Mathematical Modeling Solutions for Various Cost Functions with Reference to EOQ Model with Stock Level Dependent Demand

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Abstract

The study in the paper is about a mathematical model based on EOQ (Economic order Quantity) for holding cost using various functions with assumption of stock level dependent demand. It is to prove that the relation between cycle time and the total cost of inventory per unit is convex in nature. Two Mathematical models are established for finding optimum values of order quantity and total stock value. The models discussed refer to (i) stock dependent holding cost with assumption of instant replenishment and (ii) A quadratic function for time dependent Carrying cost with assumption of instant replenishment. The models said above are further illustrated using numerical examples. Numerical results are calculated through Mathematica 5.2 software. And, optimized solution with reference to the various variables is analyzed using sensitivity analysis done.

***IndexTerms* -** Cost, Carrying cost, holding cost, Inventory, economic order quantity, Stock-Dependent Demand, total cost, Cycle time

I. INTRODUCTION

It is a practical problem faced by production houses nowadays how to keep its perishable inventory survive longer. The critical issue of the managing the fading stock is the primary challenge faced by the supply chain industry since past few decades. One of these is to keep the depreciating stock fresh till they are delivered to the destination. Every good existing in production has a high or low rate of deterioration. Therefore, the study of inventory problem is incomplete with consideration od deterioration factor. Here, analysis of an inventory model for variation in deterioration with respect to time under demand as a slanting curve in which demand is dependent on time exponentially, specially in case of instantly produced goods for market trade. In real life, seasonal products are an example of application of such demand. The production of goods is based on proportion between inventory and

demand. In the study, we have considered that the cost of production per unit is an increasing function of demand. In case of natural calamities the production and demand are proportional. e.g. Demand and corresponding production of umbrella increases during rainy season.

Based on classical theory of inventory modeling the rate of demand is either taken as dependent variable on time or a fixed constant but it is independent of quantity of stock available. For some goods, especially the consumer friendly goods, the rate of consumption may fluctuate because of level of stock. Singh and Bhatia (2011) studied the deterministic, continuous, single unit inventory system with the available level of inventory. Alfares (2007) worked on models with varying holding cost in inventory. Mostly models assume constant holding cost throughout the inventory cycle in consideration of variation of demand with respect to availability of stock. Here the model discussed have rate of demand dependent on level of stock and variation in function for holding cost. Two types of holding

cost are studied, (i) dependent on stock (ii) quadratic ally dependent on time. Shukla *et al*. (2013) **and** Jeganathan *et al* (2015) have confided their work to the deterministic inventory type with level of inventory dependent rate of demand and the cost of holding fixed at a constant price of *h* per unit per unit time. Soni and Shah (2008) discussed a model to formulate policy for optimum order with respect to retailer specially when some part of demand is constant and remaining is stock dependent and also there is offer of successive trade credit from the supplier for account settlement. Patra et al. (2015) studied direct proportionality of sale to the amount of displayed inventory at a retail shop. Gupta and

Teng *et al.* (2013) proposed a trade credit financing model with two-levels and assumptions of (i) linear and non increasing time dependent rate of demand (ii) a permissible delay in payments is accepted and allow by the supplier and further a relative credit period is allowed to customer by retailer.

Teng (2005) established an EOQ model to allow for shortages for deteriorating items and non-zero ending inventory. He also gave an inventory model in production houses for depreciating goods with constant production rate and demand which is linearly dependent on stock. Paul *et al*. (1996) observed a deterministic inventory model with allowed shortages and complete backlogging. Tripathi and Misra (2012) developed a deterministic inventory model assumptions of stock dependent rate of demand and rate of depreciation of goods at steady rate θ. Tripathi (2011) considered value of money in terms of time and analysed a system of inventory which has shortages and rate of demand both dependent on stock. Hou (2006) proposed a model for depreciating stock on which rate of consumption is dependent and other assumptions of shortages under inflation and discounts in time under a planned timed range. Balkhi and Benkherouf (2004) proposed a model for depreciating stock with rate of demand dependent on stock and time over a fixed time.

Goyal and Chang (2009) proposed a model with property of ordering- transferring of goods from the warehouse to the display shelf, hence determining the economic order quantity and the per order quantity of goods transferred. Yang *et al*. (2010) studied an inventory model for depreciating goods with stock-dependent demand and partial backlogging. Teng *et al.* (2011) developed a model for profit optimization when inventory consumption is 100%. Soni and Shah (2008) discussed a model to formulate policy for optimum order with respect to retailer specially when some part of demand is constant and remaining is stock dependent and also there is offer of successive trade credit from the supplier for account settlement. Liao *et al*. (2000) also worked on similar concept on rate of demand dependent on stock with allowed delays in payments. Sarkar (2012) studied a model on rate of replenishment, demand dependent on stock, imperfections in production and payment delays allowed up to two consecutive times.

Considering the fact that the demand is and increasing function of time during growing levels in the life cycle of product. In this paper, we consider demand as a non-decreasing quadratic function of time instead of constant. Study in this regard is done by Teng *et al.* (2012) who proposed a model for financing under trade credit system. Sarkar (2012) developed an EOQ model for production considering rate of replenishment and rate of demand and depreciation are dependent on time. Skouri *et al*. (2011) presented an order level inventory model for deteriorating items with general ramp type demand rate under condition of permissible delay in payaments. Khanna *et al*. (2011) established an EOQ model for deteriorating items having time-dependent demand when delay in payments is permissible. Khanna and Chaudhuri (2003) and Ghosh and Chaudhuri (2006) developed inventory models considering time-quadratic demand rate which is more realistic. Singh *et al*. (2014) presented a model for

time-dependent demand with multiple productions and rework setups. Sarkar *et al*. (2013) developed an EOQ model for finite production rate and deteriorating items with time dependent increasing demand. Sarkar *et al.* (2014) presented an economic manufacturing quantity (EMQ) model for the selling price and the time dependent demand pattern in an imperfect production process.

The remaining paper is stated introduction is given in the first section. To establish the proposed model, we provide the assumptions and notations in the next section. Then we derive the mathematical formulation of the model in next section. After this, numerical examples are provided followed by the sensitivity analysis which presented to illustrate the model and its numerical example discussed. Finally, the conclusion and the scope of this study for future research of the discussed problem is stated in the end.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions are used:

1. It is a single product inventory.
2. The cost of replenishment.
3. The rate of demand is deterministic and is stock level dependent i.e. $R(q) = \alpha + \beta.q$, $\alpha \geq 0$, $0 \leq \beta \leq 1$, α is positive constant and β is the stock –dependent parameter for rate of consumption.
4. Cost of product is independent of size of order.
5. No shortages.
6. Infinite time range.
7. Negligible lead time.

NOTATIONS

Q: In hand inventory level
T: Cycle time
K: Cost of Replenishment
H: Holding cost per unit time excluding interest charges
TIC: Total relevant per unit cost of inventory
HC: Holding cost per unit time
TIC_1*: Optimal relevant inventory cost per unit time for model
T_1*: Optimal cycle time for model
Q_1*: Optimal order quantity for model

III. FORMULATION OF THE PROBLEM

Model I: Inventory Dependent Holding Cost with Instantaneous Replenishment

The level of inventory decreases quickly with the increase in quantity of demand in inventory management. As higher inventory is exhausted the size demanded $R(q) = \alpha + \beta\, q$ decreases resulting in decrease of stock. In this model, it is considered the cost of holding per unit is dq including time t is given by $hR(q)dq$. The equations used for the study are given as:

$$\frac{dq}{dt} = -\{\alpha + \beta q\}$$

with the initial conditions, $q(0) = Q$.

The solution of eqn (1) is given by:

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Also, $q(T) = 0$, eqn (2) gives, for the corresponding cycle time T:

$$T = \frac{1}{\beta}\log\left(1 + \frac{\beta Q}{\alpha}\right)$$

(1)

(2)

(3)

The quantity of order Q to minimize the total relevant inventory cost per unit time is given by the eqn:

$$TIC=\frac{R}{T}+\frac{HC}{T} \tag{4}$$

Holding cost per cycle time is integral of $\left(\alpha+\beta q\right) dq$ with limits of q from 0 to Q, to get,

$$HC=\int_0^Q h\left(\alpha+\beta q\right) dq = hQ\left(\alpha+\frac{\beta Q}{2}\right) \tag{5}$$

Hence,

$$TIC=\frac{k\beta}{\log\left[1+\frac{\beta Q}{\alpha}\right]}+\frac{h\beta Q\left(\alpha+\frac{\beta Q}{2}\right)}{\log\left(1+\frac{\beta Q}{\alpha}\right)} \tag{6}$$

Since it is not easy to find closed form optimized solution for eqn (6). Hence, log terms are expanded using Truncated Taylor's series i.e.

$$\log\left(1+\frac{\beta Q}{\alpha}\right)=\frac{\beta Q}{\alpha}-\frac{\beta^2 Q^2}{2\alpha} \quad \log\left(1+\frac{\beta Q}{\alpha}\right)=\frac{\beta Q}{\alpha}-\frac{\beta^2 Q^2}{2\alpha^2}$$

higher order terms are neglected and thus eqn (6) reduces to:

$$TIC=-\frac{2\alpha^2\left\{k+hQ\left(\alpha+\frac{\beta Q}{2}\right)\right\}}{Q\left(2\alpha-\beta Q\right)}$$

To note, the above approximation is valid only for $\alpha > \beta Q$ only.

Differentiating eqn (7) twice with respect to 'Q' , it gives,

$$\frac{d\left(TIC\right)}{dQ}=\frac{4\alpha^2\left\{k\left(\beta Q-\alpha\right)+\alpha\beta hQ^2\right\}}{Q^2\left(2\alpha-\beta Q\right)^2}$$

$$\frac{d^2\left(TIC\right)}{dQ^2}=\frac{4\alpha^2\left\{2\alpha\beta^2 hQ^3+3k\beta^2 Q^2-6\alpha\beta kQ+4k\alpha^2\right\}}{Q^3\left(2\alpha-\beta Q\right)^3}>0$$

provided $2\alpha > \beta Q$ and $h\,\alpha\beta Q^2\,(2\beta + 1) + 4k\,\alpha^2 > 3k\beta Q\,(2\alpha - \beta Q)$.

The optimum (minimised) value of Q = Q_1*, can be obtained from,

$$\frac{d\left(TIC\right)}{dQ}=0$$

Hence it gives,

$$Q = Q^* = \frac{\sqrt{k^2\beta^2 + 4\alpha^2\beta hk} - k\beta}{2\alpha\beta h}$$ (Negative sign is ignored)

and the corresponding optimal cycle time can be calculated by,

$$T = T^* = \frac{Q^*}{\alpha}\left(1 - \frac{\beta Q^*}{2\alpha}\right)$$

(7)

(8)

(9)

(10)

(11)

The minimized total relevant inventory cost can be calculated by putting eqn (11) in eqn (7), and is,

$$TIC = TIC^* = 2\alpha^2 \left\{ \frac{k}{Q^*\left(2\alpha - \beta Q^*\right)} + \frac{h\left(\alpha + \frac{\beta Q^*}{2}\right)}{\left(2\alpha - \beta Q^*\right)} \right\} \quad (12)$$

NUMERICAL EXAMPLES

To illustrate the discussed Model the parameters discussed in the algorithm have the following values in the given problem.

Example 1 for Model I

Let α =100, β = 0.06, h = 0.05, and k = 1 in their respected units, substituting these values in eqn (10), the results are, Optimal order quantity Q = Q_1^* = 14.1231 units, corresponding Optimal cycle time $T = T_1^*$ = 0.0122141 yrs and optimized inventory cost $TIC = TIC_1^*$ = Rs. 514105.

SENSITIVITY ANALYSIS OF DISCUSSED MODEL

Using the data given in above illustration, sensitivity analysis of the optimal solution with respect to each parameter is done taken with appropriate units (Table 1, Table 2, and Table 3 in the Appendix). Also, Figure 1 in the Appendix is shown as justification of the sensitivity analysis done for Model.

DISCUSSION OF RESULTS

Case I

1. When stock consumption rate 'β' increases, the order quantity $Q1^*$ decreases and total relevant inventory cost $TIC1^*$ increases. Similarly, when 'α' increases order quantity approximately constant while total relevant inventory cost $TIC1^*$ increases. That is, the change in 'β' leads negative change in $Q1^*$ and positive change in $TIC1^*$. Similarly, the change in 'α' leads positive change in optimal total relevant inventory cost $TIC1^*$ and optimal order quantity $Q1^*$ is approximately constant;
2. When holding cost per unit time 'h' and stock-consumption rate 'β' will increase, order quantity $Q1^*$ decreases and total relevant inventory cost $TIC1^*$ increases. That is, change in 'h' and 'β' will lead negative change in optimal order quantity $Q1^*$ and positive change in optimal total relevant inventory cost $TIC1^*$;
3. (When replenishment cost 'k' increases, optimal order quantity $Q1^*$ and total relevant inventory cost $TIC1^*$ increases. Similarly, if stock-consumption rate 'β' increases, the order quantity $Q1^*$ decreases and total relevant inventory cost $TIC1^*$ increases. That is, change in 'k' leads positive change in optimal order quantity $Q1^*$ and optimal total relevant cost $TIC1^*$. Also, the increase of stock consumption rate 'β' will lead negative change in optimal order quantity $Q1^*$ and positive change in optimal total relevant cost $TIC1^*$.

CONCLUSION

In this paper, We have formulated inventory models with stock dependent demand rate for two different holding cost functions (i) holding cost per cycle time is the integral of $h(\alpha + \beta q)dq$ and (ii) holding cost per cycle is the integral of $h(a + bt + ct_2)$ we have shown that the total relevant inventory cost per cycle TIC is a convex function of cycle time. Second orders approximations have been used for the logarithmic expressions for finding the closed form optimal solution of order quantity and total relevant inventory cost. We have provided the mathematical formulation of the problem to find optimal solutions. Numerical examples have been established to illustrate the both models. Finally, the sensitivity of the optimal solutions to change in the values of the different key parameters has been discussed. It is seen that change in the consumption rate 'β', ordering cost 'k', holding cost parameter 'h' etc lead to significant effects on the total relevant inventory cost TIC. From managerial point of view, we obtain the following results:

- When stock consumption rate 'β' increases, the order quantity $Q1^*$ decreases and total relevant inventory cost $TIC1^*$ increases;
- When 'α' increases order quantity approximately constant while total relevant inventory cost $TIC1^*$ increases. When holding cost per unit time 'h' and stock-consumption rate 'β' will increase, order quantity $Q1^*$ decreases and total relevant inventory cost $TIC1^*$ increases;
- When replenishment cost 'k' increases, optimal order quantity $Q1^*$ and total relevant inventory cost $TIC1^*$ increases;
- If stock-consumption rate 'β' increases, the order quantity $Q1^*$ decreases and total relevant inventory cost $TIC1^*$ increases;
- The increase of stock consumption rate 'β' will lead negative change in optimal order quantity $Q1^*$ and positive change in optimal total relevant cost $TIC1^*$;

SCOPE OF STUDY

This paper can be extended in different ways. For instance, we may extend the demand rate is a function of price and time. We may also extend the paper by considering replenishment cost, as a function of time or selling price. We could generalize the model for allowing for shortages and inflation.

REFERENCES

1. Alfares, H. K. (2007). Inventory model with stock-level dependent demand rate and variable holding cost. *International Journal of Production Economics*, *108*(1-2), 259–265. doi:10.1016/j.ijpe.2006.12.013

2. Balkhi, Z. T., & Benkherouf, L. (2004). On an inventory model for deteriorating items with stock-dependent and time varying demand rates. *Computers & Operations Research*, *31*(2), 223–240. doi:10.1016/S0305-0548(02)00182-X
3. Ghosh, S. K., & Chaudhuri, K. S. (2006). An EOQ model with a quadratic demand, time- proportional deterioration and shortages in all cycles. *International Journal of Systems Science*, *37*(10), 663–672. doi:10.1080/00207720600568145
4. Goyal, S. K., & Chang, C. T. (2009). Optimal ordering and transfer policy for an inventory with stock - dependent demand. *European Journal of Operational Research*, *196*(1), 177–185. doi:10.1016/j.ejor.2008.02.029
5. Hou, K. L. (2006). An inventory model for deteriorating items worth stock-dependent consumption rate and shortages under inflation and time discounting. *European Journal of Operational Research*, *168*(2), 463–474. doi:10.1016/j.ejor.2004.05.011
6. Jeganathan, K., Anbazhagan, N., & Vigneshwaran, B. (2015). Perishable inventory system with server interruption, Multiple server vacation and N Policy. *International Journal of Operations Research and Information Systems*, *6*(2), 32–52. doi:10.4018/ijoris.2015040103
7. Khanna, S., Ghosh, S. K., & Chaudhuri, K. S. (2003). A note on an order level inventory model for a deteriorating item with time-dependent quadratic demand. *Computers & Operations Research*, *30*(12), 1901–1916. doi:10.1016/ S0305-0548(02)00113-2
8. Khanna, S., Ghosh, S. K., & Chaudhuri, K. S. (2011). An EOQ model for a deteriorating item with time-dependent quadratic demand under permissible delay in payments. *Applied Mathematics and Computation*, *218*(1), 1–9. doi:10.1016/j.amc.2011.04.062
9. Liao, H. C., Tsai, C. H., & Su, C. T. (2000). An inventory model with deteriorating items under inflation when a delay in payments is permissible. *International Journal of Production Economics*, *63*(2), 207–214.doi:10.1016/S0925-5273(99)00015-8.
10. Patra, S. K., Lenka, T. K., & Patha, P. C. (2015). A time dependent order level inventory model for beta deterioration in two warehouse systems. *International Journal of Operations Research and Information Systems*, *6*(2), 53–69. doi:10.4018/ijoris.2015040104.
11. Sarkar, B. (2012). An EOQ model with delay in payments and time varying deterioration rate. *Mathematical and Computer Modelling*, *55*(3-4), 367–377. doi:10.1016/j.mcm.2011.08.009
12. Sarkar, B. (2012). An EOQ model with delay in payments and stock -dependent demand in the presence of imperfect production. *Applied Mathematics and Computation*, *218*(17), 8295–8308. doi:10.1016/j.amc.2012.01.053
13. Sarkar, B., Mandl, P., & Sarkar, R. S. (2014). An EMQ model with price and time dependent demand under the effect of reliability and inflation. *Applied Mathematics and Computation*, *231*, 414–421. doi:10.1016/j. amc.2014.01.004
14. Sarkar, B., Sren, S., & Wee, H. M. (2013). An inventory model with variable demand, component cost and selling price for deteriorating items. *Economic Modelling*, *30*, 306–310. doi:10.1016/j.econmod.2012.09.002
15. Shukla, D., Khedlekar, U. K., & Chandel, R. P. S. (2013). Time and price dependent demand with varying holding cost inventory model for deteriorating items. *International Journal of Operations Research and Information Systems*, *4*(4), 75–95. doi:10.4018/ijoris.2013100105
16. Singh, S. R., & Bhatia, D. (2011). An integrated vendor –Buyer model with uncertain lead time, life time under inflation and variable holding cost. *International Journal of Operations Research and Information Systems*, *2*(3), 54–63. doi:10.4018/joris.2011070104
17. Singh, S. R., Jain, S., & Pareek, S. (2014). An economic production model for time dependent demand with rework and multiple production setups. *International Journal of Industrial Engineering Computations*, *5*(2), 305–314. doi:10.5267/j.ijiec.2013.10.003
18. Skouri, K., Koustantaras, I., Papachristos, S., & Teng, J. T. (2011). Supply chain models for deteriorating products with ramp type demand rate under permissible delay in payments. *Expert Systems with Applications*, *38*(12), 14861–14869. doi:10.1016/j.eswa.2011.05.061
19. Soni, H., & Shah, N. H. (2008). Optimal ordering policy for stock-dependent demand under progressive payment scheme. *European Journal of Operational Research*, *184*(1), 91–100. doi:10.1016/j.ejor.2006.10.048
20. Teng, J. T., Krommyada, I. P., Skouri, K., & Lou, K. R. (2011). A comprehensive extension of optimal ordering policy for stock-dependent demand under progressive payment scheme. *European Journal of Operational Research*, *215*(1), 97–104. doi:10.1016/j.ejor.2011.05.056
21. Teng, J. T., Min, J., & Pan, Q. (2012). Economic order quantity model with trade credit financing for non-decreasing demand. *Omega*, *40*(3), 328–335. doi:10.1016/j.omega.2011.08.001
22. Teng, J. T., Ouyang, L. Y., & Cheng, M. C. (2005). An EOQ model for deteriorating items with power-form stock dependent demand. *Information and Management Sciences*, *16*(1), 1–16.

23. Teng, J. T., Yang, H. L., & Chern, M. S. (2013). An inventory model for increasing demand under two levels of trade credit linked to order quantity. *Applied Mathematical Modelling*, *37*(14-15), 7624–7632. doi:10.1016/j.apm.2013.02.009
24. Tripathi, R. P. (2011) . EOQ model with time dependent demand rate and time dependent holding cost function. *International Journal of Operations Research and Information Systems*, *2*(3), 79–92. doi:10.4018/joris.2011070106
25. Tripathi, R. P., & Mishra, S. S. (2012). Retailers pricing and lot sizing policy for non deteriorating items with constant demand rate under the condition of permissible delay in payments. *International Journal of Operations Research and Information Systems*, *3*(1), 74–83. doi:10.4018/IJORIS.2012010105
26. Yang, H. L., Tang, J. T., & Chen, M. S. (2010). An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. *International Journal of Production Economics*, *123*(1), 8–19. doi:10.1016/j.ijpe.2009.06.041

APPENDIX

Table 1. Sensitivity analysis in parameter change for 'α' with variation of stock consumption rate 'β'

α↓	β→	0.1	0.2	0.3	0.4
100	T_1^*	0.122141	0.00998002	0.008144990	0.0070511
	Q_1^*	14.1231	9.99	8.15497	7.06107
	TIC_1^*	514105	502002.0	502452.0	502832.0
110	T_1^*	0.0128399	0.0090744	0.00740619	0.00641174
	Q_1^*	14.133	9.99091	8.15588	7.06198
	TIC_1	606557.0	607202.0	607697.0	608115.0
120	T_1^*	0.0117712	0.00831946	0.00679027	0.00587869
	Q_1^*	14.1338	9.99167	8.15664	7.06274
	TIC_1	721698.0	722402.0	722942.0	723398.0
130	T_1^*	0.0108785	0.00768048	0.00626892	0.00542746
	Q_1^*	14.1498	9.99231	8.15728	7.06338
	TIC_1	846839.0	847602.0	848187.0	848681.0
150	T_1^*	0.0094192	0.00665779	0.00543443	0.00470516
	Q_1^*	14.1355	9.99334	8.1583	7.0644
	TIC_1	$1.1271\times10_6$	$1.128\times10_6$	$1.1286\times10_6$	$1.12925\times10_6$

Table 2. Sensitivity analysis on variation holding cost parameter h with the variation of stock consumption rate 'β'

h ↓	β →	0.1	0.2	0.3	0.4
1.0	T_1^*	0.009995	0.00706171	0.00576351	0.00499001
	Q_1^*	9.995	7.06607	5.7685	4.995
	TIC_1^*	$1.002\times10_6$	$1.00283\times10_6$	$1.00347\times10_6$	$1.004\times10_6$
1.5	T_1^*	0.0081583	0.00576684	0.00470738	0.00407582
	Q_1^*	8.16163	5.77017	4.71071	4.07915
	TIC_1	$1.5024\times10_6$	$1.50347\times10_6$	$1.50425\times10_6$	$1.5049\times10_6$
2.0	T_1^*	0.00706607	0.004995	0.00407748	0.00353053
	Q_1^*	7.06857	4.9975	4.07998	3.53303
	TIC_1	$2.0028\times10_6$	$2.00402\times10_6$	$2.0049\times10_6$	$2.00566\times10_6$
2.5	T_1^*	0.00632456	0.00446814	0.00364748	0.00315828
	Q_1^*	6.32656	4.47014	3.64948	3.16028
	TIC_1	$2.5031\times10_6$	$2.50447\times10_6$	$2.50548\times10_6$	$2.50633\times10_6$
3.0	T_1^*	0.00577017	0.00407915	0.00333	0.00288343
	Q_1^*	5.77184	4.08082	3.33167	2.88509
	TIC_1	$3.0034\times10_6$	$3.0049\times10_6$	$3.006\times10_6$	$3.00693\times10_6$

Table 3. Sensitivity analysis on the variation of replenishment cost parameters 'k' with the variation of stock consumption r ate parameter 'β'

k ↓	β→	0.1	0.2	0.3	0.4
	T_1^*	0.0148104	0.0104661	0.00854153	0.00739424
	Q_1^*	14.8214	10.4771	8.5525	7.40521
11	TIC_1	501484.0	502100.0	502572.0	502971.0
	T_1^*	0.0154679	0.0109305	0.00892031	0.0772202
	Q_1^*	15.4799	10.9425	8.93228	7.73398
12	TIC_1	501550.0	502193.0	502687.0	503103.0
	T_1^*	0.0160985	0.0113758	0.00928354	0.00803631
	Q_1^*	16.1115	11.3888	9.2965	8.04927
13	TIC_1	501614.0	502283.0	502797.0	503230.0

	T_1*	0.0167052	0.0118042	0.00963297	0.00833866
	Q_1*	16.7192	11.8182	9.64693	8.35261
14	TIC_1	501675.0	502369.0	502902.0	503352.0
	T_1*	0.0172905	0.0122175	0.00997005	0.00863032
	Q_1*	17.3055	12.2235	9.98501	8.64527
15	TIC_1	501734.0	502452.0	503005.0	503470.0

Figure 1. Between α, β, h and k vs TIC for model discussed.