Studies onThermo-FluidicCharacteristics for Flow within a Rectangular Microchannel

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Abstract : Flow through microchannels are of substantial importance as on date due to its multi-dimensional applications in modern electronic industries, Micro Electro Mechanical Systems (MEMS) as well as micro biological systems and various other applications. Hence, systematic studied are required for proper understanding of the flow behavior within a microchannel. In the present work, fluid flow and heat transfer characteristics within a microchannel of rectangular cross section is considered.The reduced form of the Navier-Stokes equation is valid for flow within microchannel as well. Thus, the Navier-Stokes and energy equations subjected to slip boundary conditions are solved byadopting various approximate analytical techniques.The velocity and temperature distributions are obtained under slip boundary conditions. The results obtained have been presented along a particular section of the channel and compared with the numerical solutions. By varying the aspect ratio of the channel, the accuracy of different approximate techniques has also been compared.It is observed that the accuracy level for the velocity as well as temperature profiles obtained in each technique depends on the aspect ratio.

***IndexTerms*–Rectangular microchannel, slip flow, analytical techniques, Knudsen Number.**

I. INTRODUCTION

With the advancement of micro electro mechanical system, its application areas like micro-heat exchangers, micro-valves and many other micro-fluidic systems have been started developing. Therefore fabrication of MEMS and understanding of flow through microchannels have gained considerable importance in modern days. Though it is difficult to precisely define the micro channels, however based on extensive literature surveys it can be concluded that, a channel having characteristics dimension in the order of 100 μm $\geq D_c >$ 10 μm may be considered as micro channels. Basic dynamic laws of Physics such as Newton's laws of motion, Newton's law of viscosity, Pascal's law etc together culminate the Navier-Stokes equation of fluid mechanics, which is basically the governing equation of fluid mechanics. Theoretically, Navier-Stokes equations are believed to be complete and any kind of fluid flow problems can be solved with the help of these equations. The classical Navier-Stokes equations fail to predict the behavior of ideal gas flows through microchannels at high Knudsen numbers [1], known as slip flow. Due to the confined area of the channel the mean free path of the fluid is comparable to the characteristic length of the channel leading to slip-like flow behavior and strong diffusion-enhanced transport of mass and momentum. To solve the slip flow problems theoretically as well as numerically, a tuning parameter is required to be incorporated, which is known as tangential momentum accommodation coefficient (TMAC). The introduction of TMAC considers the effect of slip at the wall. Deviations from classical macro scale flows include observations of higher mass flow rates through the channel [2] and nonlinear pressure drop along the channel [3].

When the molecular mean free path of the of the fluid is comparable with the system's characteristic lengths, then the fluid's molecular structure become more important and the traditional continuum assumptions does not hold good any longer. In such situations fluid exhibits non continuum behavior such as slip flow (that is non-zero velocity at the boundary) and thermodynamic temperature jump (that is fluid temperature at the boundary is different from that of the boundary itself) at the solid-fluid interface. Therefore, a non-dimensional parameter called Knudsen Number, defined as Kn = Mean free path of the fluid particle/characteristic length of the channel, plays a major role of deciding whether the flow is to be analyzed as conventional macro channel flow or a micro channel flow.

To solve the flow problems through a microchannel, either numerical or analytical methods may be adopted for solving the corresponding the Navier-Stokes equation. However, both the methods have their own merits & demerits. Like by numerical methods any kind of flow problems through any geometry may be solved but for generating proper mesh and obtaining subsequent solutions are quite laborious and cumbersome. For analytical techniques, it is very difficult to achieve the solution unless some simpler geometries and simpler initial and boundary conditions exist. The solution becomes more complicated due to the presence of slip in boundary walls.

From existing literatures, it is observed that analytical techniques are less developed for flow through micro channels. Therefore effort has been given to apply some approximate analytical techniques to determine velocity and temperature distribution for laminar fluid flow through a straight microchannel of rectangular cross section. Four approximate analytical methods, namely Integral Ritz, Integral Kantorovich, Variational Ritz andVariational Kantorovich have been considered.

II. DESCRIPTION OF THE PHYSICAL PROBLEM

For the purpose of analysis a rectangular microchannelhas been chosen. The flow of the fluid through the duct has been considered axi-symmetric. Following assumptions have also been made for the purpose of analysis. The flow is assumed to be incompressible, steady, laminar and fully developed and flowing fluid have constant thermo-physical properties. It is further assumes that body forces and axial conduction are not present and boundary walls are maintained at constant temperature. Flow is considered to be one dimensional (here in z direction).

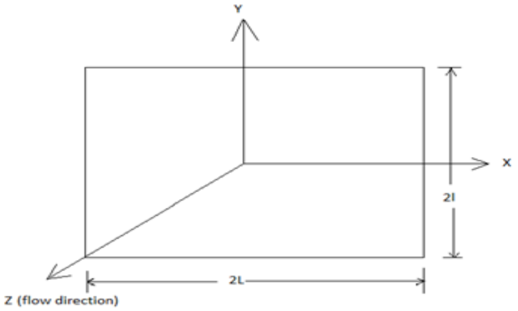

Fig. 1 Cross section of the microchannel

III. Mathematical Analysis

Governing Navier Stokes equations has been non-dimensionalized and simplified subjected to flow conditions and following equation obtained.

$$\frac{\partial^2 U}{\partial X^2} + \frac{1}{A^2}\frac{\partial^2 U}{\partial Y^2} = -1 \qquad (1)$$

Similarly non dimensionalized energy equation takes the form mentioned in Eq.2 after simplification.

$$A^2\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = \frac{A}{4}U(X,Y) \qquad (2)$$

Where X, Y, U and θ are non-dimensional forms of x, y, u (flow velocity) and T (temperature) respectively. Eq. 1 and 2 are subjected to the slip boundary conditions. Non dimensional slip velocity and temperature is given by Eq. 3 & 4 below.

$$U_S = \frac{F-2}{F}Kn\frac{\partial U}{\partial Y}_{wall} \qquad (3)$$

$$\theta_S = \frac{2-F_t}{F_t}\cdot\frac{Kn}{\Pr}\cdot\frac{2\gamma}{1+\gamma} \qquad (4)$$

Eq. 1and 2 have been solved subjected to the boundary conditions by different approximate analytical & Numerical methods. Expressions obtained by different approximate analytical methods are mentioned in following equations. Here A is the aspect ratio of the channel. Expressions obtained by applying approximate analytical methods are mentioned below

Integral Ritz Method (IR):

$$U(X,Y) = \frac{9(1-X^2)}{4(3+26A^2)}\left[\frac{2+25A^2}{(1+A^2)} + 5X^2\right](1-Y^2-2Y\cdot VF) \qquad (5)$$

$$\theta(X,Y) = (1-X^2)(a_0 + a_1X^2)(1-2Y\cdot TF - Y^2) \qquad (6)$$

Integral Kantorovich Method:

$$U(X,Y) = \frac{3}{2M_3}(1-Y^2)\left[1-(1-k_1Y^2)X_1 + (1-k_2Y^2)X_2\right] \qquad (7)$$
$$-\frac{3Y\cdot VF}{M_3}\left[1-(1-k_1Y^2)X_1 + (1-k_2Y^2)X_2\right] + \frac{3Y\cdot VF}{M_3}(1-Y^2)(k_1X_1 - k_2X_2)$$

$$\theta(X,Y) = (1-Y^2)[X_1(X) + Y^2X_2(X)] - 2Y\cdot TF\cdot X_1(X) + X_2(X)\cdot TF\cdot(2Y-4Y^3) \qquad (8)$$

Variational Ritz Method (IK):

$$U(X,Y) = \frac{9}{4}(1-X^2)(1-2Y\cdot VF - Y^2) \qquad (9)$$

$$\theta(X,Y) = -\frac{9A}{40(1+A^2)}(1-X^2)(1-2\cdot TF\cdot Y - Y^2) \qquad (10)$$

Variational Kantorovich Method (VK):

$$U(X,Y) = \frac{3}{2M_4}\left[\frac{\cosh\{\sqrt{5/2}(X/A)\}}{\cosh(\sqrt{5/2}/A)} - 1\right](1-Y^2-2\cdot VF\cdot Y) \qquad (11)$$

$$\theta(X,Y) = \frac{3A}{20}\left[\frac{\cosh\sqrt{(5/2)}(X/A)}{\cosh(\sqrt{5/2}/A)} - 1\right]\left[(1-2Y\cdot TF - Y^2)\right] \qquad (12)$$

To solve the equations numerically finite difference discritization scheme have been adopted. The entire cross section has been divided into $(m\times n)$ and therefore a total m×n grid points have been obtained. Therefore a total m×n numbers of discritized systems of liner equations have been obtained and solved by matrix method. Besides calculating the velocity and temperature profile, Poiseuille number has also been calculated by Numerical method, given as

$$Po = 2\left[\left(\sum_j\sum_i U_{i,j}\Delta X\Delta Y\right) + \left(\frac{2-F}{F}Kn\left(\frac{2A}{1+A}\right)^2\right)\right]^{-1}\left(\frac{2A}{1+A}\right)^2 \qquad (13)$$

IV. RESULTS AND DISCUSSION

Result obtained for velocity and temperature distribution using different approximate analytical methods have been compared with the results obtained by numerical methods for fluid having γ=1.4 &Prandtl Number (Pr) =0.7. For the purpose of better understanding results have been compared by plotting graphs. From the Fig. 2 and 3 of velocity and temperature obtained by applying different approximate methods, it is observed that, both velocity and temperature are strong functions of aspect ratio of the channel. From the same figures, it is evident that Integral Kantorovich method predicts the best result among different approximate methods considered. However the accuracy of the approximate solutions also varies with the channel aspect ratio. With increasing aspect ratio the accuracy levels decreases. From the same figures, it is also observed that, non-dimensional velocity and temperature at the boundary wall is non zero, which is expected for slip flow. Comparing the values for both velocity and temperature distribution obtained by the approximate analytical techniques with the values obtained by Numerical methods, it is observed that solutions obtained by approximate analytical techniques are very accurate (less than 5% error).

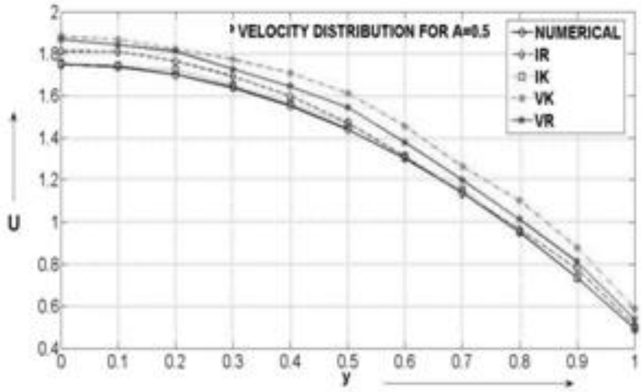

Fig. 2 Slip velocity distribution for A=0.5 of the channel

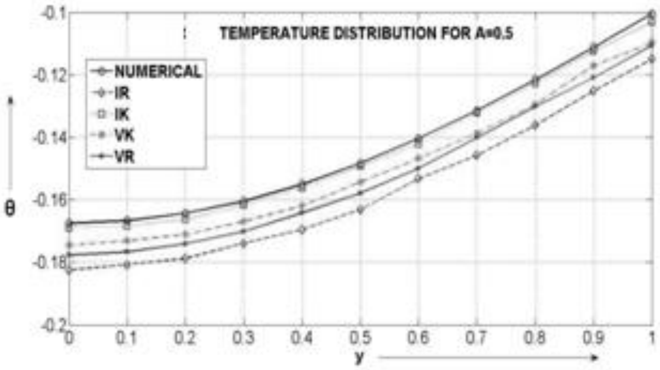

Fig. 3 Slip temperature distribution for A=0.5 of the channel

In Fig.4, the Poiseuille number is plotted with Knudsen Number at different aspect ratios and it is observed that Poiseuille number is in the range of 3.5 to 5. Such a low value of the Poiseuille number (defined as *Po=f. Re*) indicated that flow is within the laminar regime. As the Knudsen number goes on increasing, the value of *Po* goes on decreasing suggesting a decline of friction factor which is expected. Studying the variation of *Po* number, the power requirement for maintain the flow can be approximated.

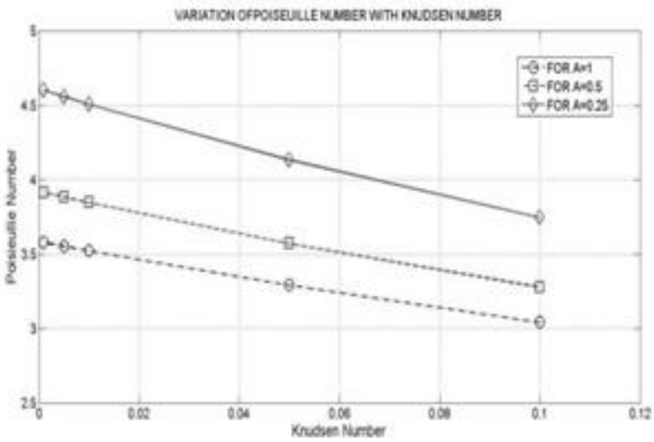

Fig. 4 Variation of Poiseuille Number with Knudsen Number

IV. CONCLUSION

Different approximate analytical methods have been considered for finding out velocity and temperature profile for hydro dynamically & thermally fully developed flow situations under slip condition. From mathematical modeling it has been observed that energy equation is coupled with velocity filed. Therefore to get best result for temperature distribution, most accurate methods adopted for finding out velocity distribution to be considered. Approximate methods are found to be a strong function of aspect ratio of the channel. Also the accuracy level of different aspect ratio depends upon the aspect ratio of the channel. Therefore proper approximate methods must be chosen, depending upon the aspect ratio of the channel. Therefore results obtained by applying approximate analytical techniques can be utilized without going for cumbersome exact analytical method or numerical method. In the present work,the Navier Stokes equation has been simplified (like viscous dissipation term neglected, one dimensional flow considered etc.) for simplification of mathematical analysis. In future,Navier Stokes equation may be solved without neglecting any term for getting solutions of more generalized flow problems.

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