(i) Relativistic equation of Planck's curve in the form of entropy changes.
(ii) Entropy of the sun at a combined effective temperature 5778 K.

Abhishek Anand
Independent Faculty & Researcher

(i) Relativistic equation of Planck's curve in the from of Entropy changes :-

Doppler Shift in wavelength of light

$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda} \qquad \text{———} \qquad (1)$$

Velocity of light in a medium

$$v = \frac{c}{\sqrt{\mu\epsilon}}$$

$$\Rightarrow \frac{v}{c} = \frac{1}{\sqrt{\mu\epsilon}} \qquad \text{———} \qquad (ii)$$

.

Dividing the equation (i) by (ii), we get

$$\frac{\frac{\cancel{v}}{\cancel{c}}}{\frac{\cancel{v}}{\cancel{c}}} = \frac{\frac{\Delta\lambda}{\lambda}}{\frac{1}{\sqrt{\mu\epsilon}}}$$

$$\Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{1}{\sqrt{\mu\epsilon}} \qquad \text{———} \qquad (iii)$$

Compton's equation refers quantum nature of light

$$\Delta\lambda = \lambda(1-\cos\theta)$$

$$\frac{\Delta\lambda}{\lambda} = (1-\cos\theta) \quad \text{——— (iv)}$$

Relativistic equation of frequency

$$\nu = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{——— (v)}$$

From eq (iii), (iv) and (v), we get :-

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{\sqrt{\mu\epsilon}} = 1-\cos\theta = \left(1-\frac{v^2}{c^2}\right)^{1/2} \quad \text{——— (vi)}$$

Again Penzias and Wilson by using a very sensitive microwave detector found that the expansion of the Universe is so great shifted that it would appear to us as a microwave radiation.

$$E = \frac{hc}{\lambda+\Delta\lambda} = \frac{hc}{\lambda+\frac{\lambda}{\mu\epsilon}}$$

$$= \frac{hc}{\lambda}\left(1-\frac{1}{\sqrt{\mu\epsilon}}\right)$$

(From Newton's binomial theorm)

$$= \frac{hc}{\lambda}\left\{1-\left(1-\frac{v^2}{c^2}\right)^{1/2}\right\} \quad \text{—— (vii)}$$

From the equation of Gibb's free energy change

$$\Delta G = \Delta H - T\Delta S - S\Delta T$$

When we combine theory of relativity with quantum mechanics, it can be shown that our Universe is energetically finite - (By Stephen Hawking)

So its internal energy remains constant

$$(\Delta T = 0)$$

$$\Delta G = \Delta H - T\Delta S$$

Matter and energy are in thermodynamic equilibrium in the black hole

$$\therefore \Delta G = 0.$$

$$\Delta H = T\Delta S$$

$$\Rightarrow \Delta S = \frac{\Delta H}{T} = \frac{hc}{\lambda T}\left\{1 - \left(1 - \frac{v^2}{c^2}\right)^{1/2}\right\} \quad (VIII)$$

Hence, this is the relativistic equation of Planck's curve in the form of entropy changes.

The above equation is dimensionally correct
When V = C, then

$$\Delta S = \frac{hc}{\lambda T_{max}} \quad \text{———} \quad (IX)$$

(ii) <u>Entropy of the Sun at an Combined effective temperature 5778K</u>
Max Planck (1858 – 1947) Germany. On the law of distribution of energy in the normal spectrum Max Planck Annalen der Physik 4 (1901) : 553

Proposition ………………….

$$E = hf$$

And also (from Einstein later, I think)………………

$$P = \frac{h}{\lambda}$$

Let's try to derive the blackbody spectrum. Planck's law is a formula for the spectral radiance of an object at a given temperature as a function of frequency (L_f) or wave length (L_λ). It was dimensions of power per solid angle per area per frequency or power per solid angle per area per wavelength. (Yuck !)

$$L_f = \frac{2hf^3}{c^2} \frac{1}{e^{hf/kT}-1} \cdot \left[\frac{W}{Sr\,m^2 Hz} \right]$$

$$L_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT}-1} \left[\frac{W}{Sr\,m^2 m} \right]$$

When these functions are multiplied by the total solid angle of a sphere (4π steradian) we get the spectral irradiance (E_f or E_λ). This function describes the power per area. Per frequency or the power per area per wavelength.

$$E_f = \frac{8\pi h f^3}{c^2} \frac{1}{e^{hf/kT}-1} \left[\frac{W}{m^2 Hz} \right]$$

$$E_\lambda = \frac{8\pi h c^2}{\lambda^5} \frac{1}{e^{hf/\lambda kT}-1} \left[\frac{W}{m^2 m} \right]$$

When either of these functions is integrated over all possible values from zero to infinity, the result is the irradiance or the power per area..

$$E = \int_0^\infty E_\lambda \, d\lambda = \int_0^\infty E_f \, d\lambda = \frac{P}{A}$$

Trust me; the solution looks like this - - -

$$\frac{P}{A} = \frac{2\pi^5 k^4}{15 h^3 c^2} T^4$$

The pile of constants in front f the temperature is known as stefan's constant.

$$\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2}$$

$$\sigma = 5.67040 \times 10^{-8} \, W/m^2K^4$$

Multiplying the irradiance by the area gives us the essence of the Stefan – Boltzmann law.

$$\frac{P}{A} = \sigma T^4$$

$$\Rightarrow P = \sigma A T^4$$

Apply the first derivative test to the wavelength form of Planck's law to determine the peak wavelength as a function of temperature.

$$\frac{d}{d\lambda} E_\lambda (\lambda_{max}) = 0.$$

Trust me, the solution looks like this ……………

$$\lambda_{max} = \frac{hc}{kx} \frac{1}{T}$$

Where X is the solution of the transcendental equation ………………….

$$\frac{xe^x}{e^x-1} - 5 = 0$$

$$x = 4.965114231744276303698759131322893944055584986797250972814\ldots$$

Combine all the constants together into Wein's Constant

$$\frac{hc}{kx} = \frac{(6.62607\times10^{-34}\,J\cdot S)(2.99792\times10^{8}\,m/s)}{(1.38065\times10^{-23}\,J/K)(4.96511)}$$

$$b = 2897.77\ \mu m\cdot k.$$

And we get the Wien displacement law …………………..

$$\lambda_{max} = \frac{b}{T}$$

$$\Rightarrow kx = \frac{hc}{\lambda T} \qquad \text{———} \qquad (x)$$

From eq (Ix) and (x), we get.

$$S = kx = \frac{hc}{\lambda_{max} T}$$

This is the entropy of the Sun at an combined effective temperature 5778K.

$$S = \frac{hc}{b}$$

$$= \frac{(6.62607\times10^{-34}\,J\cdot S)(2.99792\times10^{8}\,m/s)}{2897.77\times10^{-6}\,m/k.}$$

$$= \frac{19.864427774 \times 10^{-34+8}}{2897.77 \times 10^{-6}}$$
$$= 0.0068550739963 4 \times 10^{-20} J/K$$
$$= 6.8550739963 4 \times 10^{-23} J/K.$$

Hence its entropy approximately are in the order of 10^{-23} J/K.

SUMMARY

(i) Relativistic equation of Planck's Curve in the form of entropy change.

$$S = \frac{hc}{\lambda T}\left\{1 - \left(1 - \frac{v^2}{c^2}\right)^{1/2}\right\}$$

(ii) Different parts of the Sun are at different temperature. When combined, the Sun has an effective temperature of 5778K. you get, highest power in the visible wavelength, as shown in this graphic. Entropy of the sun at this temperature is

$$S = Kx = \frac{hc}{\lambda T} = \frac{hc}{b}$$
$$= 6.85507399634 \times 10^{-23} J/K.$$

Hence the value of its entropy are approximately in the order of 10^{-23} J/K.

(iii)