Analysis of Single Server Retrial Queue with General Retrial Time, Impatient Subscribers, Two Phases of Service and Bernoulli Schedule

[1] Elakiya ravitha.S, [1] Research scholar

[2]Revathi, , [2] Assistant Professor,

Prist University, Pudhucherry.

[1]Prist university ,Villupuram, Tamilnadu, India
email: elakiya.ravitha@gmail.com

Abstract : In this chapter, We discussed about a single sever retrial queue with impatient subscribers, general retrial time, two phases of essential service and general vacation time under Bernoulli schedule is analyzed under the condition of stability. Performance measures are also computed in steady state. Some numerical illustrations are also presented with potential applications.

I. Introduction

Retrial queueing system was discussed by the feature of arriving customers were encounter when the server busy, to join a virtual pool is called orbit. Customer in an orbit generating the stream of repeatition in the queueing system is that independent of the remaining customers in the orbit. Such queueing system played a main role in the analysis of many telephone switching systems, telecommunications networks and computer systems. The result on M/G/1 model of the retrial queueing system, Dr.Keilson is studied the method of studying the supplementary variable technique is to investigate the joining distribution of the general state of the system and the number of customers in orbit in the steady state. Aleksandrov explained the arbitrary distributed service times of the retrial time. A variant of the M/G/1 retrial queue of the system is also discussed by Neuts and Ramalhoto studied the results on M/G/1 queueing system with N-policy of the customers. The detailed overviews of the references with the system of retrial queues can be referred in the papers of Falin and Templeton survey of Artalejo, later Gautam Choudhury analyzed an M/G/1 queueing system with linear retrial policy with two phase of service and Bernoulli vacation schedule is also studied and explained. Modified vacation policy for M/G/1 retrial queueing system with balking was discussed by Ke and Chang analyzed the M/G/1 retrial queue with pre emptive resume and its feedback under N-policy to the server breakdowns and repairs. Queueing systems with batch arrivals are commonly used in many practical situations. In digital communication systems, messages which are transmitted will consist of a random number of packets.

Falin introduced the batch arrival retrial queueing model and assumed the following rule: If the server is busy at the arrival epoch of the system, then the whole batch joins the retrial group, whereas when server free then one of the arriving units starts the service and the rest of its joins the retrial group. Krishnakumar has analyzed the bulk arrival retrial queue with the multiple vacations and starting failures of the systems. Fu-Min Chang analyzed a batch retrial queueing model with J number of vacations. Senthil Kumar have analyzed the batch arrival single server retrial queue in the server of the general time which provides two phases of heterogeneous service and general vacation time under Bernoulli schedule. Choudhury has analyzed a batch arrival retrial queueing system with two phases of service and also service interruption is counted into the system.

The Mathematical Model

In this chapter, a single server retrial queueing system is considered. The primary calls arrives according to the poisson process with a rate l. If the primary call, on arrival finds the server busy, it becomes an impatient and leaves the system with probability (1□a) and with probability a, it enters into a retrying pool. The server provides the preliminary first essential service (FES) and second essential service (SES) to all the arriving customers. As soon as the SES of a call is completed and the server may go for a vacation of random length S3 with probability ‘p’(0 _ p _ 1) or it may remain to serve for the next call, if any, with probability ‘q’(= 1□ p).

To find Primary calls the free server arrival automatically get their FES. However, if a primary call finds the server busy, then it joins the orbit with probability a, to seek service again whether they finds the server free. Otherwise, it leaves the system with probability (1□a). The time between two successive repeated attempts of each service in orbit is assumed to be a general vacation.

0.0.1 Notations

The server state can be denoted as,

$$C(t) = \begin{cases} 0; \ if\ the\ server\ is\ idle \\ 1; \ if\ the\ server\ is\ doing\ FES \\ 2; \ if\ the\ server\ is\ doing\ SES \\ 3; \ if\ the\ server\ is\ on\ vacation \end{cases} :$$

Now probability functions defined as,

$$P_{0,0}(t) = Pr\{N(t)=0,\ C(t)=0\};\ n\geq 1$$

$$P_{0,n}(x,t)dt = Pr\{N(t)=n,\ C(t)=i,\ x\leq R^0(t)\leq x+dt\};\ n\geq 1$$

$$P_{i,n}(x,t)dt = Pr\{N(t)=n,\ C(t)=i,\ x\leq S_i^0(t)\leq x+dt\};\ n>0;\ i=1,2,3$$

Steady State Queue Size Distribution

These equations obtain for the queueing system, using supplementary variable technique.

$$\begin{aligned}
P_{0,0}(t+\Delta t) =& P_{0,0}(t)(1-\lambda\Delta t)+P_{3,0}(0,t)\Delta t \\
&+qP_{2,0}(0,t)\Delta t \\
P_{0,j}(x-\Delta t,\ t+\Delta t) =& P_{0,j}(x,t)(1-\lambda\Delta t)+P_{3,j}(0,t)r(x)\Delta t \\
&+qP_{2,0}(0,t)r(x)\Delta t \\
P_{1,0}(y-\Delta t,\ t+\Delta t) =& P_{1,0}(y,t)(1-\lambda\alpha\Delta t)+\lambda\Delta t P_{0,0}(t)s_1(y) \\
P_{1,j}(y-\Delta t,\ t+\Delta t) =& P_{1,j}(y,t)(1-\lambda\alpha\Delta t)+\lambda\Delta t\left[\int_0^\infty P_{0,j}(x,t)dx\right]s_1(y) \\
&+P_{0,j+1}(0,t)s_1(y)\Delta t+\lambda\alpha\Delta t P_{1,j-1}(y,t) \\
P_{2,j}(x-\Delta t,\ t+\Delta t) =& P_{2,j}(x,t)(1-\lambda\alpha\Delta t)+\lambda\alpha\Delta t P_{2,j-1}(x,t) \\
&+P_{1,j}(0,t)s_2(x)\Delta t \\
P_{3,j}(x-\Delta t,\ t+\Delta t) =& P_{3,j}(x,t)(1-\lambda\alpha\Delta t)+\lambda\alpha\Delta t P_{3,j-1}(x,t) \\
&+pP_{2,j}(0,t)s_3(x)\Delta t
\end{aligned}$$

we have Laplace-Stieljes Transform of $P_{i,j}(x)$,

$$P_{i,j}(x)=\widetilde{P}_{i,j}(\theta);\ i=0,1,2,3.$$

We have,

$$\theta\widetilde{P}_{0,j}(\theta)-P_{0,j}(0)=\lambda\widetilde{P}_{0,j}(\theta)-P_{3,j}(0)\widetilde{R}(\theta)-qP_{2,j}(0)\widetilde{R}(\theta)$$
$$\theta\widetilde{P}_{1,0}(\theta)-P_{1,j}(0)=\lambda\alpha\widetilde{P}_{1,0}(\theta)-\lambda P_{0,0}\widetilde{S}_1(\theta)$$
$$\theta\widetilde{P}_{1,j}(\theta)-P_{1,j}(0)=\lambda\alpha\widetilde{P}_{1,j}(\theta)-\lambda\int_0^\infty P_{0,j}(x)dx\widetilde{S}_1(\theta)$$
$$-P_{0,j+1}(0)\widetilde{S}_1(\theta)-\lambda\alpha\widetilde{P}_{1,j-1}(\theta)$$
$$\theta\widetilde{P}_{2,j}(\theta)-P_{2,j}(0)=\lambda\alpha\widetilde{P}_{2,j}(\theta)-\lambda\alpha\widetilde{P}_{2,j-1}(\theta)-P_{1,j}(0)\widetilde{S}_2(\theta)\theta\widetilde{P}_{3,j}(\theta)$$

The Probability Generating Functions (PGF)

We have,

$$\widetilde{P}_0(z,\theta)=\sum_{j=1}^{\infty}\widetilde{P}_{0,j}(\theta)z^j,$$
$$P_0(z,0)=\sum_{j=1}^{\infty}P_{0,j}(0)z^j,$$
$$\widetilde{P}_i(z,\theta)=\sum_{j=0}^{\infty}\widetilde{P}_{i,j}(\theta)z^j,$$
$$P_i(z,0)=\sum_{j=0}^{\infty}P_{i,j}(0)z^j;\ i=1,2,3.$$

$$(\theta-\lambda)\widetilde{P}_0(z,\theta)=P_0(z,0)-\widetilde{R}(\theta)$$
$$[P_3(z,0)+qP_2(z,0)-\lambda P_{0,0}]$$
$$(\theta-\lambda\alpha+\lambda\alpha z)\widetilde{P}_1(z,\theta)=P_1(z,0)-\left[\lambda P_{0,0}+\lambda\widetilde{P}_0(z,0)+\frac{1}{z}P_0(z,0)\right]\widetilde{S}_1(\theta)$$

$$(\theta-\lambda\alpha+\lambda\alpha z)\widetilde{P}_2(z,\theta)=P_2(z,0)-P_1(z,0)\widetilde{S}_2(\theta)$$
$$(\theta-\lambda\alpha+\lambda\alpha z)\widetilde{P}_3(z,\theta)=P_3(z,0)-pP_2(z,0)\widetilde{S}_3(\theta)$$

$$P_0(z,0)=\widetilde{R}(\lambda)[P_3(z,0)+qP_2(z,0)-\lambda P_{0,0}]$$
$$P_1(z,0)=\left[\lambda P_{0,0}+\lambda\widetilde{P}_0(z,0)+\frac{1}{z}P_0(z,0)\right]\widetilde{S}_1(\alpha\lambda-\alpha\lambda z)$$
$$P_2(z,0)=P_1(z,0)\widetilde{S}_2(\lambda\alpha-\lambda\alpha z)$$
$$P_3(z,0)=pP_2(z,0)\widetilde{S}_3(\lambda\alpha-\lambda\alpha z)$$

$$\widetilde{P}_0(z,0)=\frac{1-\widetilde{R}(\lambda)}{\lambda}[P_3(z,0)+qP_2(z,0)-\lambda P_{0,0}]$$

$$(\theta - \lambda\alpha + \lambda\alpha z)\widetilde{P}_1(z,\theta) = \left(\widetilde{S}_1(\lambda\alpha - \lambda\alpha z) - \widetilde{S}_1(\theta)\right)\left[\lambda P_{0,0} + \frac{[1-\widetilde{R}(\lambda)]z + \widetilde{R}(\lambda)}{z\widetilde{R}(\lambda)} P_0(z,0)\right]$$

$$\widetilde{P}_1(z,0) = \frac{(\widetilde{S}_1(\lambda\alpha - \lambda\alpha z) - 1)}{(-\lambda + \lambda z)\alpha}\left[\lambda P_{0,0} + \frac{[1-\widetilde{R}(\lambda)]z + \widetilde{R}(\lambda)}{z\widetilde{R}(\lambda)} P_0(z,0)\right]$$

$$\widetilde{P}_2(z,0) = \frac{\widetilde{S}_1(\lambda\alpha - \lambda\alpha z)(\widetilde{S}_2(\lambda\alpha - \lambda\alpha z) - 1)}{(-\lambda + \lambda z)\alpha}\left[\lambda P_{0,0} + \frac{[1-\widetilde{R}(\lambda)]z + \widetilde{R}(\lambda)}{z\widetilde{R}(\lambda)} P_0(z,0)\right]$$

$$\widetilde{P}_3(z,0) = \frac{\widetilde{S}_1(\lambda\alpha - \lambda\alpha z)\widetilde{S}_2(\lambda\alpha - \lambda\alpha z)p(\widetilde{S}_3(\lambda\alpha - \lambda\alpha z) - 1)}{(-\lambda + \lambda z)\alpha}\left[\lambda P_{0,0} + \frac{[1-\widetilde{R}(\lambda)]z + \widetilde{R}(\lambda)}{z\widetilde{R}(\lambda)} P_0(z,0)\right]$$

It should noted that the probability generating function P(z) of no. of customers in orbit at the arbitrary epoch can expressed as follows,

$$P(z) = P_{0,0} + \widetilde{P}_0(z,0) + \widetilde{P}_1(z,0) + \widetilde{P}_2(z,0) + \widetilde{P}_3(z,0)$$

$$P(z) = \frac{P_{0,0}\left[\alpha\widetilde{R}(\lambda)[z - K(z)] + (K(z) - 1)\widetilde{R}(\lambda)\right]}{\alpha\left[z - K(z)(\widetilde{R}(\lambda) + (1 - \widetilde{R}(\lambda))z)\right]}$$

where

$$K(z) = [\widetilde{S}_1(\lambda\alpha - \lambda\alpha z)\widetilde{S}_2(\lambda\alpha - \lambda\alpha z)(p\widetilde{S}_3(\lambda\alpha - \lambda\alpha z) + q)]$$

It can be immediately follows that the steady state condition of the system is,

$$\rho\alpha = \lambda\alpha(E(S_1) + E(S_2) + pE(S_3)) < \widetilde{R}(\lambda)$$

$$i.e. \quad \rho = \lambda(E(S_1) + E(S_2) + pE(S_3)).$$

and

$$P_{0,0} = \frac{(\widetilde{R}(\lambda) - \rho\alpha)}{\widetilde{R}(\lambda)[\rho(1-\alpha) + 1]}$$

(a). The mean number of customers in the orbit of the system.

$$L = E[N(t)] = \lim_{z\to 1}\frac{d}{dz}P(z)$$

$$L = \left[\frac{\beta_2(1-\alpha)}{2\alpha(1+\rho(1-\alpha))} + \frac{\beta_2 + 2\rho\alpha(1-\widetilde{R}(\lambda))}{2(\widetilde{R}(\lambda)-\rho\alpha)}\right].$$

where

$$\beta_2 = \lambda^2(E[S_1^2]+E[S_2^2]+pE[S_3^2]+2E[S_1]E[S_2] + 2E[S_1]pE[S_3]+2E[S_2]pE[S_3])$$

0.2 Numerical Illustration

Here, we presented the some numerical results to study the effect of Bernoulli vacation probability ‘p’ and the effect of persistent probability of the system a on mean orbit size and the mean waiting time of the system. Mail system used Simple Mail Transfer Protocol (SMTP) to deliver messages between mail servers at time to time. Typically, contacting messages arrive at the mail server following the Poisson stream of the system. If all packets of a message are arrived to the mail server, the server starts to do service in FCFS. When all messages comes to the mail server, a packet is selected to serve and the rest will join to the buffer in a retrial group of the system. In the buffer of the system, each packet waits a certain amount of time and retries the service again and again.

It is verified for the effect of the following parameters ‘p’ and retrial rate ‘v’ with mean number of the buffer in the packets LQ and with average waiting time of a packet in buffer. In this section the effect of the parameters of the system is analysed numerically with these assumptions.

(i) Average arrival rate of messages in the system l = 1:5

(ii) Exponential retrial rate in the system n

In this table 1, for vacation time parameter s3 = 20 & s3 = 30, the mean waiting time is compared with varying values of a when FES, SES and Vacation time distribution follow Exponential distribution, Erlangian of order two. It is observed that the rate of increase in orbit size L for vacation parameter in the system s3 = 20 is faster than that for s3 = 30.

Also, as a increases, mean waiting time is also increased in the system (ie., as probability of impatience decreases, mean waiting time is increased).

Table 1. Mean waiting time (W)

a	W Exponential distribution		W Erlangian of order two	
	$s_3 = 20$	$s_3 = 30$	$s_3 = 20$	$s_3 = 30$
0.1	0.350579	0.325612	0.766359	0.710337
0.2	0.362952	0.336291	0.825275	0.760937
0.3	0.376212	0.34768	0.893602	0.818997
0.4	0.390457	0.359852	0.97373	0.886254
0.5	0.405799	0.37289	1.068924	0.965025
0.6	0.422368	0.386888	1.183754	1.058465
0.7	0.440315	0.401956	1.32482	1.170975
0.8	0.459816	0.418217	1.502005	1.30889
0.9	0.481078	0.435819	1.730791	1.481651
1	0.504348	0.45493	2.036842	1.703968

Bibliography

[1] Madan, K. C. and Choudhury, G. (2005), 'A single server queue with two phases of heterogeneous service under Bernoulli schedule and a general vacation time', International Journal of Information and Management Sciences, Vol. 16, No. 2, pp. 1-16.
[2] Medhi, J. (2002), 'A single server Poisson input queue with a second optional channel', Queueing Systems, Vol. 42, pp. 239-242.
[3] Senthil Kumar, M., and Arumuganathan, R. (2008), "On the Single Server Batch Arrival Retrial Queue with General Vacation Time under Bernoulli Schedule and Two Phases of Heterogeneous Service", Quality Technology and Quantitative Management, Vol. 5, pp. 145-160.
[4] Wang J. and Li J. (2008a), 'A repairable M/G/1 retrial queue with Bernoulli vacation and two phase service