# AN INVESTIGATION ON THE DYNAMICS OF GROUNDHOOK DAMPED DVA ATTACHED TO A NONLINEAR SYSTEM

[1]Visakh S Kumar, [2]Dr Y V K S Rao
[1]MTech Scholar, [2]Rted. Professor in Mechanical Engineering
[1]Department of Mechanical Engineering,
[1]Mar- Baselios College of Engineering, Trivandrum, India

***Abstract :*** In this study the nonlinear analysis of 2 DOF vibration system with weakly nonlinear spring is studied. Different from conventional way, the DVA damper is connected between the absorber mass and the ground. A simple Ground hook damped DVA is thus employed to suppress the nonlinear vibrations of forced nonlinear system for the primary resonance conditions. The effects of the linked spring, the damper and the attached mass on the reduction of nonlinear vibration are being studied with the help of frequency response curves, time plots and phase planes.

***IndexTerms* –Degrees of Freedom, Dynamic Vibration Absorber.**

## I. INTRODUCTION

The nonlinear analysis becomes necessary whenever finite amplitudes of motion are encountered. In a SDOF nonlinear structure subjected to external excitation, a small amplitude excitation may produce a relatively large amplitude response under primary resonance conditions. A simple mass- spring-damper [1] vibration absorber is thus employed to suppress the nonlinear vibrations. Kefu Liu and Gianmarc Coppola [2] proposed an optimum design of damped DVA for effectively reducing vibrations. They designed an absorber in which damper is connected directly to the ground instead of primary mass. The characteristics of the nonlinear system attached by the linear absorber change only slightly in terms of the values of its new linearized natural frequency, damping coefficient and frequency interval. Studying a system with nonlinear spring will reduce the vibration amplitude that can be obtained by selecting the parameters proposed by J C Ji and N Zhang [3]. Various methods of solving the nonlinear vibration problems are Lindstedt's perturbation method, the iterative method and the Ritz- Galerkin method. One of the perturbation methods known as straightforward expansion is used to obtain the first order approximate solutions to primary resonance vibrations of the forced nonlinear structure.

Our main aim in this paper is to analyze the nonlinearity of a vibration system with weakly nonlinear spring. The approximate analytical solution for nonlinear system with nonlinear spring is carried out. The time plots with displacement are also plotted. A comparison of plots is made with the computer generated solution obtained from MATLAB to ensure the validation of the solution.

***Abbreviations***

DOF- Degrees of freedom
DVA- Dynamic Vibration Absorber

## II. RESEARCH METHODOLOGY

### 2.1 Mathematical Modeling Using Nonlinear Spring

It is assumed here that a 1dof weakly nonlinear system may be described as one which consists of a mass subjected to a periodic excitation. A significantly lighter mass $m_a$ (in comparison with the primary mass M), which will be referred as a small attachment, connected to the nonlinear system through a massless spring. The secondary system referred as the damped vibration absorber. It is noted that the mass less damper is connected directly to the ground. Hence the damped DVA is sometimes called ground hook damped DVA. The addition of the absorber mass to the nonlinear primary system results in a new 2dof nonlinear system. The mass M is attached to a rigid boundary through a viscous damper and a spring of linear plus nonlinear characteristic, as shown in **Figure 2.1**.

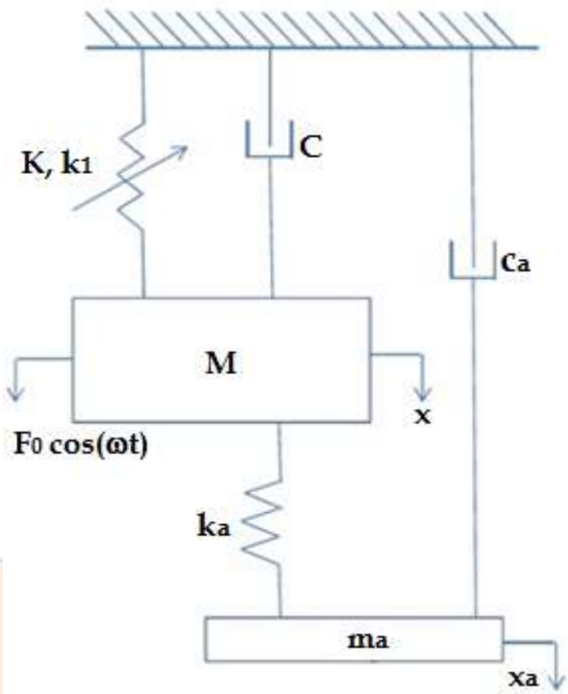


**Fig.2.1 System with Nonlinear spring**

The displacement of the nonlinear primary system & the linear absorber system are denoted by x & $x_a$ respectively. By applying Newton's second law of motion, two equations of motion for the new system composed of the nonlinear primary system incorporated by a linear absorber system may be written as

$$M\ddot{x} + Kx + k_1x^3 - k_a(x_a - x) + C\dot{x} = F_0\cos\omega t \qquad (1)$$

$$m_a\ddot{x}_a + k_a(x_a - x) + c_a\dot{x}_a = 0 \qquad (2)$$

Where M, K, $k_1$, C and $m_a$,$k_a$, $c_a$ are the system parameters for primary nonlinear system and secondary absorber system respectively. Dividing M on both sides of Eq.1 & dividing $m_a$ on both sides of Eq.2 & then rewriting the resultant equations yields the following equations.

$$\ddot{x} + \left(\frac{K+k_a}{M}\right)x + \left(\frac{k_1}{M}\right)x^3 - \left(\frac{k_a}{M}\right)x_a + \left(\frac{C}{M}\right)\dot{x} = \left(\frac{F_0}{M}\right)\cos\omega t \qquad (3)$$

$$\ddot{x}_a + \left(\frac{k_a}{m_a}\right)(x_a - x) + \left(\frac{c_a}{m_a}\right)\dot{x}_a = 0 \qquad (4)$$

$$\ddot{x} + \omega_1^2x + \epsilon x^3 - m\omega_a^2x_a + \mu_1\dot{x} = F\cos\omega t \qquad (5)$$

$$\ddot{x}_a + \omega_a^2(x_a - x) + \mu_2\dot{x}_a = 0 \qquad (6)$$

The solution x of our problem is a function of the independent variable t and the parameter, $\epsilon$ i.e. x = x(t;$\epsilon$). One of the perturbation method known as the straight forward expansion is used to expand the above equations to determine the analytical solution. The straight forward expansion in the form of a power series in $\epsilon$ is given by

$$x(t;\epsilon) = x_0(t) + \epsilon x_1(t) + \epsilon^2x_2(t) + \epsilon^3x_3(t) + \cdots \qquad (7)$$

Here only the first term in the correction series is considered and neglecting the higher order terms, so that the approximate solution in the form

$$x(t;\epsilon) = x_0(t) + \epsilon x_1(t) \qquad (8)$$

Even if Eq.6 looks like linear, it may have the effect of non-linearity due to the nonlinearity of primary system. So that both of these equations Eq.5 and Eq.6 are to be expanded simultaneously with straight forward expansion.
Substituting Eq.8 into Eq.5 & Eq.6,

$$(\ddot{x}_0 + \epsilon\ddot{x}_1 + \cdots) + \omega_1^2(x_0 + \epsilon x_1 + \cdots) + \epsilon(x_0 + \epsilon x_1 + \cdots)^3$$

$$+\mu_1(\dot{x}_0 + \epsilon\dot{x}_1 + \cdots) = F\cos\omega t + m\omega_a^2(x_{a0} + \epsilon x_{a1} + \cdots) \qquad (9)$$

$$(\ddot{x}_{a0} + \epsilon\ddot{x}_{a1} + \cdots) + \omega_a^2(x_{a0} + \epsilon x_{a1} + \cdots) + \mu_2(\dot{x}_{a0} + \epsilon\dot{x}_{a1} + \cdots)$$

$$= \omega_a^2(x_0 + \epsilon x_1 + \cdots) \qquad (10)$$

Equating each of the coefficients of $\epsilon^0$ & $\epsilon^1$ to zero

$$\ddot{x}_0 + \omega_1^2 x_0 + \mu_1\dot{x}_0 = F\cos\omega t + m\omega_a^2 x_{a0} \qquad (11)$$
$$\ddot{x}_1 + \omega_1^2 x_1 + {x_0}^3 + \mu_1\dot{x}_1 = m\omega_a^2 x_{a1} \qquad (12)$$
$$\ddot{x}_{a0} + \omega_a^2 x_{a0} + \mu_2\dot{x}_{a0} = \omega_a^2 x_0 \qquad (13)$$
$$\ddot{x}_{a1} + \omega_a^2 x_{a1} + \mu_2\dot{x}_{a1} = \omega_a^2 x_1 \qquad (14)$$

Since Eq.11 is inhomogeneous, its general solution can be obtained as the sum of a homogeneous solution and any particular solution. It is observed that Eq.11 & Eq.13 are linear equations, so that the homogeneous solution can be expressed as

$$x_0 = X_0\cos(\omega t - \phi) \;\&\; x_{a0} = X_{a0}\cos(\omega t - \phi) \qquad (15)$$

Using trigonometric relations, $\cos(\omega t - \phi) = \cos\omega t\cos\phi + \sin\omega t\sin\phi$ &
$$\sin(\omega t - \phi) = \sin\omega t\cos\phi - \cos\omega t\sin\phi \qquad (16)$$

Substituting Eq.15 & using Eq.16 in Eq.11 & Eq.13 & equating the coefficients of cos(ωt) & sin(ωt), the amplitude can be determined as

$$X_0 = \frac{F}{\left[\left\{(\omega_1^2-\omega^2) - \frac{1}{\omega_a^2-\omega^2}m\omega_a^4\right\}^2 + \mu_1^2\omega^2\right]^{\frac{1}{2}}} \qquad (17)$$

The particular solution is also expected to be harmonic, therefore the steady state solution is

$$x_p(t) = \frac{F}{\left[\left\{(\omega_1^2-\omega^2) - \frac{1}{\omega_a^2-\omega^2}m\omega_a^4\right\}^2 + \mu_1^2\omega^2\right]^{\frac{1}{2}}}\cos(\omega t - \phi) \qquad (18)$$

& the phase angle, $\phi = \tan^{-1}[\frac{\mu_1\omega}{\{(\omega_1^2-\omega^2)-\frac{1}{\omega_a^2-\omega^2}m\omega_a^4\}}]$ (19)

The steady state vibration of the system is shown in **Figure 2.2**. The complete solution is given by $x_0(t) = x_h(t) + x_p(t)$. For an underdamped system $x_h(t)$ is given by

$$x_h(t) = Ae^{(-\xi\omega_1 t)}\cos(\omega_d t - \psi) \quad (20)$$

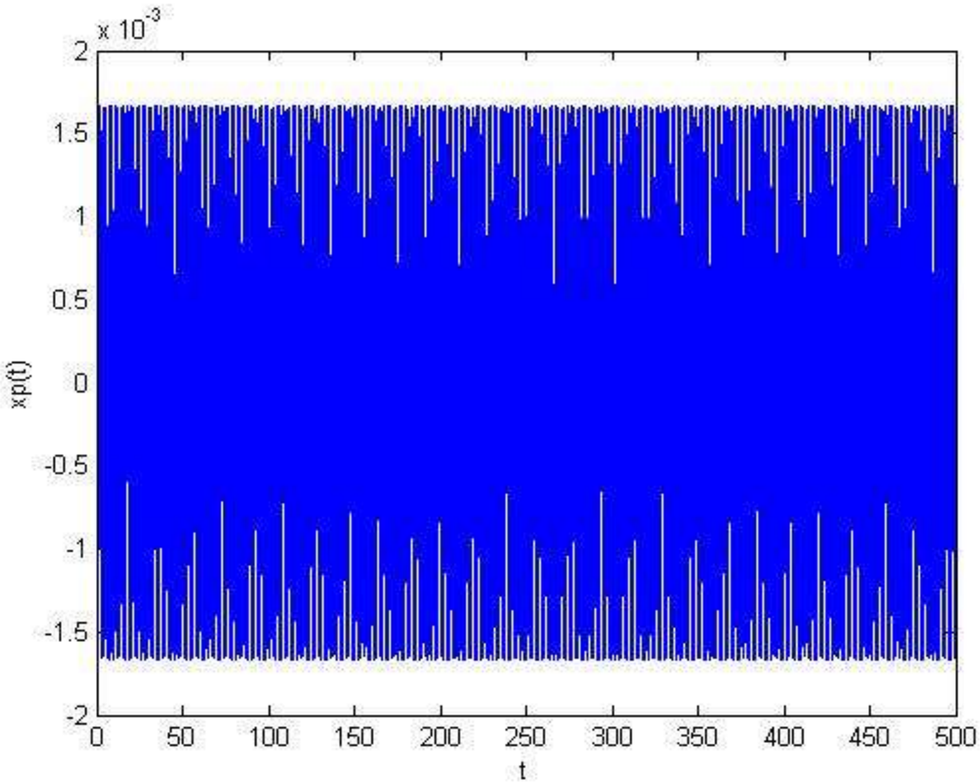


**Fig.2.2**

**Steady state solution**

$$x_0(t) = x_h(t) + x_p(t)$$

$$x_0(t) = Ae^{(-\xi\omega_1 t)}\cos(\omega_d t - \psi) + X_0\cos(\omega t - \phi) \quad (21)$$

$$A = \sqrt{A_1 + A}_2$$

$X_0$ & $\phi$ are given by Eq.17 & Eq.19 respectively, A, $A_1$, $A_2$ & $\psi$ can be determined from the initial conditions, $x(t=0) = x_0$ & $\dot{x}(t=0) = \dot{x}_0$.

$$A_1 = x_0 \ \& \ A_2 = \frac{\dot{x}_0 + \xi\omega_1 x_0}{\sqrt{1-\xi^2}\omega_1} \quad (22)$$

$$\psi = \tan^{-1}\left(\frac{A_2}{A_1}\right) \quad (23)$$

The motion described by Eq.20 is a damped harmonic motion of angular frequency $\sqrt{1-\xi^2}\omega_1$, but because of the factor $e^{(-\xi\omega_1 t)}$, the amplitude decreases exponentially with time as shown in **Figure 2.3**. The quantity $\sqrt{1-\xi^2}\omega_1$, is called the frequency of damped vibration. It can be seen that the frequency of damped vibration $\omega_d$ is always less than $\omega_1$.The under-damped case is very important in the study of mechanical vibrations, as it is the only case that leads to an oscillatory motion.

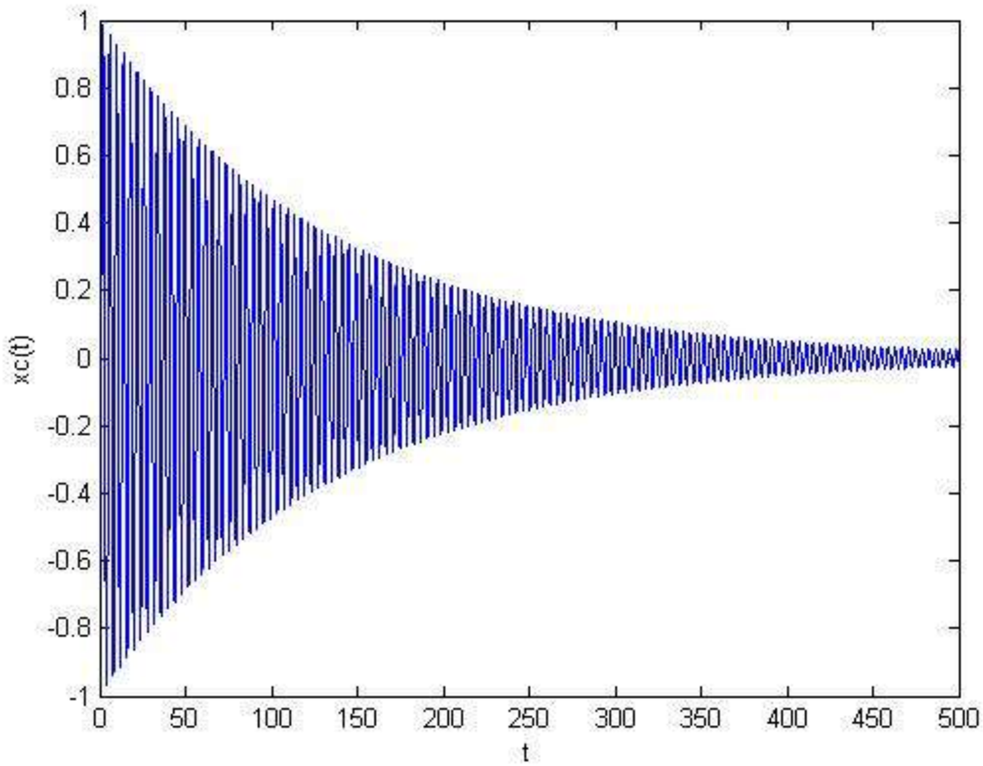


**Fig.2.3 Response of Complementary function**

To find $x_1(t)$, solve for unknowns $x_1$ & $x_{a1}$. Eq.12 & Eq.14 on rearranging gives

$$\ddot{x}_1 + \omega_1^2 x_1 + \mu_1 \dot{x}_1 - m\omega_a^2 x_{a1} = -({x_0}^3) \quad (24)$$

$$\ddot{x}_{a1} + \omega_a^2 x_{a1} + \mu_2 \dot{x}_{a1} - \omega_a^2 x_1 = 0 \quad (25)$$

$$(D^2 + \omega_1^2 + D\mu_1)x_1 - m\omega_a^2 x_{a1} = -(x_0^3) \quad (26)$$

$$(D^2 + \omega_a^2 + D\mu_2)x_{a1} - \omega_a^2 x_1 = 0 \quad (27)$$

In order to find the solution of $x_1$ & $x_{a1}$ from the above Eq..26 & Eq.27, multiply Eq.26 by $\omega_a^2$ & Eq.27 by $(D^2 + \omega_1^2 + D\mu_1)$ and then adding the resulting equations. will give

$$(D^2 + \omega_1^2 + D\mu_1)(D^2 + \omega_a^2 + D\mu_2)x_{a1} - m\omega_a^4 x_{a1} = -(x_0^3)\omega_a^2 (28)$$

$$(D^4 + D^2(\omega_a^2 + \omega_1^2 + \mu_1\mu_2) + D^3(\mu_1 + \mu_2) + D(\mu_1\omega_a^2 + \mu_2\omega_1^2) + \omega_1^2\omega_a^2 - m\omega_a^4)x_{a1} = -(x_0^3)\omega_a^2$$

$$(D^4 + C_1D^3 + C_2D^2 + C_3D + C_4)x_{a1} = -(X_0^3 \cos^3(\omega t - \phi))\ \omega_a^2$$

$$(D^4 + C_1D^3 + C_2D^2 + C_3D + C_4)x_{a1} = C_5 \cos^3(\omega t - \phi) \quad (29)$$

Using trigonometric relation

$$\cos^3(\omega t - \phi) = \frac{3}{4}\cos(\omega t - \phi) + \frac{1}{4}\cos 3(\omega t - \phi) \quad (30)$$

Substituting Eq.30 into Eq.29, will give

$$x_{a1} = \frac{1}{(D^4+C_1D^3+C_2D^2+C_3D+C_4)}(\frac{3C_5}{4}\cos(\omega t - \phi) + \frac{C_5}{4}\cos 3(\omega t - \phi)$$

$$x_{a1p} = \frac{1}{(D^4+C_1D^3+C_2D^2+C_3D+C_4)}(C_6 \cos(\omega t - \phi) + C_7 \cos 3(\omega t - \phi)(31)$$

$$x_{a1p} = PI_1 + PI_2 \quad (32)$$

$$PI_1 = \frac{1}{(D^4+C_1D^3+C_2D^2+C_3D+C_4)}(C_6\cos(\omega t-\phi)) \quad (33)$$

Put $D^2 = -\omega^2$

$$PI_1 = \frac{1}{(\omega^4 - C_2\omega^2 + C_4 + (C_3 - C_1\omega^2)D)}(C_6\cos(\omega t-\phi)$$

$$PI_1 = \frac{1}{(C_8 + C_9D)}(C_6\cos(\omega t-\phi)$$

Multiply both numerator and denominator by the conjugate of $(C_8 + C_9D)$ & on solving gives

$$PI_1 = \frac{1}{(C_8^2+C_9^2\omega^2)}(\omega C_6C_9\sin(\omega t-\phi) + C_6C_8\cos(\omega t-\phi)) \quad (34)$$

$$PI_2 = \frac{1}{(D^4+C_1D^3+C_2D^2+C_3D+C_4)}(C_7\cos 3(\omega t-\phi)) \quad (35)$$

Put $D^2 = -9\omega^2$

$$PI_2 = \frac{1}{(81\omega^4 - 9C_2\omega^2 + C_4 + (C_3 - 9C_1\omega^2)D)}(C_7\cos 3(\omega t-\phi))$$

$$PI_2 = \frac{1}{(C_{10}+C_{11}D)}(C_7\cos 3(\omega t-\phi)) \quad (36)$$

Multiply both numerator and denominator by the conjugate of $(C_{10} + C_{11}D)$ & on solving gives

$$PI_2 = \frac{1}{(C_{10}^2+9C_{11}^2\omega^2)}(C_7C_{10}\cos 3(\omega t-\phi) + 3\omega C_7C_{11}\sin 3(\omega t-\phi)) \quad (37)$$

$$x_{a1p} = \frac{1}{(C_8^2+C_9^2\omega^2)}\left(\omega C_6C_9\sin(\omega t-\phi) + C_6C_8\cos(\omega t-\phi)\right) + \frac{1}{(C_{10}^2+9C_{11}^2\omega^2)}(C_7C_{10}\cos 3(\omega t-\phi) + 3\omega C_7C_{11}\sin 3(\omega t-\phi)) \quad (38)$$

$$x_{a1p} = C_{12}\sin(\omega t-\phi) + C_{13}\cos(\omega t-\phi) + C_{14}\cos3(\omega t-\phi) + C_{15}\sin3(\omega t-\phi) \quad (39)$$

Substitute Eq.39 into Eq.27 & on solving gives

$$x_{1p}(t) = \frac{\omega^2}{\omega_a^2}(-C_{12}\sin(\omega t-\phi) - C_{13}\cos(\omega t-\phi) - 9C_{14}\cos3(\omega t-\phi) - 9C_{15}\sin3(\omega t-\phi) + \left(C_{12}\sin(\omega t-\phi) + C_{13}\cos(\omega t - \phi) + C_{14}\cos3(\omega t-\phi) + C_{15}\sin3(\omega t-\phi)\right) + \frac{\mu_2\omega}{\omega_a^2}(C_{12}\cos(\omega t-\phi) - C_{13}\sin(\omega t-\phi) - 3C_{14}\sin3(\omega t-\phi) + 3C_{15}\cos3(\omega t-\phi) \quad (40)$$

The complete solution is given by $x_1(t) = x_{1h}(t) + x_{1p}(t)$. For an underdamped system $x_{1h}(t)$ is given by

$$x_{1h}(t) = Ae^{(-\xi\omega_1 t)}\cos(\omega_d t-\psi) \quad (41)$$

Thus the approximate solution can be obtained by substituting Eq.21, Eq.40 & Eq.41 into Eq.8

$$x(t;\epsilon) = x_0(t) + \epsilon x_1(t) + \cdots$$

$$= Ae^{(-\xi\omega_1 t)}\cos(\omega_d t-\psi) + \frac{F}{\left[\left\{(\omega_1^2-\omega^2) - \frac{1}{\omega_a^2-\omega^2}m\omega_a^4\right\}^2 + \mu_1^2\omega^2\right]^{\frac{1}{2}}}\cos(\omega t-\phi) + \epsilon\left[\frac{\omega^2}{\omega_a^2}\left(-C_{12}\sin(\omega t-\phi) - C_{13}\cos(\omega t-\phi) - 9C_{14}\cos3(\omega t-\phi) - 9C_{15}\sin3(\omega t-\phi) + \left(C_{12}\sin(\omega t-\phi) + C_{13}\cos(\omega t - \phi) + C_{14}\cos3(\omega t-\phi) + C_{15}\sin3(\omega t-\phi)\right) + \frac{\mu_2\omega}{\omega_a^2}\left(C_{12}\cos(\omega t-\phi) - C_{13}\sin(\omega t-\phi) - 3C_{14}\sin3(\omega t-\phi) + 3C_{15}\cos3(\omega t-\phi)\right)\right) + Ae^{(-\xi\omega_1 t)}\cos(\omega_d t-\psi)\right] \quad (42)$$

## III. VALIDITY OF THE SOLUTION

### 3.1 Analytical Solution

This section deals with the comparison of the time plots obtained from analytical solution with the plot directly obtained from the mat lab, so that the validity of the solution can be assessed. The time plots obtained from the analytical solutions for the system parameters M=10kg, $m_a$=0.8kg, K=15N/m, $k_a$=10N/m, C=0.15Ns/m, $c_a$=0.03Ns/m, $k_1$=8N/$m^3$, $c_1$=0.1Ns/$m^3$, $F_0$=5.5 are shown below

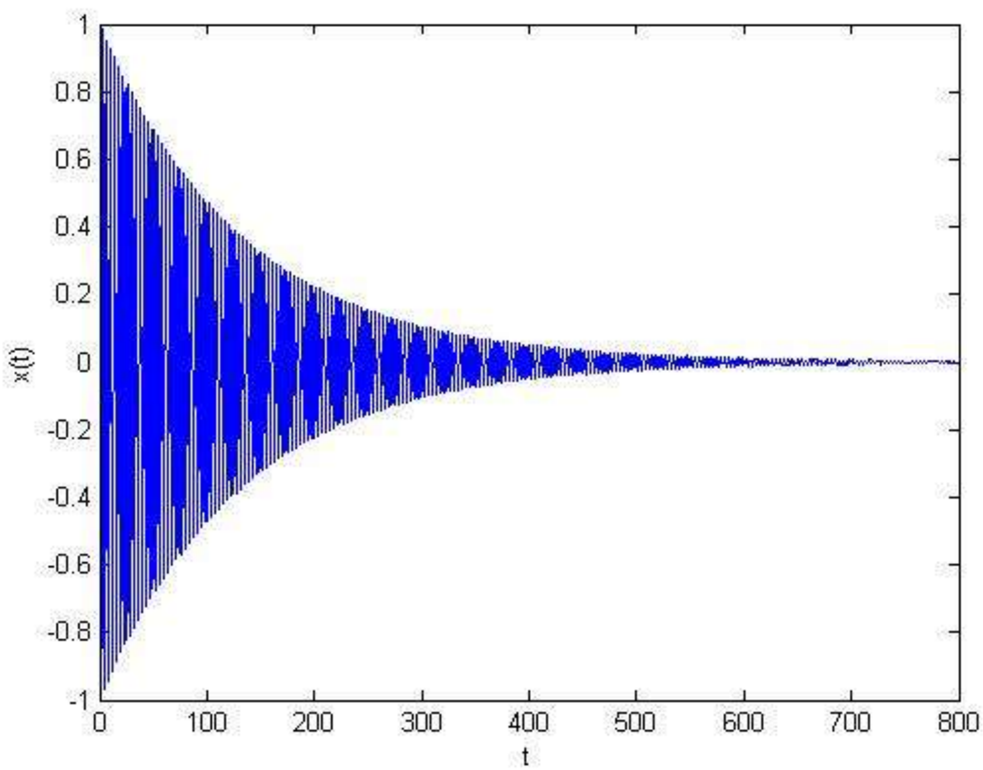


**Fig.3.1 Response of the system with nonlinear spring**

### 3.2. MAT-LAB Solution

In equations of motion, take $x = y(1)$, $\dot{x} = y(2)$, $x_a = y(3)$ & $\dot{x}_a = y(4)$, so that the derivatives of y(1), y(2), y(3) & y(4) are y(2), $\ddot{x}$, y(4) & $\ddot{x}_a$. For obtaining the time plots, y(2), $\ddot{x}$, y(4) & $\ddot{x}_a$ are taken as functions f(1), f(2), f(3) & f(4) respectively. By using these functions different time plots with displacement, velocity, acceleration of the system can be plotted.

For the system with nonlinear spring, say

$$f(1) = y(2),$$

$$f(2) = \frac{F_0}{M}\cos\omega t - \frac{K + K_a}{M}y(1) - \frac{k_1}{M}y(1)^3 + \frac{k_a}{M}y(3) - \frac{C}{M}y(2),$$

$$f(3) = y(4),$$

$$f(4) = -\frac{k_a}{m_a}\big(y(3) - y(1)\big) - \frac{c_a}{m_a}y(4)$$

The time plot obtained from these functions using the same system parameters as shown in **Figure 3.2**.

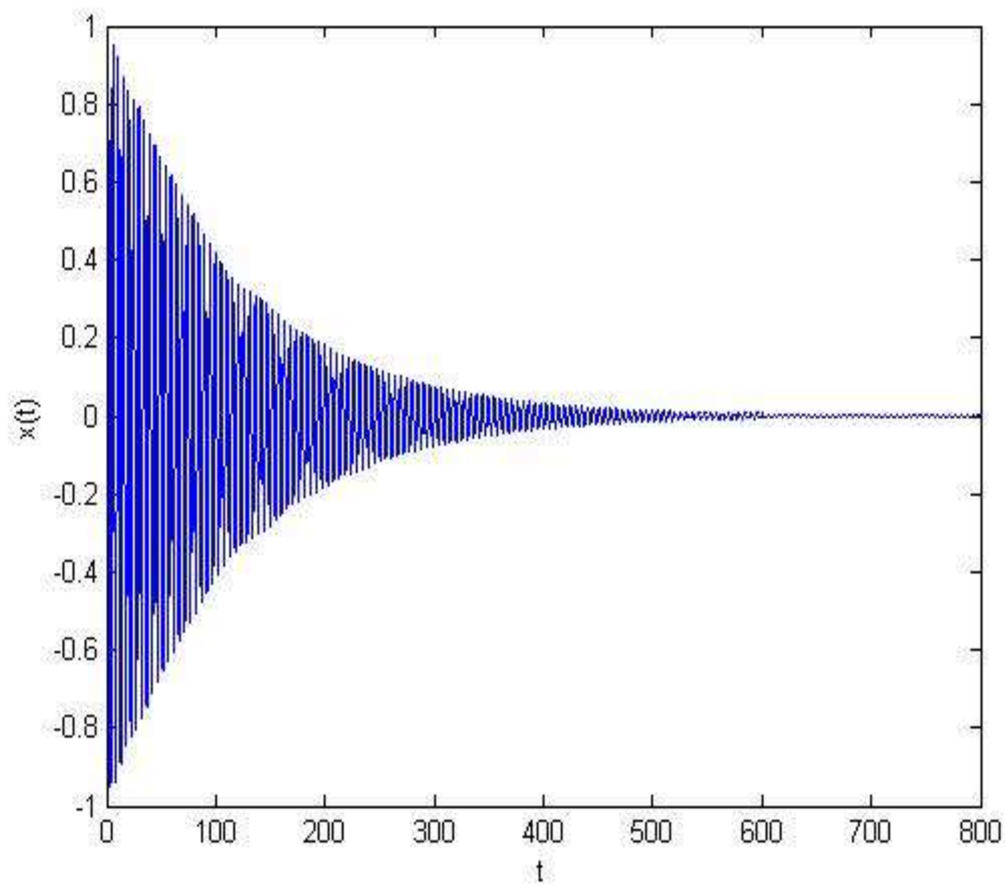


**Fig.3.2 Response of the system with nonlinear spring**

It is seen that the plots are almost identical in various situations which ensures the validity of the analytical solutions. The slight discrepancy seen in the graphs are due to the approximations that already taken earlier in Eq.8.

## IV. RESULTS AND DISCUSSION

### 4.1. Numerical Simulations

This section presents illustrative examples to show the effectiveness of the linear vibration absorber for suppressing the nonlinear vibrations of the nonlinear oscillator under primary resonance conditions. The effects of the linked spring and damper and the attached mass on the reduction of nonlinear vibration will be interpreted by the frequency –response curves, time plots and phase plane.

Numerical simulations have been performed under the following values of the system parameters shown in **Table 4.1**. This combination of system parameters indicates that the mass ratio, m=0.06(i.e., $m_a/M$).This set of system parameters confirms a small mass attachment to the nonlinear primary system. The linearized natural frequencies of the nonlinear primary system before and after being attached by the vibration absorber are found to be approximately, $\omega_{10}$=2.098 rad/sec, $\omega_1$=2.280 rad/sec and natural frequency of the absorber be $\omega_a$=3.651 rad/sec.

The selection of the parameters of the linear vibration absorber made in the present paper is thus distinct from the one for controlling the linear vibrations of linear systems in the sense that for controlling linear vibrations, the natural frequencies of the resulting system composed of the linear system attached by vibration absorber are designed to be away from the excitation frequency. For the nonlinear system considered in the present paper, due to its distinct nature in primary resonances from the dynamics of linear system, there is no need to shift the linearized natural frequency of the nonlinear primary system away from the excitation frequency.

The nonlinear vibrations of the nonlinear oscillator under primary resonance conditions can be significantly reduced by adding a small attachment, which is expected to be feasible in practical applications.

**Table 4.1 System parameter values**

| Primary mass, M (kg) | Absorber mass, $m_a$ (kg) | Linear stiffness, K (N/m) | Absorber stiffness, $k_a$ (N/m) | Primary damping, C (Ns/m) | Absorber damping, $c_a$ (Ns/m) | Nonlinear stiffness, $k_1$ ($N/m^3$) | External Excitation, $F_0$ (N) |
|---|---|---|---|---|---|---|---|
| 10 | 0.6 | 44 | 8 | 0.1 | 0.08 | 2 | 4.5 |

Using the system parameters given above, the displacements of the primary system for different time period have been plotted as shown in **Figure 4.1.1**. It is observed that for a small value of damping the amplitude goes on decreasing with time. For time t=450s, the amplitude almost reaches zero.

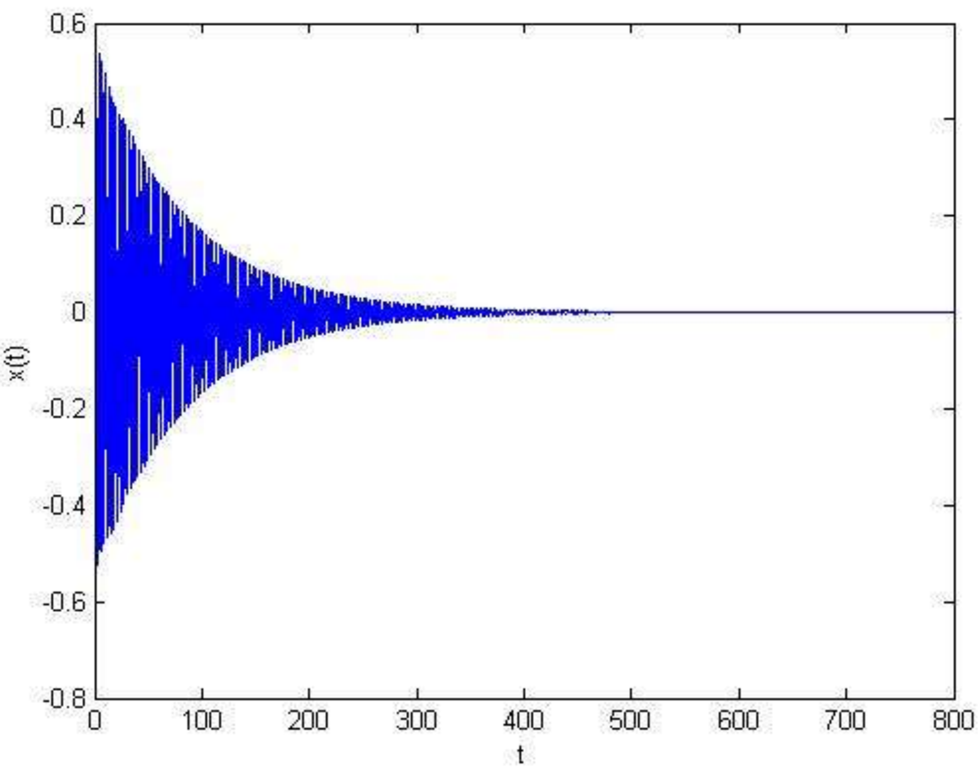


Fig.4.1.1 Response of the nonlinear system at $k_1$=2

The response of the nonlinear system for the same system parameters without absorber is shown in **Figure 4.1.2**. It is observed that addition of vibration absorber will suppress the vibration of the primary nonlinear system to a great extent.

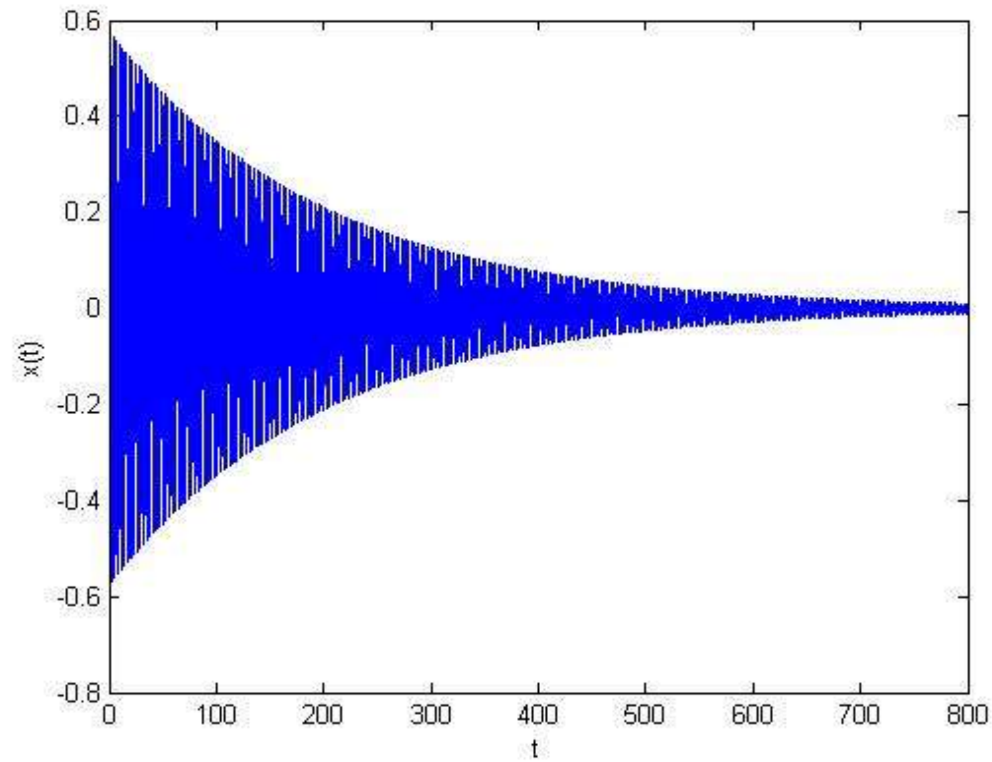


**Fig. 4.1.2 Response of the 1dof system**

The response of the system for different values of $k_1$ = 0, 8, 15, 30 are shown in **Figure 4.1.3**. When $k_1$=30N/$m^3$, the amplitude decreases. So it is clear from the plot that increasing the nonlinear stiffness values will gradually reduce the amplitude of the system and hence the vibration can be reduced with the effect of nonlinear spring.

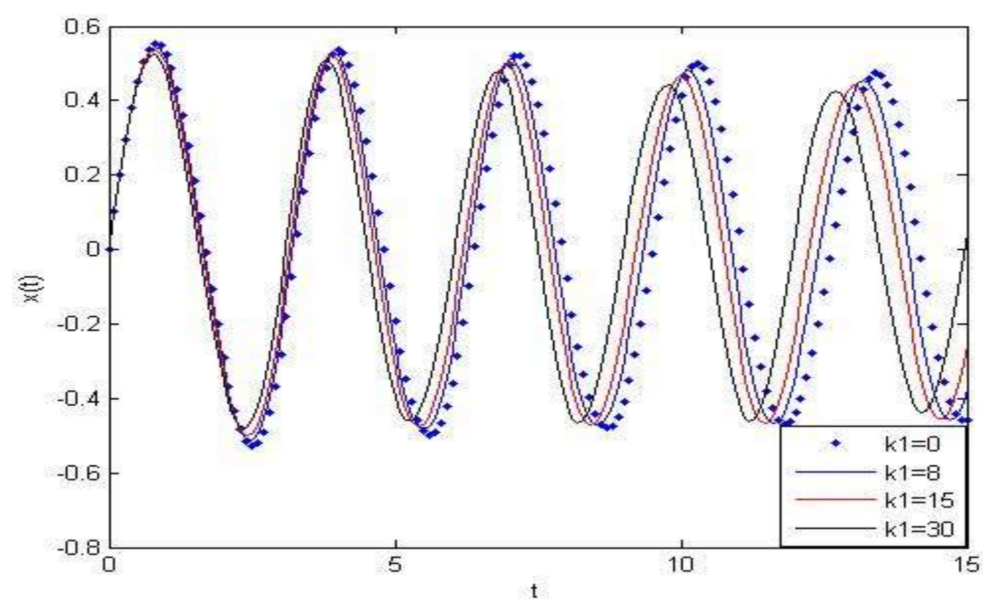


**Fig.4.1.3 Response of the nonlinear system at $k_1$=0, 8, 15 & 30**

### 4.2. Stability of the system

Graphical methods can be used to obtain qualitative information about the behaviour of the linear or nonlinear system and also to integrate the equations of motion. For any degree of freedom system, two parameters are needed to describe the state of motion completely. These parameters are usually taken as the displacement and velocity of the system. When the parameters are used as coordinate axes, the resulting graphical representation of the motion is called the phase plane representation. Thus each point in the phase plane represents a possible state of the system. As time changes, the state of the system changes. A typical point in the phase moves and traces a curve known as the trajectory. The trajectory shows how the solution of the system varies with time. Chaos represents the behaviour of a system that is inherently unpredictable. In other words, chaos refers to the dynamic behaviour of a system whose response, although described by a deterministic equation, becomes unpredictable because the nonlinearities in the equation enormously amplify the errors in the initial conditions of the system.

The **Figure 4.2.1** shows a computer generated solution in the phase plane for a nonlinear system without absorber.

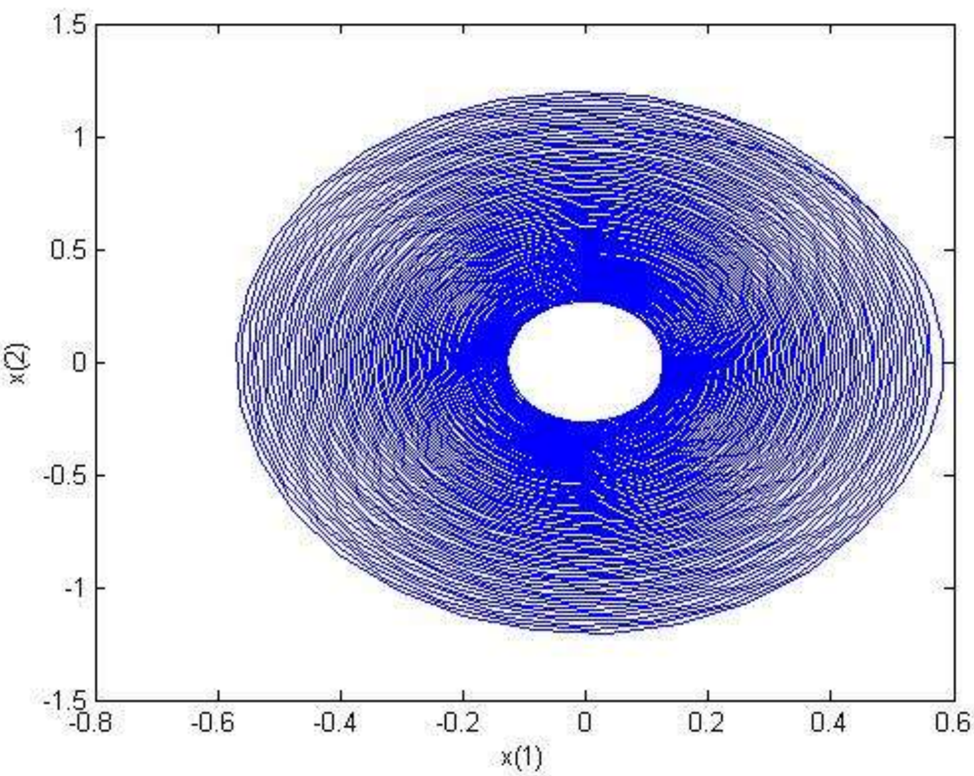


**Fig. 4.2.1 Phase plane for the nonlinear system without absorber**

The trajectory is a slowly elliptical spiral; it takes many cycles for the amplitude to decrease substantially. When the nonlinear system is attached with linear absorber, the no of cycles required for the amplitude to decrease substantially becomes reduced as shown in **Figure 4.2.2**. The amplitude of oscillation decreases gradually due to the damping of the system, which means that the system loses part of its energy in each cycle and eventually comes to a rest position. This shows that all the trajectories tend to the origin as t →infinity and hence the origin is called a stable node and the system behaviour is stable.

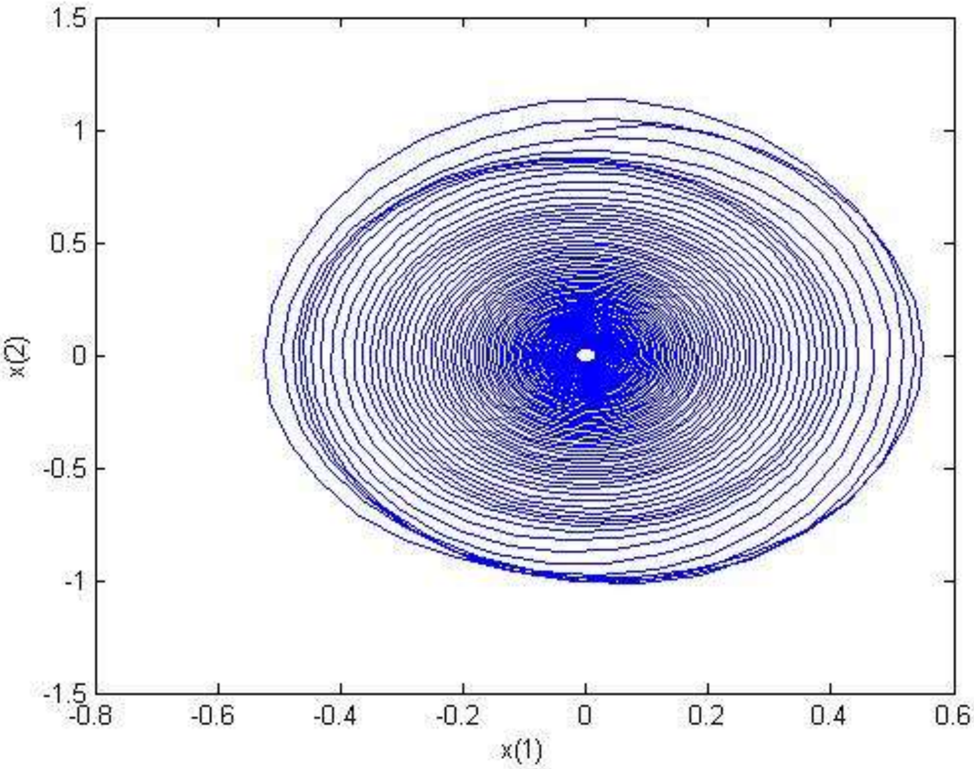


**Fig. 4.2.2 Phase plane for the nonlinear system with $k_1$=2**

But when the nonlinear stiffness coefficient is increased, the system remains stable & tuned with some chaos as shown in **Figure 4.2.3**.

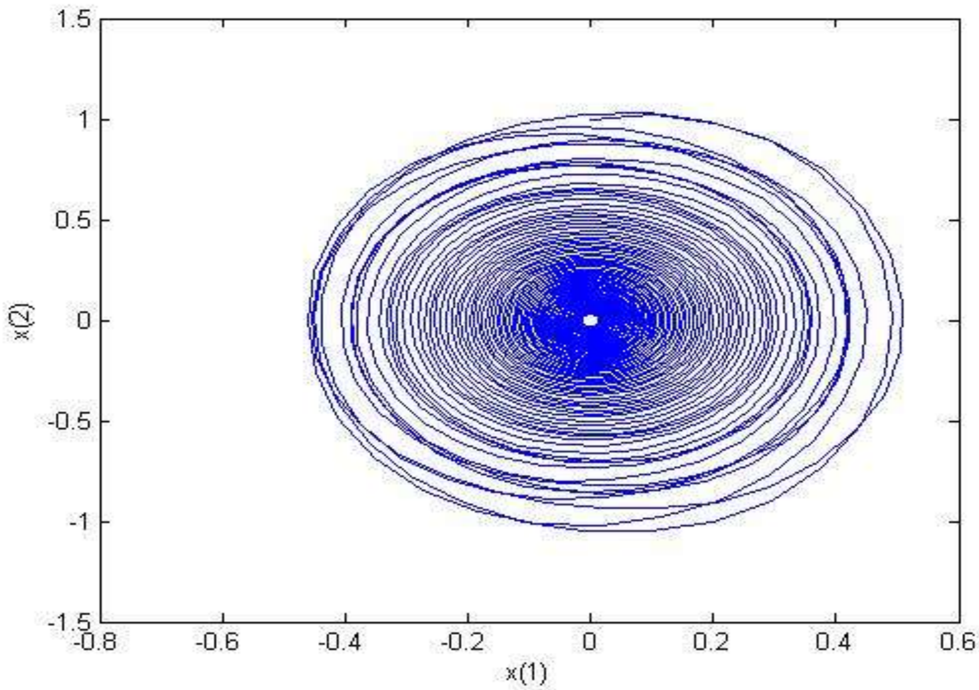


**Fig. 4.2.3 Phase plane for the nonlinear system with $k_1$=50**.

### 4.3. Frequency Response Curves

The amplitude spectrum for the system with nonlinear spring with the function of frequency as shown in **Figure 4.3.1**.The amplitude spectrum for the nonlinear system without absorber is also plotted in **Figure 4.3.2**. It is seen that the amplitude of nonlinear vibrations of the nonlinear primary oscillator has been greatly suppressed by adding the linear vibration absorber. The amplitude of the nonlinear system with absorber is not zero, but its amplitude is very small in comparing with the amplitude of vibrations of the nonlinear oscillator alone.

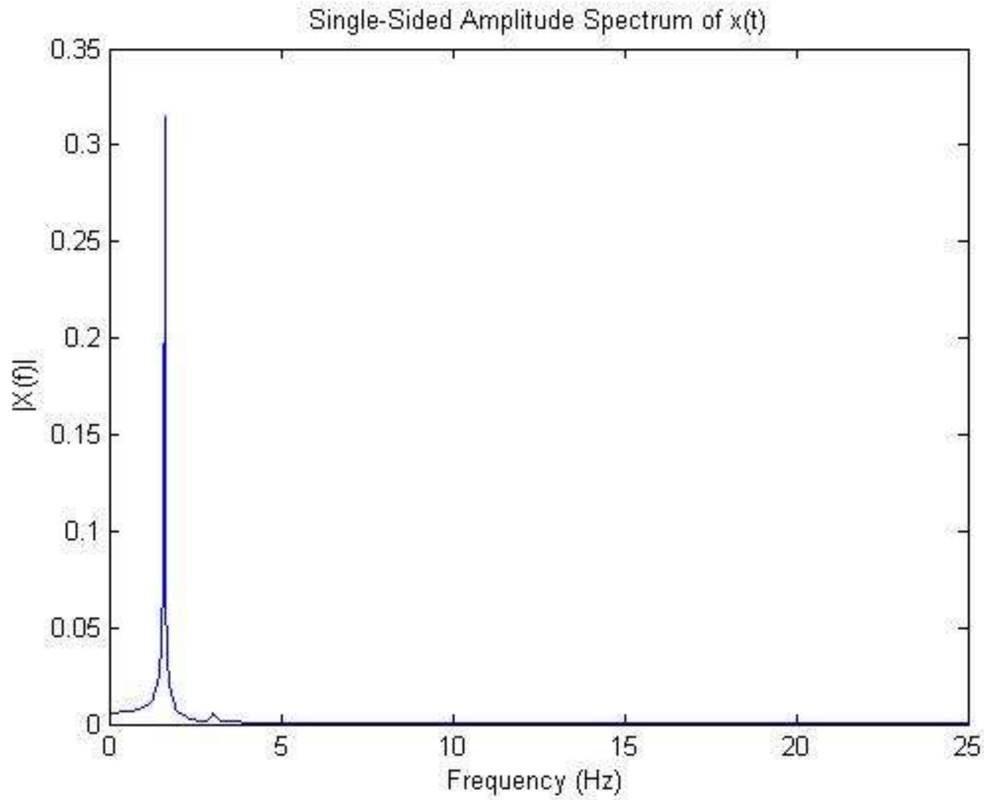


**Fig. 4.3.1 Single-sided Amplitude spectrum of x(t) (Nonlinear spring)**

It is seen that a small amplitude peak is observed. This is because the proposed nonlinear system is a 2DOF system so that the system has two linearized natural frequencies.

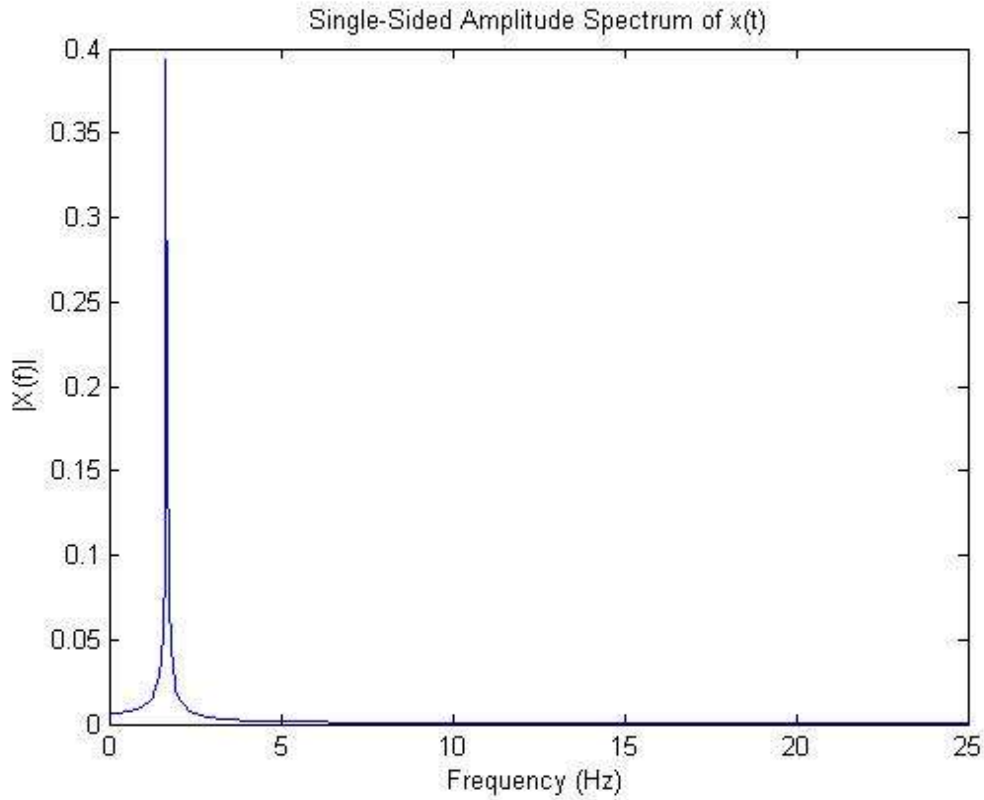


**Fig4.3.2. Single-sided Amplitude spectrum of x(t) (without absorber)**

It is noted that for a given set of system parameters, increase of absorber mass results in decrease in amplitude of the primary frequency as well as an increase in amplitude in the neighborhood of primary resonance frequencies. The two peaks in amplitude shift to the left is due to the increase in absorber mass as shown in **Figure 4.3.3**. This is because the natural frequency of the absorber decreases with an increase of absorber mass.

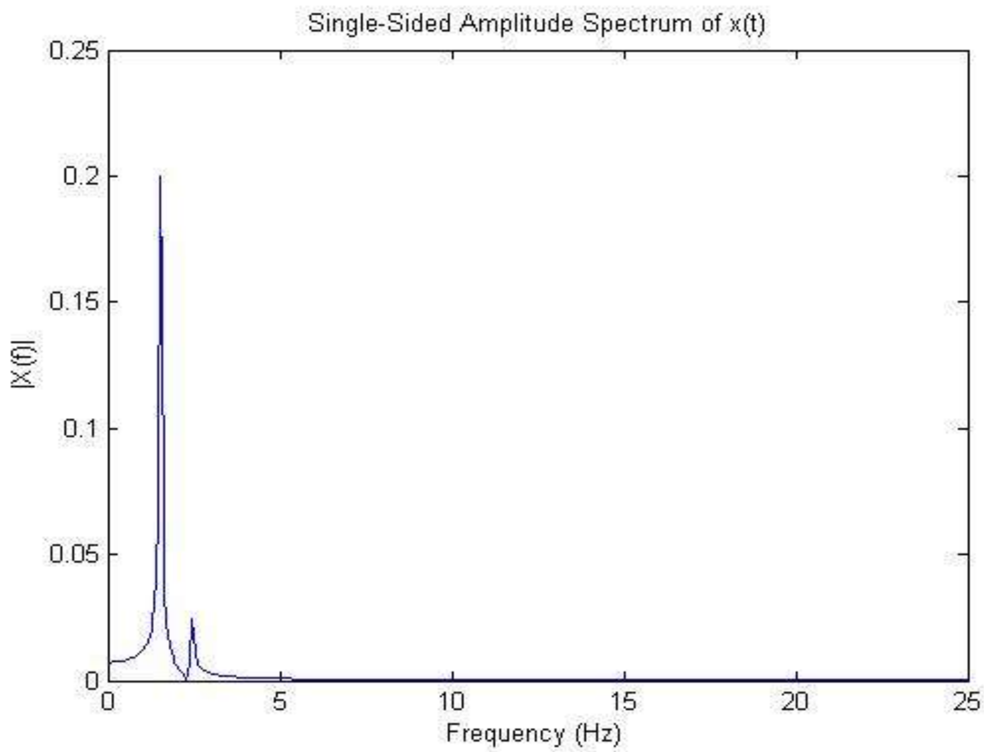


**Fig. 4.3.3 Single-sided Amplitude spectrum of x(t) for $m_a$=1**

## V. CONCLUSION

In this paper, the nonlinear analysis of a vibration system with weakly non-linear spring is studied. A linear vibration absorber is attached to the nonlinear system which results in a new 2DOF weakly nonlinear system. One of the perturbation methods known as straight forward expansion is used to find the first order approximate solutions to primary resonance vibrations of the forced nonlinear structure.

Neglecting the higher order terms in the expansion, the approximate analytical solution for the proposed model is derived. The time plots with displacement are also plotted. A comparison of plots is made with the computer generated solution obtained from mat lab. It is found that the plots are almost identical in various situations which ensures the validity of the analytical solutions. The slight discrepancy seen in the graphs are due to the approximations that already taken earlier in Eq.8.

The effects of the linked spring and damper and the attached mass on the reduction of nonlinear vibration have been studied with the help of time plots, phase plane and frequency response curves. It is found that the nonlinear system with nonlinear spring will reduce the amplitude of the primary system in comparison with that of the linear system.

The stability and the response of the system is also studied with the help of phase plane. It is found that all the trajectory in the phase plane approaches to zero due to damping & the system is stable and also some dynamic chaos are seen on higher values of the nonlinear parameters. It is also found that the amplitude of nonlinear vibrations of the nonlinear primary oscillator has been greatly suppressed by adding the linear vibration absorber.

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