{y}_{m-1}), \qquad (2.2.11)$$

$$L_1[y_m(t) - \chi_m y_{m-1}(t)] = \bar{h}_2 H_2(t) R_{2m}(\vec{x}_{m-1}, \vec{y}_{m-1}),$$

Let us consider $H_1(t) = H_2(t) = 1$ and the initial conditions $x_0(t) = x(t=0) = x_0 \quad y_0(t) = y(t=0) = y_0$ in above equations

$$R_{1m}(x_{m-1}, y_{m-1}) = \frac{1}{(m-1)!} \frac{d^{m-1}}{dp^{m-1}} N[x(t,p)] = \frac{d}{dt} x_{m-1}(t) - a_1 x_{m-1} + \alpha_{11} \sum_{n=1}^{m} x_n(t) x_{m-n-1}(t) - \alpha_{12} \sum_{n=0}^{m-1} x_n(t) y_{m-n-1}(t)$$

$$R_{2m}(x_{m-1}, y_{m-1}) = \frac{1}{(m-1)!} \frac{d^{m-1}}{dp^{m-1}} N[y(t,p)] = \frac{d}{dt} y_{m-1}(t) - a_2 y_{m-1} + \alpha_{22} \sum_{n=1}^{m} y_n(t) y_{m-n-1}(t) \qquad (2.2.12)$$

The solution of mth order deformation equation is given by for m ≥1

$$x_{1,m}(t) = \chi_m x_{1,m-1}(t) + hL^{-1}\left[R_{1,m}(x_{m-1}, y_{m-1})\right]$$

$$y_{1,m}(t)=\chi_m y_{1,m-1}(t)+hL^{-1}\left[R_{2,m}(x_{m-1},y_{m-1})\right]$$

and $\chi_m=\begin{cases}0, m\le 1,\\ 1, m>1.\end{cases}$ (2.2.13)

The analytic solution of the model (2.1) using polynomial base function can be expressed as

$$x(t)=\sum_{m=1}^{+\infty}a_m(h)t^m,\; y(t)=\sum_{m=1}^{+\infty}b_m(h)t^m \tag{2.2.14}$$

First approximation for the model (2.1) is given by

$$L_1\left(x_1(t)-\chi_1 x_0(t)\right)=h\left[-a_1x_0(t)+\alpha_{11}x_0^2(t)-\alpha_{12}x_0(t)y_0(t)\right]$$

$$x_1(t)=h\left[-a_1x_0+\alpha_{11}x_0^2-\alpha_{12}x_0y_0\right]t$$

$$x_1(t)=hk_1t$$

$$L_1\left(y_1(t)-\chi_1 y_0(t)\right)=h\left[-a_2y_0(t)+\alpha_{22}y_0^2(t)\right]$$

$$y_1(t)=h\left[-a_2y_0+\alpha_{22}y_0^2\right]t$$

$$y_1(t)=hk_2t \tag{2.2.15}$$

where

$$k_1=\left[-a_1x_0+\alpha_{11}x_0^2-\alpha_{12}x_0y_0\right] \tag{2.2.16}$$

$$k_2=[-a_2y_0+\alpha_{22}y_0^2]$$

The following second approximations for the system (2.1) is given by

$$L_1\left(x_2(t)-\chi_2 x_1(t)\right)=h\left[\frac{d}{dt}x_1(t)-a_1x_1(t)+\alpha_{11}\sum_{n=0}^{1}x_n(t)x_{1-n}(t)-\alpha_{12}\sum_{n=0}^{1}x_n(t)y_{1-n}(t)\right]$$

$$x_2(t)=(hk_1t+h^2k_1t+l_1h^2t^2)$$

$$L_1\left(y_2(t)-\chi_2 y_1(t)\right)=h\left[\frac{d}{dt}y_1(t)-a_2y_1(t)+\alpha_{22}\sum_{n=0}^{1}y_n(t)y_{1-n}(t)\right]$$

$$y_2(t)=(hk_2t+h^2k_2t+l_2h^2t^2)$$

where

$$l_1=\left[\frac{-1}{2}a_1k_1+x_0\alpha_{11}k_1-\frac{1}{2}y_0\alpha_{12}k_1-\frac{1}{2}x_0\alpha_{12}k_2\right] \tag{2.2.17}$$

$$l_2=\left[\frac{-1}{2}a_2k_2+y_0\alpha_{22}k_2\right]$$

The third approximations for system (2.1) is given by

$$L_1\left(x_3(t)-\chi_3 x_2(t)\right)=h\left[\frac{d}{dt}x_2(t)-a_1x_2(t)+\alpha_{11}\sum_{n=0}^{2}x_n(t)x_{1-n}(t)-\alpha_{12}\sum_{n=0}^{2}x_n(t)y_{1-n}(t)\right]$$

$$x_3(t)=hk_1t+2h^2k_1t+2l_1h^2t^2+h^3k_1t+2l_1h^3t^2+m_1h^3t^3$$

$$L_1\left(y_3(t)-\chi_3 y_2(t)\right)=h\left[\frac{d}{dt}y_2(t)-a_2y_2(t)+\alpha_{22}\sum_{n=0}^{2}y_n(t)y_{1-n}(t)\right]$$

$$y_3(t)=hk_2t+2h^2k_2t+2l_2h^2t^2+h^3k_2t+2l_2h^3t^2+m_2h^3t^3$$

where $m_1=\left[\frac{-1}{3}a_1l_1+\frac{2}{3}x_0\alpha_{11}l_1+\frac{1}{3}\alpha_{11}k_1^2-\frac{1}{3}y_0\alpha_{12}l_1-\frac{1}{3}x_0\alpha_{12}l_2-\frac{1}{3}\alpha_{12}k_1k_2\right]$ (2.2.18)

$$m_2 = \left[\frac{-1}{3} a_2 l_2 + \frac{2}{3} y_0 \alpha_{22} l_2 + \frac{1}{3} \alpha_{22} k_2^2 \right]$$

The two terms approximation to the solution will be considered as

$$x(t) \approx x_0 + x_1(t) + x_2(t) \qquad (2.2.19)$$
$$y(t) \approx y_0 + y_1(t) + y_2(t)$$
$$x(t) \approx x_0 + 2k_1 ht + k_1 h^2 t + l_1 h^2 t^2 \qquad (2.2.20)$$
$$y(t) \approx y_0 + 2k_2 ht + k_2 h^2 t + l_2 h^2 t^2$$

The three terms approximation to the solution will be considered as

$$x(t) \approx x_0 + x_1(t) + x_2(t) + x_3(t) \qquad (2.2.21)$$
$$y(t) \approx y_0 + y_1(t) + y_2(t) + y_3(t)$$
$$x(t) \approx x_0 + 3hk_1 t + 3k_1 h^2 t + 3l_1 h^2 t^2 + k_1 h^3 t + 2l_1 h^3 t^2 + m_1 h^3 t^3$$
$$y(t) \approx y_0 + 3hk_2 t + 3k_2 h^2 t + 3l_2 h^2 t^2 + k_2 h^3 t + 2l_2 h^3 t^2 + m_2 h^3 t^3$$

2.3 HAM Solution as polynomial functions of the model (2.1) for different auxiliary parameter h_i (for i=1,2)

By choosing the different auxiliary parameter values of h_i (i =1,2), with initial conditions, linear operator are described by equations (2.3.2) to (2.3.4)

The solution of m^{th} order deformation equation is given by for m ≥1

$$x_{1,m}(t) = \chi_m x_{1,m-1}(t) + h_1 L^{-1}\left[R_{1,m}(x_{1,m-1}, y_{1,m-1}) \right]$$
$$y_{1,m}(t) = \chi_m y_{1,m-1}(t) + h_2 L^{-1}\left[R_{2,m}(x_{1,m-1}, y_{1,m-1}) \right]$$

and $\chi_m = \begin{cases} 0, m \le 1, \\ 1, m > 1. \end{cases}$ (2.3.1)

The analytic solution of the model (2.1) is expressed as

$$x(t) = \sum_{m=1}^{+\infty} a_m t^m, \; y(t) = \sum_{m=1}^{+\infty} b_m t^m \qquad (2.3.2)$$

First approximation for the model (2.1) is given by

$$L_1\left(x_1(t) - \chi_1 x_0(t)\right) = h_1\left[-a_1 x_0(t) + \alpha_{11} x_0^2(t) - \alpha_{12} x_0(t) y_0(t) \right]$$
$$x_1(t) = h_1\left[-a_1 x_0 + \alpha_{11} x_0^2 - \alpha_{12} x_0 y_0 \right] t$$
$$L_1\left(y_1(t) - \chi_1 y_0(t)\right) = h_2\left[-a_2 y_0(t) + \alpha_{22} y_0^2(t) \right]$$
$$y_1(t) = h_2\left[-a_2 y_0 + \alpha_{22} y_0^2 \right] t$$

The following second approximations for the system (2.1) is given by

$$L_1\left(x_2(t) - \chi_2 x_1(t)\right) = h_1\left[\frac{d}{dt} x_1(t) - a_1 x_1(t) + \alpha_{11} \sum_{n=0}^{1} x_n(t) x_{1-n}(t) - \alpha_{12} \sum_{n=0}^{1} x_n(t) y_{1-n}(t) \right]$$

$$x_2(t) = (h_1 + h_1^2)\left[-a_1 x_0 + \alpha_{11} x_0^2 - \alpha_{12} x_0 y_0 \right] t + \frac{h_1 x_0}{2} \left[\begin{array}{r} h_1\left(-a_1 + \alpha_{11} x_0 - \alpha_{12} y_0\right)(-a_1 + 2\alpha_{11} x_0 - \alpha_{12} y_0) \\ - h_2 \alpha_{12}\left(-a_2 y_0(t) + \alpha_{22} y_0^2(t)\right) \end{array} \right] t^2$$

$$L_1\left(y_2(t)-\chi_2 y_1(t)\right)=h_2\left[\frac{d}{dt}y_1(t)-a_2 y_1(t)+\alpha_{22}\sum_{n=0}^{1}y_n(t)y_{1-n}(t)\right]$$

$$y_2(t)=(h_2+h_2^2)\left[-a_2 y_0+\alpha_{22}y_0^2\right]t+\frac{h_2^{\,2}y_0}{2}\left(a_2^2-3a_2\alpha_{22}y_0+2\alpha_{22}^2y_0^2\right)\frac{t^2}{2} \tag{2.3.4}$$

The third approximations for system (2.1) is given by

$$L_1\left(x_3(t)-\chi_3 x_2(t)\right)=h_1\left[\frac{d}{dt}x_2(t)-a_1 x_2(t)+\alpha_{11}\sum_{n=0}^{2}x_n(t)x_{1-n}(t)-\alpha_{12}\sum_{n=0}^{2}x_n(t)y_{2-n}(t)\right]$$

$$\begin{aligned}x_3(t)&=(1+h_1)(h_1+h_1^2)k_1t\\&+\begin{bmatrix}-a_1h_1(h_1+h_1^2)k_1+2\alpha_{11}x_0(h_1+h_1^2)k_1h_1-h_1\alpha_{12}x_0k_3\\-h_1\alpha_{12}y_0(h_1+h_1^2)k_1)\end{bmatrix}\frac{t^2}{2}\\&+\begin{bmatrix}h_1(1+h_1)k_2-\frac{1}{2}a_1h_1^2k_2+h_1\alpha_{11}x_0k_2+\frac{1}{2}\alpha_{11}h_1^3k_1^2\\-\frac{1}{2}\alpha_{12}h_1x_0k_4-\frac{1}{2}\left(\frac{\alpha_{12}h_1^2h_2}{(h_2+h_2^2)}k_1k_3+\frac{h_1}{2}\alpha_{12}y_0k_2\right)\end{bmatrix}\frac{t^3}{3}\end{aligned}$$

$$L_1\left(y_3(t)-\chi_3 y_2(t)\right)=h_2\left[\frac{d}{dt}y_2(t)-a_2 y_2(t)+\alpha_{22}\sum_{n=0}^{2}y_n(t)y_{2-n}(t)\right]$$

$$\begin{aligned}y_3(t)&=(1+h_2)k_3t+\left[(1+h_2)k_4-a_2h_2k_3+2h_2k_3\alpha_{22}y_0\right]\frac{t^2}{2}\\&+\left[\left(-\frac{1}{2}a_2h_2k_4+h_2\alpha_{22}k_4y_0+\frac{\alpha_{22}h_2^3k_3^2}{(h_2+h_2^2)^2}\right)\right]\frac{t^3}{3}\end{aligned} \tag{2.3.5}$$

where

$$p_1=\begin{bmatrix}h_1(1+h_1)k_2-\frac{1}{2}a_1h_1^2k_2+h_1\alpha_{11}x_0k_2+\frac{1}{2}\alpha_{11}h_1^3k_1^2\\-\frac{1}{2}\alpha_{12}h_1x_0k_4-\frac{1}{2}\left(\frac{\alpha_{12}h_1^2h_2}{(h_2+h_2^2)}k_1k_3+\frac{h_1}{2}\alpha_{12}y_0k_2\right)\end{bmatrix}$$

$$p_2=\left(-\frac{1}{2}a_2h_2k_4+h_2\alpha_{22}k_4y_0+\frac{\alpha_{22}h_2^3k_3^2}{(h_2+h_2^2)^2}\right)$$

$$k_1=\left[-a_1x_0+\alpha_{11}x_0^2-\alpha_{12}x_0y_0\right]$$

$$k_2=\frac{h_1x_0}{2}\left[h_1\left(-a_1+\alpha_{11}x_0-\alpha_{12}y_0\right)(-a_1+2\alpha_{11}x_0-\alpha_{12}y_0)-h_2\alpha_{12}\left(-a_2y_0+\alpha_{22}y_0^2\right)\right]$$

$$k_3=(h_2+h_2^2)\left[-a_2y_0+\alpha_{22}y_0^2\right]$$

$$k_4=\frac{h_2^{\,2}y_0}{2}\left[\left(a_2^2-3a_2\alpha_{22}y_0+2\alpha_{22}^2y_0^2\right)\right] \tag{2.3.6}$$

The second order approximation to the solution will be given by

$$x(t)\approx x_0+x_1(t)+x_2(t)$$

$$y(t)\approx y_0+y_1(t)+y_2(t)$$

$$x_1(t)\approx x_0+\left[2h_1+h_1^2\right]k_1t+l_1\frac{t^2}{2} \tag{2.3.7}$$

$$y_1(t) \approx y_0 + \left[2h_2 + h_2^2\right]k_2 t + l_2 \frac{t^2}{2}$$

The third order approximation to the solution will be considered as

$$x(t) \approx x_0 + x_1(t) + x_2(t) + x_3(t), \quad y(t) \approx y_0 + y_1(t) + y_2(t) + y_3(t)$$

$$x(t) \approx x_0 + \left[3h_1 + 3h_1^2 + h_1^3\right]k_1 t + (l_1 + m_1)\frac{t^2}{2} + \frac{t^3}{3}p_1$$

$$y(t) \approx y_0 + \left[3h_2 + 3h_2^2 + h_2^3\right]k_2 t + (l_2 + m_2)\frac{t^2}{2} + \frac{t^3}{3}p_2 \qquad (2.3.8)$$

Convergence region can be determined using h-curves and the accuracy of the analytical solution can be improved by finding higher order approximations.

Using h-curves, valid regions of a convergent series solution can be determined by increasing the order of approximation the results are more accurate.

2.5 Numerical Examples:

Example: x_0=5; y_0=10; a_1=5; a_2=5; x_{11}=0.10; x_{22}=0.90; x_{12}=0.80;

h-curves for the system of equations (2.1) via polynomial base functions

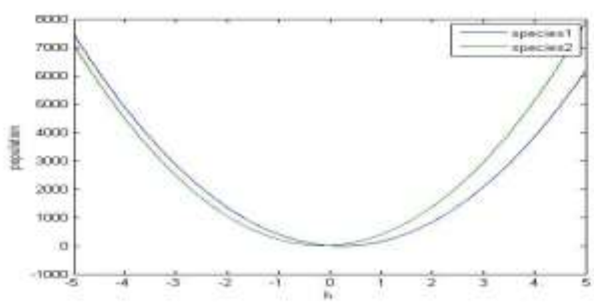

Fig..2.5.1

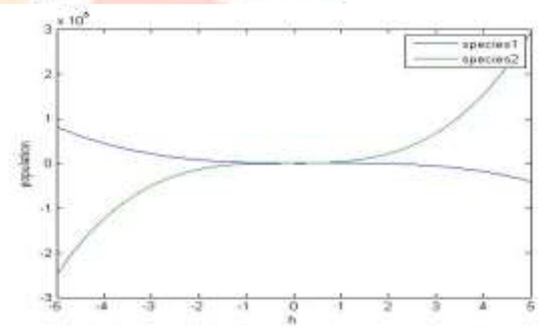

Fig..2.5.2

The Fig. 2.5.1 shows the h-curve of second order approximation for the system of equations (2.1). The valid region of h, is $-0.5 < h < 0.2$, where the series approximation solutions are convergent.

The Fig. 2.5.2 shows the h-curve of model (2.1) of third order approximation, the valid region of h, corresponding to the line segments parallel to the horizontal axis. Convergence region is $-1 < h < 1.2$, where the series approximation solutions are convergent.

From the above example the region of convergence is improved by third order approximation, so accuracy can be improved by increasing the order of approximations.

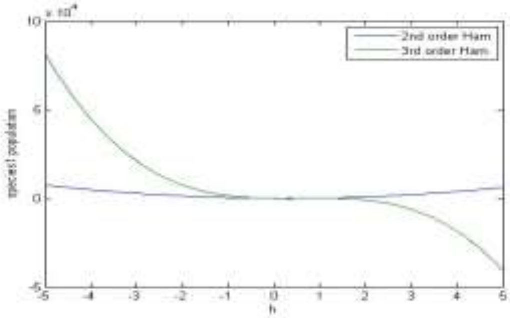

Fig.2.5.3

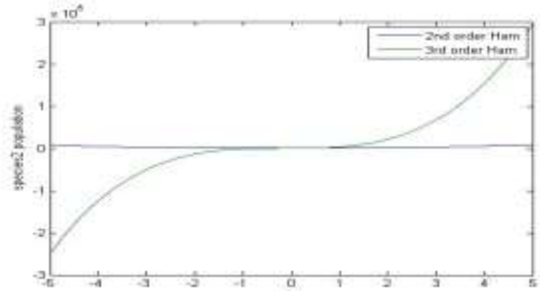

Fig..2.5.4

The solution curve can be obtained by fixing an auxiliary parameter ‘h’ for the above mentioned parametric values in example 1.

Fig. 2.5.3 & 2.5.4 shows the variation of species1 and species2 of second and third order HAM approximations respectively.

t-curves are plotted for the above mentioned parametric values for the model (2.1) .The t-curve are population curves over time.

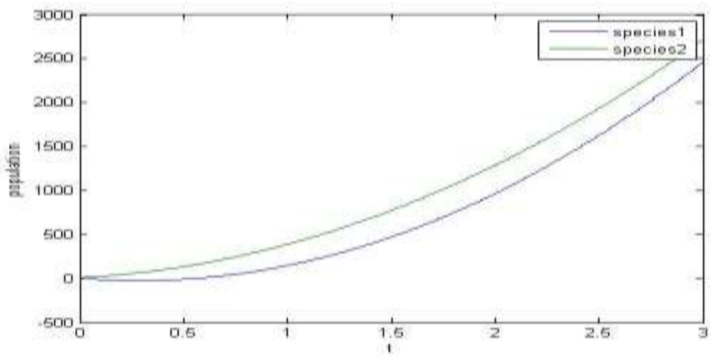

Fig.2.5.5

Fig.2.5.5: variation of commensal and species2 with respect to time (t) for second order HAM solution

The Fig. 2.5.5 shows the t-curves with respect to time, from the above Fig,2.55 the initial population of species1 increases due to commensal effect

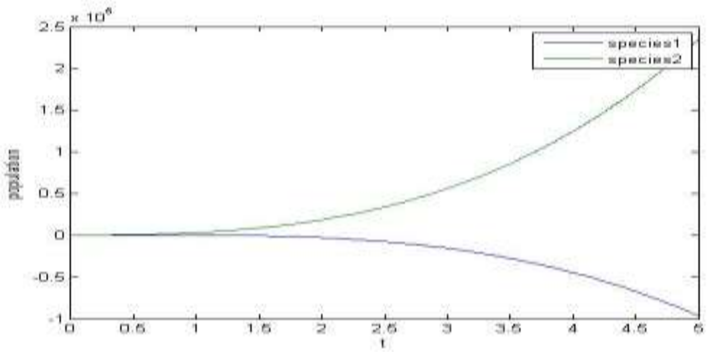

Fig.2.5.6

Fig.2.5.6: variation of commensal & species2 with respective time (t) for third order HAM solution

From the above example it is evident that the efficiency is increased by finding higher order terms. Clearly the population of species1 increases from the equilibrium point

Conclusion:

In this chapter a two species commensalism model with limited resources for both the species was taken up for analytic study. The series solutions of this model are obtained by Homotopy analysis method by taking polynomial as base function. The convergence region is identified by h-curves. The solution curves of this model are discussed by t-curves. The h-curves and t-curves of Second and third order HAM series solutions are derived and found that the higher order HAM solutions are improve the efficiency of the model it clearly supported by numerical examples.

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