The real life applications of knot theory and how it is connected with the graph theory

T.kalaiselvi M.Phill., M. Ramesh Kumar
Mathematics, Associate Dean
Prist university, Tanjore(puducherry branch),india.

Abstract :

This paper introduces some fundamental ideas of knot theory in a way that is accessible to non-mathematicians. It summaries a few important examples of ways in which knot theory can be used to modern real world phenomena. Perticularly the role of knot theory in DNA Research including the importance of topology to the pharamaceutical industry and also use discuss the intersection of knot theory and graph theory it provide a new field graph knotted.

Keywords :

Knot, graph, linking numbers, graph knotted, Intrinsically Knotted, Embedding.

Introduction:

The first think that most people think of upon hearing the word knot is probably a troublesome tangle in a piece of string of perhaps to contortion of shoelaces that prevents shoes from falling off. In mathematics such messes are not Knots, However since they have loose ends floating around. If we take the two loose ends and glue them together then we have what mathematician will agree to call a knot of course we really cannot define a knot as " a tangled piece of string with it's ends glued together". And hope to do any serioues mathematics using this definition.

The survey considers three current areas of the steady that combine the field of graph theory and knot theory. Recall that a graph consist of a set of vertices and a set of edges that connect them.

A fairly recent development has been the investigation of graph that have non-trivial links and knots in every spatial embedding. We say that a graph is intrinsically linked if it contains a pair of cycles that form a non-splittable links in every spatial embedding similarly we say that a graph is intrinsically knotted if it contains a cycle that forms a non trivial knot in every spatial embedding and also we finding graphs in which every embedding a more complex structure such a finding other minor-minimal intrinsically knotted graphs.

***Definition* 1:** A knot is basically a mathematical model of a "tangled up loop of rope" that lives in three dimensions.

***Definition* 2:** Mathematically knot is defined as follows.

A knot is a subset $K \ C \ R^3$ of the form k=f(R)

Where f: R→ R^3 is a function satisfying.

1) $f(f_1) - f(f_2) \leftrightarrow t_1\text{-}t_2 \in Z$
2) d^nf/dt^n exist for all n
3) (df/dt) (t) ≠ 0 for all t

Where f is well defined function. Thus why we say that knot is the one of the branch of topology and also we said, it is the part of algebraic topology.

***The fundamental ideas of knot theory*:**

Simply we define A knot is a simple close polygonal curve in R^3. Seems like we went through a lot of effort to come up with this definition. But having done things properly here that has given us a taste of the worrying that we must do (and we have omitted a great deal of the optional worrying that the biggest worriers will do) when using mathematics to model a real world entire. If we fail to impose enough restrictions on the mathematical model we can find ourselves dealing with a model that admits objects that cannot exist in the real world. Now we can put all of this worrying behind us and remain content that we have a solid idea of what a knot is ,

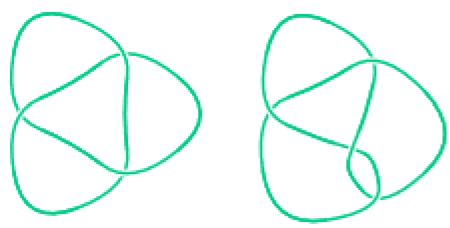

Fig 1. Trefoil and figure 8 knot.

From now on we will generally not draw our knots as simple closed polygonal curves, but rather as smooth curves such as can be seen in Figure 1 where we show that trefoil knot and the figure-8 knot.

In our figures, we will use a break in the drawing to indicate the portion of the knot that is passing under another portion of the knot such a representation of the knot will be called a projection or diagram.

A few additional bits of knot theory jargon will be useful for reading the remainder of this paper. First a link is a generalization of a knot in which there may be multiple distinct simple closed polygonal curves referred to as components, that links around each other. A knot then becomes a link with one component. So sometimes we will usually use "link" in a sense that includes knots. A knot is said to be alternating if it has projection in which the crossing alternate between over and under as the knot is traced out. Finally just as with the integers. We have an idea of the prime knot. Intensively a knot is prime if it does not consist of two knots tied in the same piece of string Figure 2 shows a composite knot ((ie) one that is not prime) consisting of two trefoils connected (or added) together.

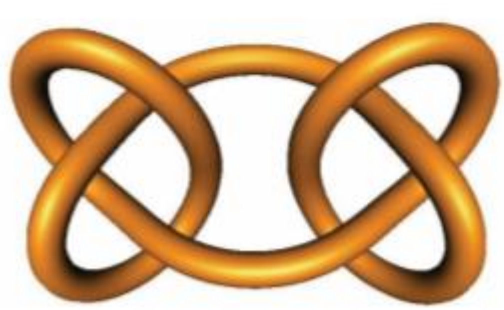

Fig 2. A composite knot

The early history of knot theory was motivated by applications it will be difficult to avoid any reference to application of knot theory in this process. However discussion of modern application of knot theory will be reserved the following passage. Which graph theory can be used to study knots. We will conclude with some original research conducted by the author.

Modern real world application of knot theory :

Knot can be related to statistical mechanics in particular one can look at a knot as a sort of lattice (in the sense of physics/chemistry) and derive knot invariants from studying models of statistical measure on lattices. For example the Jones polynomial comes from a representation of the braid group into the Temperley-lieb algebra, which naturally appears in the study of ice-type models on lattices. I don't know if there are direct applications from knot theory to lattice models. But the approach has inspired the use of knot theoretic tools in more high falutin physics.

There are also application in chemistry and biology. Dorotly buck at inperical college of London studies the knotting of DNA has conjectured that the unknotting number of a knotted strand of DNA should by 1. This makes it easy to unknot the strand for reproduction.

Applications of knot theory in DNA research:

What is DNA:

DNA is a nucleic acid that contains the instructions required to construct other cellular components often it is called the blue print of life it does not act upon other molecules. It is acted upon by enzymes which control replication of other DNA process.

How can knot theory help us understand DNA?

Among other things DNA double helix can be modelled as a ribbon or belt lk=tw+wr, 2 DNA molecules may have some sequence of bore pair but different linking numbers. They behave differently knot theory principles critical in understanding topoisomerases.

Supercoiling:

Human DNA is extremely long and packed into cell nucleic-not neatly word! Like tangled fishing wire stuffed inside a ball when DNA is twisted the strands become more tightly wound supercoiling coils must be relaxed for critical processes like replication to occur topoisomerases. DNA as a belt of ribbon: lk=tw+wr.

(i.e) linking number=Twise+writhe, Twise and writhe can be modelled using a belt. Two edges of belt are stands dead centre is axis in space Tw=twist how the 2 edge of belt (strands) wind about each other in space wr= writhe if belt is buckled without being untwisted then relaxed twist converted into writhe.

knot theory is used to calculating lk, Tw and wr:

(i.e) Linking number lk is same as in knot theory half (1/2) the same of all the crossing of 2 backbone components of DNA strand.

Twist: when axis flat in the plane without crossing itself, twist=1/2 the sum of the +1 -1's of the crossing between the axis and a particular one of the two stands bounding the axis. When axis not flat in the plane Twist=The integral of the backbone about the axis, integrated as the axis in traversed once we can calculate writhe using signed crossover numbers or the sum of all the +1 and -1 crossing where the axis crosses itself. But writhe is not a topological invariant, so we must calculate the average value of the signed crossover number over every possible projection of the axis.

The Topoisomerases:

Enzymes which modify DNA topology to unknot, unlike and maintain proper supercoiling specifically. They cut a strand of DNA, allow another then reseal it there are two main type of Topoisomerases is called type 1 and type 2. Without the topoisomerases, crucial DNA processes like replication are not possible.

Type 1 Topoisomerases **:**

It can break only a single backbone stand the sole function is to regulate supercoiling converting twist into writhe. It has a crucial replication DNA helix is unzipped, and if supercoiling becomes too tight DNA molecule breaks and cell dies.

Type 2 Topoisomerases:

It can break both backbone strands in type1, type2 we can add/remove supercoils, but primary function is to change DNA knot or link type preferentially Type2's unknot/unlike DNA –topological simplification at the end of replication process daughter cells must be completely disentangled (unlinked) before chromosome division can occur otherwise, cell cannot replicate and dies

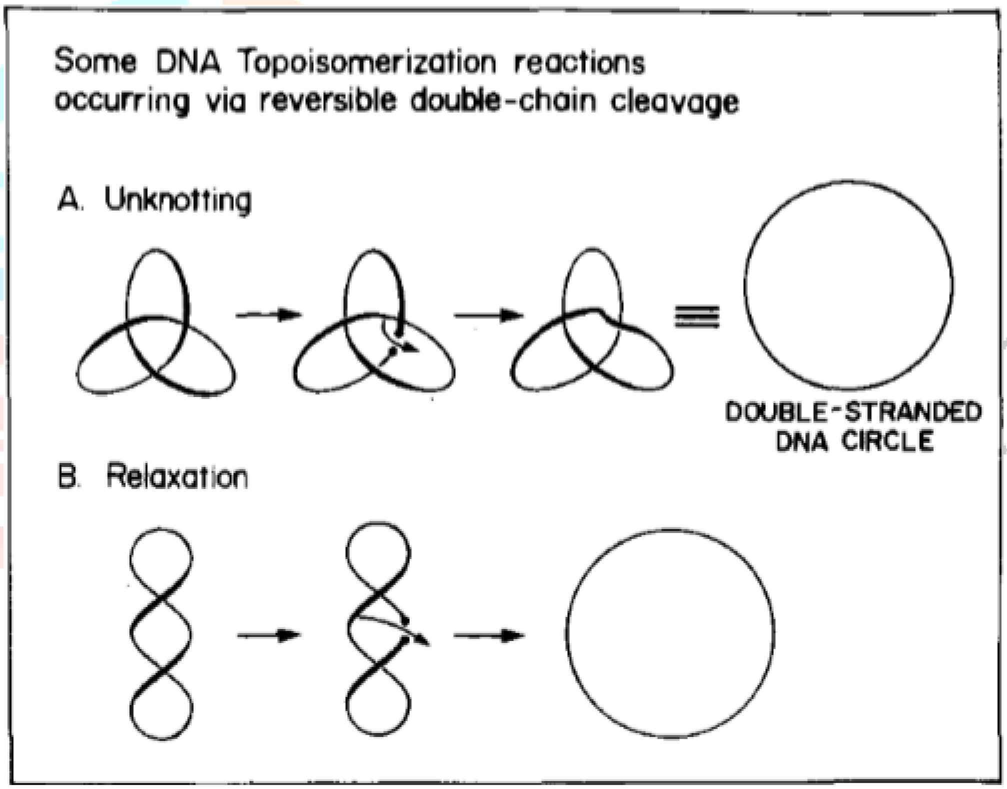

Fig 3.

Knot theory is used to develop life-saving drug:

Many antibiotics and chemotherapy drug target type2 typoisomerases the idea is to prevent disease cells from replicating halting the illness in it's path.

Unknotting number is critical to drug development:

If unknotting number of a DNA molecule is known can accurately estimate howlong it will take for a topoisomerase to unknot it is quickly disease will spread /progress and it quickly drugs will work. Topoisomerases can change only one crossing numbers at the time the unknotting action of type2 topoisomerases is directly related to the goal of classifying all knot with unknotting number.

***When knot theory meets graph theory* :**

On the graph of knots:

A projection of a knot or a link on a 2-dimensional plane divides the plane into several regions. It is frequently used method to separate these regions into two classes. White regions and black regions in the study of knot theory. Simplifying this method C.Bankuiz introduced the motion of graph of knot in his study of alternating knots

This passage contains a method of a graphical treatment of knots and links, and as on application a sufficient condition is given that a knot is amphibious.

***The graph of knots*:**

Let π be a regular normed projection a knot k on a 3-dimensional sphere S. If π has n double point D_1, D_2 …D_n, then it divides S into n+2 regions each of which is homeomorphic to an open disk. Now separate these region into two classes B_1 and B_2 in such a way that each segment of π (ie) an arc from a double point to the next one, is always the common boundary of a region of B_1 and that of B_2 we assume for convenience that the point at infinity is contained in the region of class B_2

Let A_1, A_2…A_n be the region of class B_1 take points pi ϵ Ai (i=1,2…∞) and correct these points by n non-intersecting arcs d_1,d_2…d_k in such a way that each d_k corresponds to D_k (k=1,2….n) and Pi and Pj are connected by d_k, if and only if Ai and Aj have a common double point D_k on their boundaries

The region of class B_1 in all can be considered as a projection of a surface spanning k which is twisted 180^0 at each double point of π from this consideration define the sign of d_k(k=1,2…n) in such a way that it is +ve or –ve according as the surface is twisted at D_k as the right screw or the left one respectively.

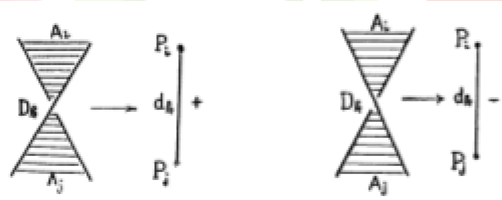

Fig 4.

Then we have a linear graph of vertices P_1,P_2….P_∞ and n signed arcs d_1, d_2…d_n each vertex of which corresponds to a region of B_1 and each signed arc corresponds to a normed double point we call this linear graph. The graph of the projection π and denote it by g(π)

From the same consideration about the class B_1 we get another graph g'(π) we call this the dual graph of g(π) we get the dual graph g'(π) directly from a given graph g(π) as follows. Let B_1', B_2'… B_β' be the region of S- g(π). Take point θ i ϵ Bi' (i=1,2…β) and connect these points by n non-intersecting arcs. d_1', d_2...$_n$d' in such a way that each dw' corresponds to d_k of g(π) (k=1,2..n) and θi and θj are connected by d_k' if and only if d_k is the common boundary of Bi' and Bj'. Each arc d_k' takes the sign opposite to the sign of d_k thus we have a graph g'(π) which consists of vertices θ_1, θ_2….θ_β and arcs d_1', d_2...$_n$d'

The following examples show several graphs of the trefoil knot.

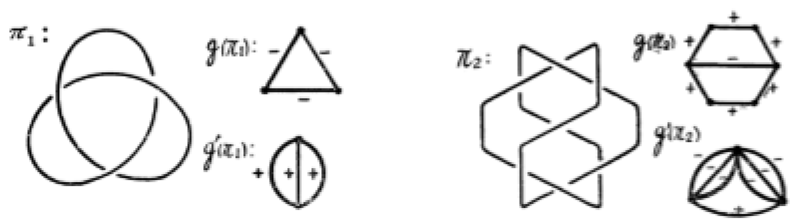

Fig 5.

Remark 1: For a given projection π of a knot or a link we have two graph g(π) and g'(π) conversely for a given graph, whichever g(π) or g'(π), there is a uniquely determined projection π

Remark 2: for the sake of simplicity we may omit one of the sighs + or – from graphs.

Remark 3: Let π and π* be projections of knots k and k* respectively, which are mirror images of each other. Then g(π) and g(π*) are of the same type and have opposite signs we call g(π*) the conjugate graph of g(π)

Remark 4 : R.J Aumann defined the canonical from k' from a graph g(π) of a knot k. k' is a knot equivalent to k situating on an orientable 2-manifold t. In general T-k' is not connected. However we can prove that at least one of T-k' and T'-k'' is connected where k'' is the canonical from g'(π) and T' is the orientable 2-manifold with respect to k''

Knotted graph:

When we introduced knot theory, we defined a knot as an embedding of the circle into space. What about other 1-dimensional objects, like...a graph?

A graph knot or knotted graph is a tame embedding of some graph G into space. Such embeddings are considered equivalent to ambient isotopy. As with knots we may consider diagrams of knotted graphs, such as the following diagram of K6.

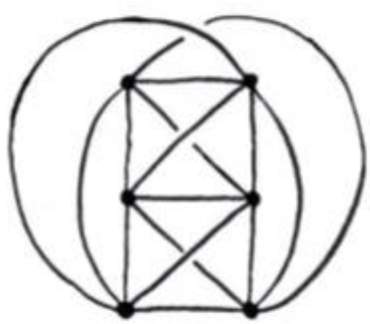

Fig 6

And, just as with knots, we have a Reidemeister-like theorem for manipulating knotted graphs.

Theorem 1: Two graph embeddings are equivalent if and only if their diagrams are related by a sequence of planar isotopies, R1, R2, R3, R4, and R5 moves.

An R4 move allows one to pull a strand over a vertex.

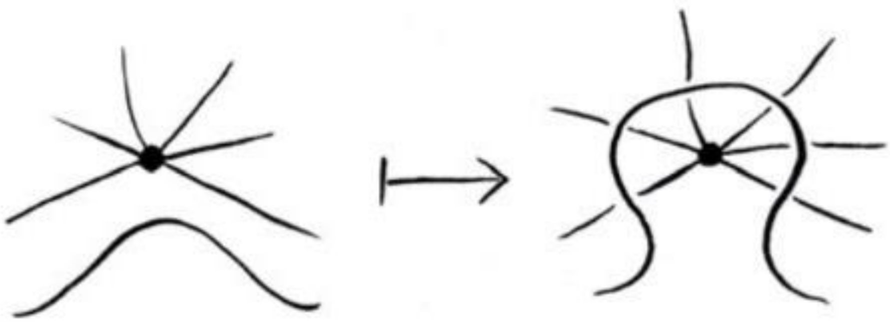

Fig 7

An R5 move allows one to “twist” strands emanating from a vertex.

Fig 8

Within the theory of knotted graphs, we recover traditional knot theory by considering embeddings of the triangle K3.For this reason it would appear that we have made our lives harder. However, in the process of this “complexification” we have made some previously trivial questions non-trivial. For instance, given a graph G, can we embed G into space such that all simple cycles in G are unknotted?

Definition 3. A graph G is said to be intrinsically knotted if for all embeddings of G, there exists some knotted (simple) cycle of G in the given embedding. We say G is IK for short or NIK (not intrinsically knotted) in the opposite case. K3is clearly NIK, confirming our suspicion that intrinsic knotting is not an interesting question in traditional (non-graph) knot theory. Furthermore, some graphs are almost certainly IK, such as K1000. The far more interesting graphs to ponder are the simplest ones that are still knotted. For instance, it turns out that K7is intrinsically knotted, while all of its proper minors are not.

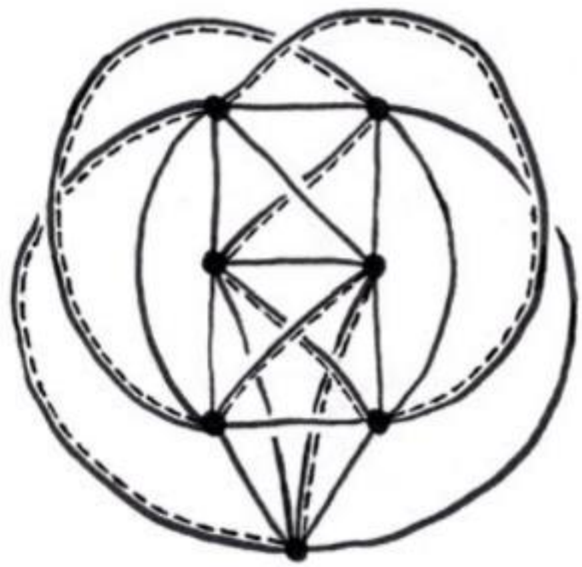

Figure 8: embedding of K_7 with exactly one knotted cycle (indicated)

By way of contrast the following picture of K_6 contains no knots.

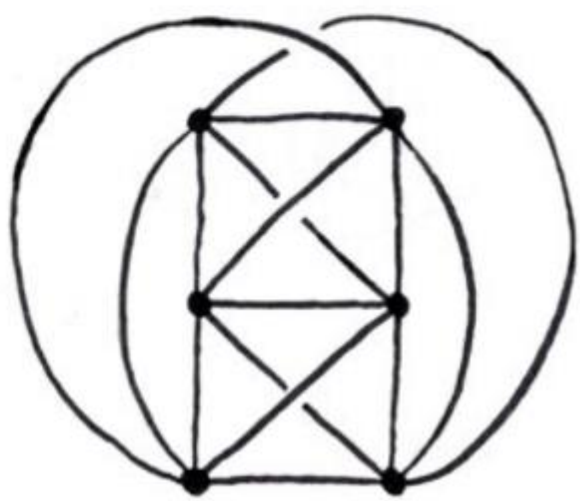

Fig 9

Therefore, we may conclude that K_6 is NIK. This demonstrates our first key observation about intrinsic knotting: It is far easier to prove that a graph is NIK than IK, just as it's easier to prove that a particular diagram is unknotted (just show a sequence of Reidemeister moves) than it is to prove that it's knotted. Even though K_6 is not intrinsically knotted, it is intrinsically linked.

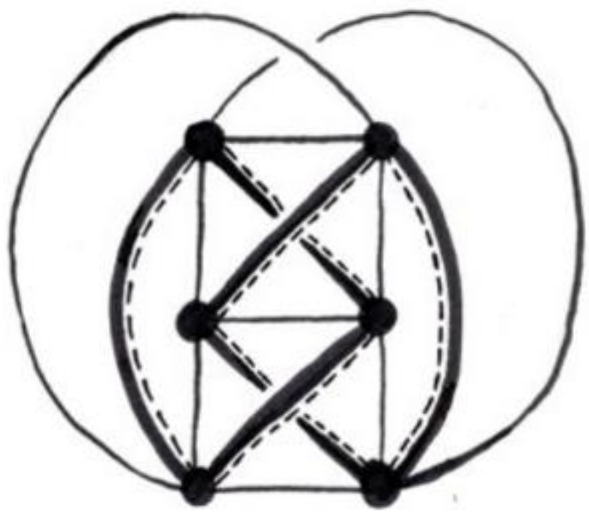

Fig 10

Embedding of K6with exactly one pair of linked cycles (indicated)

Definition 3. A graph G is intrinsically linked (IL) if for all embeddings of G there exists some non-trivial link in the given embedding (on a vertex disjoint union of simple cycles). We say a graph is not intrinsically linked (NIL)

***When is graph intrinsically knotted*:**

The goal of this topic is to develop a systematic method of proving that graphs are intrinsically knotted. To understand how we would go about such a systematization. Let's first focus on the simpler problem of proving that a graph G is IL. To behind, let's write our desired statement and unpack it. The graph is IL

This is equivalent to saying $\leftrightarrow$ For every embedding E of G there is some pair (A,B) of vertex disjoint simple cycle (VDSC$_s$) such that (A,B) is a non-trivial link in k.

Using logical quantifiers to abbreviate, we get $\leftrightarrow$ embedding E : ∃ (A,B) ∍ VDSC$_s$(A,B) is non-trivial in E.

we made use of the linking number mod 2 to detect linked cycles logically speaking we used to statement "lk_E(AB) =1 mod 2 → (A,B) is a non-trivial link in E"

Applying this implication here, it suffices to prove that ⇐=∀ embeddings E : ∃ (A,B)VDSCs: lk_E(A,B) = 1 mod 2

Now, observe that the inner most quantified statement "lk_E (A,B) = 1 mod 2" only refers to the embedding E via lk_E. This suggests one last rewriting.
⇐⇒ ∀lk_E :∃ (A,B) VDSCs: lk_E(A,B) = 1 mod 2

Finally let's codify this lemma

Lemma 1. Let G be a graph. If ∀ lk_E :∃(A,B) VDSCs : lE (A,B) = 1 mod 2 then G is intrinsically linked. Although non-obvious, Robertson and Seymour's results on intrinsic linking imply that the converse is also true. Unsurprisingly, we can execute a similar chain of logic for intrinsic knotting.

The graph G is IK.

Unpacking the definition... ⇐⇒ For every embedding E of G there is some simple cycle C such that C is knotted in E.

Switching to quantifiers... ⇐⇒ ∀ embeddings E :∃cycle C : C is knotted in E Let's focus on the inner statement here: ∃ cycle C : C is knotted in E . In Foisy's proof that K3,3,1,1 is IK we didn't look for knotted cycles just anywhere. Rather, we found some Foisy minor of G such that the sum of Arf invariants α = 1 mod 2in E, implying the existence of some knotted cycle.

And we did that by instead showing that F is "doubly linked" (lk(A,B) = 1 and lk(C,D) = 1), subsequently appealing to Foisy's lemma (F is "doubly linked" =⇒ knotted cycle in F). Using this argument it suffices to prove that ⇐=∀ embeddings E : ∃Foisy minor F : F is "doubly linked" in E or if we expand the definitions of these concepts...

⇐⇒ ∀ embeddings E :∃ Foisy pair of pairs ((A,B),(C,D)) : lk_E (A,B) = 1 mod 2 and lk_E(C,D) = 1 mod 2 of course we can just replace our quantification over E with a quantification over lk_E as before. Combined with some other tidying, we get

⇐⇒ ∀ lk_E :∃ Foisy pair of pairs ((A,B),(C,D)) : lk_E(A,B)·lk_E(C,D) = 1 mod 2 Again, we can codify the argument into a lemma. Lemma 3.0.2. Let G be a graph. If ∀ lk_E :∃ Foisy pair of pairs ((A,B),(C,D)) : lk_E(A,B)· lk_E (C,D) = 1 mod 2 then G is intrinsically knotted. We will further refine our two current lemmas until their hypotheses become tractably computable statements. At that point the hypotheses will resemble linear algebra problems, and we'll be able to write a computer program that can ascertain their truth for arbitrary graphs G.

Because Robertson and Seymour's result implies that the converse of our intrinsic linking lemma is true, the program we develop for intrinsic linking will always decide intrinsic linking for a graph G. That is the program will always determine "yes, G is IL" or "no, G is not IL." By contrast, the program we develop for intrinsic knotting will only sometimes decide intrinsic knotting. That is, the program will sometimes answer "yes, G is IK" but might just say "sorry, G may or may not be IK." However, if the converse of the intrinsic knotting lemma is true, then our program always determines intrinsic knotting for intrinsically knotted graphs. This gives us the first open question generated by this paper.

Conclusion :

In these paper we discuss a brief introduction of knot theory and it's morden real life applications of knot theory. Particularly knot theory in DNA research. And also the relavent subject to knot theory is a graph theory. In these paper we pointed when knot theory meets the graph theory and it's further development.

References :

1) The knot book, C.Adams text American mathematical society
2) M.Epple, topology mater and space, Topological nutions in 19^{th} century natural philosophy
3) J.C Maxwell , The scientific letters and papers of James clerk Maxwell vol 2, combridge university.
4) P.T England, J. Chen , N.R cozzarelli, J.H White the Topology of the KInetoplast DNA network