Intersubband electromagnetically induced transparency in a GaAlAs/GaAs/GaAlAs semiconductor quantum well

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Abstract

Two atomic sub-levels are considered within the frame work of compact density matrix formulism. The occurrence of electromagnetically induced transparency in a quantum well is observed in the present work. The intersubband electromagnetically induced transparency in the two level single quantum well is investigated theoretically. The taken system is GaAs sandwiched between the GaAlAs semiconducting material. The confinement potential and the well size of the quantum well are fixed. The conduction band subband energies and the eigenfunctions of the single electron and the hole in a $Ga_{0.7}Al_{0.3}As/GaAs/Ga_{0.7}Al_{0.3}As$ *quantum well are computed. The intersubband energies are calculated. The optical susceptibilities, the Rabi frequency and the detuning parameters are investigated in the present work. The interaction between the electromagnetic field and the taken semiconducting system is defined within the rotating wave approximation. The dependence of the nonlinearities on the Rabi frequency and detuning parameter is brought out.*

Keywords: Electromagnetically induced transparency, density matrix differential equations Rabi frequency, two level system

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1. Introduction

The observation of efficient electromagnetically induced transparency (EIT) in any low dimensional semiconductor system is possible with the slow light propagation. EIT is a quantum interference phenomenon which transforms an initially absorbing medium into a transparent medium for a particular frequency and it considerably enhances the medium's dispersion. In fact, it modifies the optical properties drastically near an atomic resonance with the application of external field. The dispersion will be created in the window leading to slow light. The phenomenon of EIT is associated with the reduction of speed of light in an atomic system and it has been observed earlier in an ultracold atomic gas [1]. EIT which is an interesting and nonlinear optical property occurs due to the atomic coherence and the interference effects. The atomic coherence and interference effects considerably influence on the absorption and the refractive index. The atomic coherence will modify the nonlinear optical properties of an atomic medium. In fact, the quantum interference enhances the refractive index. If any external sources interact with the atoms in resonant the EIT occurs. It possesses the controlled manipulation of atomic optical properties coupled with the resonant optical signal field

[2]. Better transparency will be obtained with the stronger field producing MIT. It has the unique property of passing through the opaque medium without any attenuation.

The interaction of light with the matter particularly with the subatomic levels brings out a lot of interesting quantum phenomena. The quantum well having discrete energy levels compared to that of a bulk material, has the large electronic dipole moments and thereby the high optical nonlinearities can be obtained. Electromagnetically induced transparency has been performed theoretically [3-5] and experimentally earlier [6-8]. It is applied in varieties of applications such as electro-optical devices, efficient nonlinear mixing , nonlinear optics, optical switching, lasing without inversion, sharp dispersion to control speed of light, quantum sensor technologies and quantum information process.

The steep anomalous and normal dispersion have been perceived in a two level atomic system [9]. The transition energies and the exciton wave functions can be altered with the application of external magnetic fields [10]. Since, the optical properties are modified with the application of external fields, the electromagnetically induced transparency finds various potential applications especially in quantum optics. Practical realization of quantum memory using slow and stored light is an exciting phenomenon in EIT.

In the present work, the two atomic sub-levels are considered and the system is analyzed using a density matrix approach. The occurrence of electromagnetically induced transparency in a quantum well is observed. The intersubband electromagnetically induced transparency in the two level single quantum well is investigated theoretically. The taken system is GaAs sandwiched between the GaAlAs semiconducting material. The confinement potential and the well size of the quantum well are fixed. The conduction band subband energies and the eigenfunctions of the single electron and the hole in a $Ga_{0.7}Al_{0.3}As/GaAs/Ga_{0.7}Al_{0.3}As$ quantum well are computed. The intersubband energies are calculated. The optical susceptibilities, the Rabi frequency and the detuning parameters are investigated in the present work. The interaction between the electromagnetic field and the taken semiconducting system is defined within the rotating wave approximation. The theoretical approach followed is presented in Section 2, the results and discussion are presented in Section 3. At the end, the main conclusions are explained.

2. Theoretical Formulism

The taken Hamiltonian is the combination of free and interaction Hamiltonian. For any two level system, the time dependent Schrödinger equation, in the resonant oscillating electric field, is expressed as [11]

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_{\text{int}}(t) \qquad (1)$$

where $\hat{H}_0$ denotes the unperturbed Hamiltonian with the two available eigen states referred as $|a\rangle$ and $|b\rangle$. For a single two level system, the basis of the states is given by

$$|a\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ and } \quad |b\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

Then, the atomic state is expressed as

$$|\psi\rangle = A_{|a\rangle}|a\rangle + B_{|b\rangle}|b\rangle = \begin{pmatrix} B_{|b\rangle} \\ A_{|a\rangle} \end{pmatrix} \tag{3}$$

where $A_{|a\rangle}$ and $B_{|b\rangle}$ are the population amplitudes of the respective states.

In the absence of electric field, it is expressed as

$$\hat{H}_0 = E_1|a\rangle\langle a| + E_2|b\rangle\langle b| \tag{4}$$

with $E_b = \hbar\omega_{ba}$ defines the energy of the excited state $|b\rangle$ and it is considered with respect to the ground state $|g\rangle$. Here, the ground state is measured as the origin of the energy axis and $\hbar\omega_{ba}$ is the atomic transition frequency. Hence, the ground state energy is zero energy. Further, the energy of the excited state is $\hbar\omega_{ba}$. So, the Eq.(2) becomes

$$\hat{H}_0 = \hbar\omega_{ba}|b\rangle\langle b| \tag{5}$$

In Eq.(1), the interaction Hamiltonian is given by

$$\hat{H}_{\text{int}}(t) = \beta\big[|a\rangle\langle b| + |b\rangle\langle a|\big] \tag{6}$$

Here, β denotes the strength of the interaction. The interaction term includes the hermiticity of the Hamiltonian operator.

The interaction Hamiltonian is expressed as

$$\hat{H}_{\text{int}}(t) = -\hat{p}E(z,t) \tag{7}$$

where $\hat{p}$ is the oriented in the direction of the applied electric field. $E(z,t)$ which is the oscillating electric field is valued at the position of the atom and it is given by

$$E(z,t) = E_0(z,t)\exp(i\omega t) + E_0^*(z,t)\exp(-i\omega t) \tag{8}$$

where E_0 is the peak amplitude of the applied electric field and E_0^* is the corresponding complex conjugate. The frequencies are connected with the detuning, Δ, of the external source, as $\omega = \omega_{|b\rangle} - \omega_{|a\rangle} + \Delta$.

The dipole moment operator is expressed as

$$\hat{p} = D\left[\left|a\right\rangle\left\langle b\right| + \left|b\right\rangle\left\langle a\right|\right] \qquad (9)$$

where D is the modulus of the dipole moment interacting the states defined as $\langle a|x|b\rangle$. It is defined as the coupling strength of the laser with respect to the transition of the atom.

Combining Eq.(6) and Eq.(7), the interaction term becomes,

$$\hat{H}_{\text{int}}(t) = -e\left[\vec{E}_0(z,t)\exp(i\omega t) + \vec{E}_0^*(z,t)\exp(-i\omega t)\right]\cdot\vec{D}\left[\left|a\right\rangle\left\langle b\right| + \left|b\right\rangle\left\langle a\right|\right] \qquad (10)$$

Therefore,

$$\hat{H}_{\text{int}}(t) = -\begin{bmatrix} e\vec{E}_0\cdot\vec{D}\exp(i\omega t)\left|a\right\rangle\left\langle b\right| + e\vec{E}_0\cdot\vec{D}\exp(i\omega t)\left|b\right\rangle\left\langle a\right| + e\vec{E}_0^*\cdot\vec{D}\exp(-i\omega t)\left|a\right\rangle\left\langle b\right| \\ + e\vec{E}_0^*\cdot\vec{D}\exp(-i\omega t)\left|b\right\rangle\left\langle a\right| \end{bmatrix} \qquad (11)$$

Thus, the interaction Hamiltonian, using interaction picture and the time evolution operator, becomes,

$$\hat{H}_{\text{int}}(t) = \begin{bmatrix} e\vec{E}_0\cdot\vec{D}\exp(2i\omega t)\left|a\right\rangle\left\langle b\right| + e\vec{E}_0\cdot\vec{D}^*\left|b\right\rangle\left\langle a\right| + e\vec{E}_0^*\cdot\vec{D}\left|a\right\rangle\left\langle b\right| \\ + e\vec{E}_0^*\cdot\vec{D}^*\exp(-2i\omega t)\left|b\right\rangle\left\langle a\right| \end{bmatrix} \qquad (12)$$

We have neglected the fast oscillating term in the Hamiltonian to the time evolution of the system and it is generally known as rotating wave approximation. So the Eq.(9) becomes,

$$\hat{H}_{\text{int}}(t) = \frac{1}{2}\hbar\left[\Omega\left|b\right\rangle\left\langle a\right| + \Omega^*\left|a\right\rangle\left\langle b\right|\right] \qquad (13)$$

where Ω is well known as Rabi frequency and it is defined as the measure of strength of the applied oscillating electric field defined as

$$\Omega = \frac{2e\vec{E}_0\cdot\vec{D}^*}{\hbar} \qquad (14)$$

The EIT arises due to the interaction of strong controlled frequency (ω_c) and the amplitude of the field (E_0) with the atomic system. Two atomic levels and the laser couples the states. Thus, the Rabi frequency becomes, $\Omega_c = (\Omega^2 + \Delta_c^2)^{1/2}$ with $\Delta_c = \omega_c - \omega_{ba}$. Here, ω_{ba} is the atomic transition frequency as explained earlier. Thus, the controlled frequency, $\omega_c = \omega_{ba}$ couples level $|a\rangle$ to $|b\rangle$ with Rabi frequencies.

The equations of evolution for the density matrix elements are attained by employing a standard approach. The density matrix elements for a linear system of coupled differential equations of first order, are given by

$$\dot{\rho}_{aa} = \gamma\rho_{bb} + \frac{i}{2}(\Omega^* \tilde{\rho}_{ba} - \Omega\tilde{\rho}_{ab})$$

$$\dot{\rho}_{bb} = -\gamma\rho_{bb} + \frac{i}{2}(\Omega\tilde{\rho}_{ab} - \Omega^* \tilde{\rho}_{ba})$$

$$\dot{\tilde{\rho}}_{ab} = -\frac{\gamma}{2}\tilde{\rho}_{ab} + \frac{i}{2}[\Omega^*(\rho_{bb} - \rho_{aa}) - 2\delta\tilde{\rho}_{ab}]$$

$$\dot{\tilde{\rho}}_{ba} = -\frac{\gamma}{2}\tilde{\rho}_{ba} + \frac{i}{2}[\Omega(\rho_{aa} - \rho_{bb}) + 2\delta\tilde{\rho}_{ba}] \qquad (15)$$

where the spontaneous decay rate, γ, is defined as [12]

$$\gamma = \frac{\omega^3 e^2 D^2}{3\pi\varepsilon_0 \hbar c^3} \qquad (16)$$

Taking into account the real values of Rabi frequency, using the conservation of population, $\rho_{aa} + \rho_{bb} = 1$ and the optical coherence, $\rho_{ba} = \rho^*_{ab}$, Eq.(15) has been reduced to ,

$$\dot{\rho}_{ba} = -\left(\frac{\gamma}{2} - i\delta\right)\rho_{ba} + \frac{i\omega\Omega}{2}$$

and the difference in population is given by

$$\rho_{aa} - \rho_{bb} = -\gamma\omega - i(\Omega\rho^*_{ba} - \Omega^* \rho_{ba}) + \gamma \qquad (17)$$

Using the polarization of the atomic medium, the linear susceptibility of the two level system is given by [13,14]]

$$\chi_{21} = \frac{|\mu|^2 n_0}{\hbar\varepsilon_0} \frac{\Delta + i\frac{\gamma}{2}}{\left(\frac{\gamma}{2}\right)^2 + \Delta^2} \qquad (16)$$

The two level system executes oscillations between the ground and excited states when there is no spontaneous emission and these oscillations will be modified in the case of resonance. The real and imaginary part of the susceptibility gives the dispersion and absorption of the probe in the medium respectively. The absorption coefficient and the refraction from the optical susceptibility are expressed as

$$\alpha(\omega_c) = \frac{4\pi\omega_{ab}}{c} \operatorname{Im}\chi_{21}(\omega_c) \quad (17)$$

and

$$\operatorname{Re}\chi_{21}(\omega_c) = n(\omega_c) - 1 \quad (18)$$

The group velocity associated with the derivative of the refractive index, in the absence of detuning, is given by [15]

$$\frac{v_g}{c} = \frac{1}{1 + \frac{2\Omega|\mu|^2 n_0}{\hbar\varepsilon_0\Gamma^2}} \quad (19)$$

where c is the velocity of light, μ represents the dipole matrix element of the transition and n_0 is the density of the atoms.

Using density matrix equations of motion [16], the third order non linear susceptibility is given by [17]

$$\chi^3(\omega_s) = \frac{8N|eD|^4}{(\hbar\Gamma)^3} \frac{(A^* - \delta)[(\delta + i)\eta_0^2 + \Omega_c(\chi_2^{(2)} - \chi_0^{(2)*})] + \frac{1}{2}(|\Omega_c|^2\eta_0^2 - \Omega_2^2\eta_2^2)}{(A+\delta)(A^* - \delta)(\delta + i) + (\delta + i\vec{\gamma})|\Omega_c|^2} \quad (20)$$

in which the expressions for $\chi_2^{(2)}, \chi_0^{(2)}$ and $\eta_2^{(2)}$ are taken from Ref.[6] with

$A = (\Delta_c + i\gamma)/\Gamma, \delta = (\omega_s - \omega_c)/\Gamma, \omega_0 = -|A|^2/|A|^2 + \gamma|\Omega_c|^2$ $\vec{\gamma} = \gamma/\Gamma$ and

$p_0 = -\Omega_c A^*/2(|A|^2 + \gamma|\Omega_c|^2)$.

3. Results and discussion

Numerical calculations are carried out to obtain the lowest optical transition energies which are directed to lead to find out the different nonlinear optical properties. The two atomic sub-levels are performed within the frame work of compact density matrix formulism. The occurrence of electromagnetically induced transparency in a quantum well is investigated. Thus, the intersubband electromagnetically induced transparency in the two level single quantum well is computed theoretically.

Fig.1 shows the variation of real and imaginary parts of optical susceptibility. The curves which are associated with the optical susceptibilities are known predications of optical properties of any two level system. The absorption is found to be maximum when the resonance occurs and the phase shift is found to be zero. It is also observed that the absorption and the phase shift vanish for larger detunings. Also, it is observed that the peaks of real part of the susceptibility take place at exactly the frequencies in which the imaginary part disappears. For any two-level system, the atoms absorb photons partially resulting the existence of two-level atomic medium which enhances the effective decay [18].

Fig.2 displays the variation of linear susceptibility, absorption spectra, as a function of detuning factor. It is found to be symmetric with respect the applied detuning parameter. This can lead to find the possible application of group velocity. This trend is similar to the earlier observation [19,20]. The splitting of these two transparency windows is found to be equal.

The group velocity with the laser frequency is shown in Fig.3. The transparency can be seen clearly. One resonant peak is observed. The distribution is found to be normalized. It is found that the group velocity is less than the velocity of light. When a light passes through the medium, it suffers a compression by the factor c/v_g. It is observed that the group velocity is controlled by varying the pump laser intensity [21,22]. Thus, it is clear that the group velocity can be modified by altering the external parameters.

Fig.4 shows the real part of third nonlinear optical susceptibilities as a function of signal filed and curve brings out the nonlinear optical properties of the two level system. The nonlinear optical susceptibility is based on the quantum interference. The tuned controlled field is applied to the system. This is an important condition for the occurrence of electromagnetically induced transparency. Figure clearly brings out that light passes through the medium slowly and retains more time in the medium. Both real and imaginary parts of the nonlinear optical susceptibilities disappear in the presence of two photon resonance. The result of coherent evolution is characterized by the Rabi flopping [23]. Further, it is observed that the optical linearities are found to be enhanced.

4. Conclusion

In conclusion, the two atomic sub-levels have been measured within the frame work of compact density matrix formulism. The occurrence of electromagnetically induced transparency in a quantum well has been investigated in the present work. The intersubband electromagnetically induced transparency in the two level single quantum well has been studied theoretically. The taken system is GaAs sandwiched between the GaAlAs semiconducting material. The confinement potential and the well size of the quantum well have been fixed. The conduction band subband energies and the eigenfunctions of the single electron and the hole in a $Ga_{0.7}Al_{0.3}As/GaAs/Ga_{0.7}Al_{0.3}As$ quantum well have been calculated. The intersubband energies have been found. The optical susceptibilities, the Rabi frequency and the detuning parameters have been investigated. The interaction between the electromagnetic field and the taken semiconducting system has been defined within the rotating wave approximation. The group velocity can be controlled by changing the intensity. The large nonlinear susceptibilities are observed it is because the light travels slowly spending more time interacting with the material. It is brought out that the lowest optical transition energies are controlled by the excitonic effects which lead to different nonlinear optical properties. Further, the results show that the Rabi frequency and detuning parameter can alter the optical susceptibilities with the application of slow light. The results can be applied for building optical switching devices in near future.

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