Divisibility Theorem; Introduction of Quantization in number system and its application in deriving square root of any number (Aditya's Method).

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Abstract: The age old method to calculate the square root of any given integer manually is done using trial and error approach and statistically the chances of getting the correct result is very low. In this paper it is discussed how by applying the concept of quantization from quantum theory we can solve this age old algebraic problem and can view the number theory in a different approach. This approach is supported by experimental data and backed by theoretical derivations leading to the formation of a new theory and certain new concepts in the field of algebra in general and number theory in particular. It should be noted that this theorem is devised to have a perspective of the quantum nature of number line and as of now this concept is used to find the square root of an integer, helping any person to find them more efficiently while working manually only.

***Index Terms* – Number theory, algebra, quantum mechanics, quantization, conceptual mathematics, square root function.**

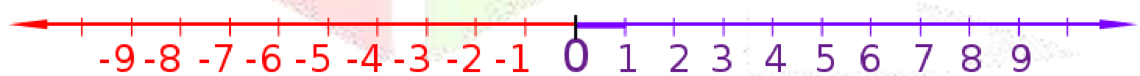
<u>Keywords</u>
NLP integer *– Nearest lowest perfect integer; a perfect square number which is less than and nearest to our given integer.*
Perfect square number- *A number which is the square of a same integer and hence on applying the square root function will give an integer as the result.* [a]
Divisibility constant- *A constant devised to help in simplifying the derivation of the divisibility theorem.*

I. <u>INTRODUCTION</u>

In this section certain important concepts are discussed as per their application in the development of the theory (discussed below).

THE NUMBER LINE

In mathematics we define number line as a method to map numbers on a graduated straight line. Every point on the number line represents a real number [1]. Hence it serves as a cogitative tool for understanding and interpretation of the numeric world [2].
A normal number line looks as follows: [3]



Every point on the number line represents a real number. The point zero on the line is considered as the origin point as per the Cartesian coordinate system, with reference to this point along the right side of the axis lies the positive numbers and on the left side lies the negative numbers.[4]-[6]

QUANTUM THEORY AND CONCEPT OF QUANTIZATION

It is worth mentioning that in this theory the idea and application of quantum theory lies in its very origin of quantization concept of quantum mechanics. The word quantum theory is often inscribed with physics and sub-atomic particles because of its role in specifying a particular branch of physics known as the quantum mechanics which deals mostly with subatomic particles and their behaviors [7]. Although this is almost true but that is not the entire picture of quantum theory. In fact the origin of quantum theory can be traced back to the proposition of the wave theory of light by Hook, Huygens and Euler. Following their footsteps many eminent physicists including Planck, Schrödinger and Albert Einstein also worked on this theory [8]-[9]. Although all of them have different application of the theory, there basic principle lies in the fact of quantization of photons (many times referred as the quanta). This is the origin of the word quantum. [10]- [12]
The concept of quantization lies in the idea that in any given system, the quantity that can act as the smallest individual component of that system and repeat itself in an integral fashion to build the entire system as a whole [13]-[14], for example photons can be considered as the quantized particle in a light wave. This building block unit is said as the quanta of the system and the process is called as

quantization. [15]- [17] Although the theory of quantization is exclusively used in physical sciences, this concept is used here in developing the entire theorem.
From the various experimental data collected for various cases of the theorem it is found that there is a small unit which is almost constant and keep repeating itself throughout the series. It behaves like cells of our body, almost identical and act as a building block. In each of the derivation it is explained which is the repetitive unit and how quantization is maintained. However it should be noted that since this is a theoretical approach [18], hence we have to consider some approximation along with the application of the correspondence principle [19] and hence there is always a chance of small error, which is true for almost every scientific theorem. [20]

CONCEPT OF ORDER OF INTEGER

"Integral numbers are the fountainhead of all mathematics" was once said by the famous German mathematician H.Minkowski. It is one of the oldest branches of mathematics ever created and ranges back to the Babylonians and ancient Egyptians for a core of information about the properties of the natural numbers.[21] – [23], [26]
A striking fact that was found in one of the ancient culture of the Amazonian tribes is the importance given to number 10. [24] It can be noted that in many instances that this number serves as a marker in the number line in deciding one key factor; the order of the number.[25] This is something all of us and can define from our basic understanding of mathematics, which includes single, double, triple digits and so on. [27]- [28] A single digit number is the one which is less than 10 ($<10^1$), similarly a double digit is one which is less than 100 (10^2) and so one. This particular idea comes from a general sense and doesn't require much of an elaboration, however this particular idea is one of the building block of the theorem.
The following diagram can illustrate on how the number 10 can act as a milestone or indicator in the number line.

Fig. - 1: Illustration of number 10 as a marker in the number line.
In the above figure the purple square mark each power of the number 10, each power of 10 represents the pattern of digit before it. From the picture and our general knowledge we can predict one more important and rather interesting pattern, which is while increasing the order of the number we can observe an increase in the number of integers between them. For example, in case of 10 (10^1), the number of single digit integers are 9, for 100 (10^2), the number of double digit integer is 99 and so one. A person may wonder that what is the significance of these details, but that is where the concept of quantization comes into and help in developing the theorem. If we observe them closely we can gather the following information

Order of the Digit (n)	**Exponential Power of 10 = 10^{n-1}**	**Number of integers within the range= 10^n-1**
SINGLE DIGIT (n=1)	10^0	9 (FROM 1 TO 10)
DOUBLE DIGIT (n=2)	10^1	99 (FROM 1 TO 100)
TRIPPLE DIGIT (n=3)	10^2	999 (FROM 1 TO 1000)
QUADRIPPLE DIGIT (n=4)	10^3	9999 (FROM 1 TO 10000)

From the above table we can say that number of integers within a particular order of the number system = ***10^n-1***. This denotes that we can quantify the entire number system because just by altering the value of **n** we can find the number of integers between two different orders of 10. Hence we can say that ***10^n-1*** is the quanta of the number line, and from henceforth will be termed as the **quanta of the divisibility theorem.**

SQUARE ROOT FUNCTION

Mathematically square root of an integer x is y, if

$y^2 = x$

Or, $y = \sqrt{x}$

Which means that a number when multiplied by itself will give a new value which is its squared value and by using the square root function we try to find back the original number. [29]

Every positive number has two square root, one positive and the other is negative [30]. Hence we can write the square root of 4 as $\sqrt{4} = \pm 2.$

However the square root of a negative number is defined as per the parameters of a complex number [31] and is not required to be cover as per the necessity of this paper. The idea of this theorem is to devise a simple method to replace the tiresome and unscientific trial and error method used today to calculate the square root of a given integer.

II. <u>THE THEOREM (Experiments and Observations)</u>

Let us consider that there is an integer **M** whose square root is **A.** The nearest and lowest perfect square number to **M** is **N.** The square root of **N** is **S.** Mathematically speaking,

$\sqrt{M} = A$ [Eq.-1]

$\sqrt{N} = S$ [Eq.-2]

Here divisibility theorem states that the difference between **A** & **S** is directly proportional to that of the difference between **M** & **N.**

<u>"The difference between any given number and its nearest lowest perfect square integer is directly proportional to the difference between their corresponding square roots"</u>

Which means that more the difference of the given number from the NLP number, more will the actual square root shift towards the next higher perfect square. This statement is proved in this paper with logical reasoning and with actual experimental data which are mentioned below¥.

Mathematically speaking,

$(A - S) \propto (M - N)$

Or, $(A - S) = K\,(M - N)$ [Eq.-3]

Or, K = $\frac{A-S}{M-N}$ [Eq.-3a]

Where K is a constant, which will be termed as <u>the divisibility constant</u> for all future references.

Or in other way we can say that;

<u>"The ratio of the difference between the square roots of any given number & its nearest lowest perfect square to the difference between the numbers approximately remains constant, provided other conditions of the theorem are satisfied"</u>

The whole idea from henceforth will be to derive the divisibility constant and the various methods equipped to do so.

For the simplicity of the theorem and its application, it is divided into two cases which are mentioned below.

- **Single digit natural numbers including fractional digits between 2 to 10**
- **Higher order of natural numbers**

Before proceeding, it should be noted that there are certain exception which are discussed under different corollaries. Two such corollaries are as follows:

1. **When the nearest, lowest perfect integer (NPL) will be 1.**
2. **When the given number is less than one but more than zero.**

¥ This statement is only valid when other conditions associated with it are satisfied which are mentioned under the various cases and corollaries. From the experiments we have proved beyond doubt that there is an approximate proportionality as stated by the theorem and using this we can derive the required equations.

- **Case 1: Single digit natural number**

Condition to be satisfied: 1. The given number (including fractions) must lie between 4 and 10.
This is to ensure that the lowest NLP integer is never 1 (which is an exception discussed later).

EXPERIMENT

In order to study the pattern and establish the idea of a single constant value which will act as a building block of the entire series, we collected the following data of numbers which satisfy the above condition, making sure that the NLP is not equal to 1.

Hence the number selected for this experiment starts from 4.1 and ends at 9.9. In the actual experiments we even had considered changing the second decimal place and find that the result holds well, hence for simplicity of calculation purpose only the first decimal point is altered. All the data are shown in **table 1** below.

OBSERVATION

We can study table 1 given below and make the following observations:

- The theorem holds good with its first statement as we can observe that by increasing the difference between the given number and its nearest perfect square root, there is a gradual increase in the difference between their square roots and vice versa.
- The second statement of the theorem says that if other conditions governing a series are satisfied than the ratio of the square roots and the given numbers (including the NLP integer) should approximately remain constant. This is also found to be true from the data and is approximately equal to 0.2 (from the last column).
- Hence for this particular case when the number is a single digit number and have a NLP which is more than 1, we can say that the divisibility constant **K** can be approximately taken to be **0.2**

MATHEMATICAL EQUATION

Hence from the observation made and the statement of the theorem we can say that equation 3a for this particular scenario can be modified as follows:

$$K=\frac{A-S}{M-N} \qquad \text{(from eq.-3a)}$$

or, $$0.2 = \frac{A-S}{M-N}$$

or, $$A= 0.2(M-N)+S \qquad \text{[Equation 4]}$$

INFERENCE

From the experiment for case 1 the following conclusion can be drawn:

- The statement of the theorem holds good and we have established the idea of quantization in the series by implying that the ratio of (A-S): (M-N) is almost constant and act as a building block in repeating itself and forwarding the series.
- Since this is a theoretical approach we have to make certain assumption which is true for all scientific theories and hence as a result there is a small percentage of error in the outcome which is almost negligible. To prove this some solved example for this particular condition are mentioned below.

Value of given number (M)	Value of the NLP integer (N)	M-N	Actual sq.root of M (A)	Sq, root of the NLP integer (S)	A-S	K=[(A-S)/(M-N)]
4.1	4	0.1	2.024845673	2	0.0248456731316584	0.248456731316584
4.2	4	0.2	2.049390153	2	0.0493901531919194	0.246950765959597
4.3	4	0.3	2.073644135	2	0.0736441353327719	0.24548045110924
4.4	4	0.4	2.097617696	2	0.0976176963403028	0.244044240850757
4.5	4	0.5	2.121320344	2	0.121320343559642	0.242640687119284
4.6	4	0.6	2.144761059	2	0.144761058952722	0.24126843158787
4.7	4	0.7	2.167948339	2	0.16794833886788	0.239926198382686
4.8	4	0.8	2.19089023	2	0.190890230020664	0.23861278752583
4.9	4	0.9	2.213594362	2	0.213594362117866	0.237327069019851
5	4	1	2.236067977	2	0.23606797749979	0.23606797749979
5.1	4	1.1	2.258317958	2	0.258317958127243	0.234834507388403
5.2	4	1.2	2.28035085	2	0.280350850198276	0.233625708498563
5.3	4	1.3	2.302172887	2	0.302172886644267	0.232440682034051
5.4	4	1.4	2.323790008	2	0.32379000772445	0.231278576946036
5.5	4	1.5	2.34520788	2	0.345207879911715	0.23013858660781
5.6	4	1.6	2.366431913	2	0.366431913239846	0.229019945774904
5.7	4	1.7	2.387467277	2	0.387467277262664	0.227921927801567
5.8	4	1.8	2.408318916	2	0.408318915758459	0.226843842088033
5.9	4	1.9	2.42899156	2	0.428991560298222	0.225785031735906
6	4	2	2.449489743	2	0.449489742783176	0.224744871391588
6.1	4	2.1	2.469817807	2	0.469817807045692	0.223722765259853
6.2	4	2.2	2.48997992	2	0.489979919597745	0.222718145271702
6.3	4	2.3	2.50998008	2	0.509980079602224	0.221730469392271
6.4	4	2.4	2.529822128	2	0.529822128134701	0.220759220056125
6.5	4	2.5	2.549509757	2	0.54950975679639	0.219803902718556
6.6	4	2.6	2.569046516	2	0.569046515733024	0.218864044512702
6.7	4	2.7	2.588435821	2	0.588435821108955	0.217939193003317
6.8	4	2.8	2.607680962	2	0.607680962081058	0.217028915028949
6.9	4	2.9	2.626785107	2	0.626785107312737	0.216132795625082
7	4	3	2.645751311	2	0.645751311064589	0.21525043702153
7.1	4	3.1	2.664582519	2	0.664582518894844	0.214381457708014
7.2	4	3.2	2.683281573	2	0.683281572999746	0.213525491562421
7.3	4	3.3	2.701851217	2	0.701851217221257	0.212682187036745
7.4	4	3.4	2.720294102	2	0.720294101747087	0.211851206396202
7.5	4	3.5	2.738612788	2	0.738612787525829	0.21103222500738
7.6	4	3.6	2.75680975	2	0.756809750418042	0.210224930671678
7.7	4	3.7	2.774887385	2	0.77488738510232	0.209429023000627
7.8	4	3.8	2.792848009	2	0.792848008753786	0.208644212829944
7.9	4	3.9	2.810693865	2	0.810693864511038	0.207870221669497
8	4	4	2.828427125	2	0.828427124746189	0.207106781186547
8.1	4	4.1	2.846049894	2	0.84604989415154	0.206353632719888
8.2	4	4.2	2.863564213	2	0.863564212655269	0.205610526822683
8.3	4	4.3	2.880972058	2	0.880972058177585	0.204877222831996
8.4	4	4.4	2.898275349	2	0.898275349237884	0.204153488463155
8.5	4	4.5	2.915475947	2	0.915475947422647	0.203439099427255
8.6	4	4.6	2.93257566	2	0.932575659723033	0.202733839070225
8.7	4	4.7	2.949576241	2	0.949576240750522	0.202037498032026
8.8	4	4.8	2.966479395	2	0.966479394838262	0.201349873924638
8.9	4	4.9	2.983286778	2	0.983286778035256	0.200670771027603
9.1	9	0.1	3.016620626	3	0.0166206257996713	0.166206257996713
9.2	9	0.2	3.033150178	3	0.0331501776206169	0.165750888103084
9.3	9	0.3	3.049590136	3	0.0495901363953779	0.16530045465126
9.4	9	0.4	3.065941943	3	0.065941943351175	0.164854858377937
9.5	9	0.5	3.082207001	3	0.0822070014844849	0.16441400296897
9.6	9	0.6	3.098386677	3	0.0983866769659305	0.163977794943217
9.7	9	0.7	3.1144823	3	0.114482300479484	0.16354614354212
9.8	9	0.8	3.130495168	3	0.130495168499702	0.163118960624627
9.9	9	0.9	3.146426545	3	0.146426544510452	0.162696160567169

TABLE 1: EXPERIMENTAL DATA FOR CASE 1 OF DIVISIBILITY THEOREM

Q1- Find the square root of the following numbers using divisibility theorem.

- **a) 7.5**
- **b) 8**

Sol. - a) Here the given number M = 7.5. The nearest lowest perfect square; N= 4, whose square root; S=2. Now using the divisibility theorem we can say that:

$$A = (M - N) \times 0.2 + S$$

Therefore, A = (7.5 - 4) x 0.2 + 2 = **2.70**
The actual value of the square root of 7.5 is 2.738.
Hence percentage of accuracy = 98.61%

b) In this question, the given information are as follows:
M=8, N= 4, S= 2.
Hence, $A = (M - N) \times 0.2 + S$

A= (8 -4) x 0.2 + 2 = **2.8**.
Actual value= 2.838
Hence percentage of accuracy= 98.66%
If considered up to one decimal place we can say that the theorem yields 100% accurate results and that to in a very short and simple manner, instead of the traditional tiresome trial and error method.

- **Case 2: Higher order of natural number**

Although the theorem is universally true throughout the number system but as we move high in the number line then the idea of quantized factors of the increasing number between each order of the number line comes into play which is described in the introduction to quantization topic and this lead to a deviation from the standard.[32] Even when conducting the various experimental studies it is being observed that this quanta of divisibility theorem plays a vital role in determining the divisibility constant, which can be observed in the data collected in the table below. To eliminate such deviation the formula is modified as follows.

Conditions to be satisfied: Any given number (including fractions/decimals) which is more than or equal to 10; that is it can be double digit, triple digit or higher.

EXPERIMENT

In order to find a correlation between the numbers of various order, hundreds of numbers are studied along with their square roots but in order to save, only a handful of them are shown in the observation table. However it should be noted that we can observe all those individual properties even in these few given numbers. In table 2 a variety of numbers are selected for the experiment to diversify the input numbers and obtain an accurate results.

TABLE 2: Data Collected to identify the divisibility constant for higher order of numbers

M	N	(M-N)	A	S	(A-S)	K=(A-S)/(M-N)	K X S =K'
10	9	1	3.162278	3	0.162278	0.16227766	0.486833
11	9	2	3.316625	3	0.316625	0.158312395	0.474937
12	9	3	3.464102	3	0.464102	0.154700538	0.464102
13	9	4	3.605551	3	0.605551	0.151387819	0.454163
14	9	5	3.741657	3	0.741657	0.148331477	0.444994
15	9	6	3.872983	3	0.872983	0.145497224	0.436492
16	16	0	IT IS A PERFECT SQUARE AND HENCE NOT APPLICAI				NA
17	16	1	4.123106	4	0.123106	0.123105626	0.492423
18	16	2	4.242641	4	0.242641	0.121320344	0.485281
19	16	3	4.358899	4	0.358899	0.119632981	0.478532
20	16	4	4.472136	4	0.472136	0.118033989	0.472136
21	16	5	4.582576	4	0.582576	0.116515139	0.466061
22	16	6	4.690416	4	0.690416	0.115069293	0.460277
23	16	7	4.795832	4	0.795832	0.113690218	0.454761
24	16	8	4.898979	4	0.898979	0.112372436	0.44949
25	NA	NA	NA	NA	NA	NA	NA
26	25	1	5.09902	5	0.09902	0.099019514	0.495098
27	25	2	5.196152	5	0.196152	0.098076211	0.490381
28	25	3	5.291503	5	0.291503	0.097167541	0.485838
29	25	4	5.385165	5	0.385165	0.096291202	0.481456
30	25	5	5.477226	5	0.477226	0.095445115	0.477226
31	25	6	5.567764	5	0.567764	0.094627394	0.473137
32	25	7	5.656854	5	0.656854	0.093836321	0.469182
33	25	8	5.744563	5	0.744563	0.093070331	0.465352
34	25	9	5.830952	5	0.830952	0.092327988	0.46164
35	25	10	5.91608	5	0.91608	0.091607978	0.45804
36	NA	NA	NA	NA	NA	NA	NA
37	36	1	6.082763	6	0.082763	0.08276253	0.496575
38	36	2	6.164414	6	0.164414	0.082207001	0.493242
39	36	3	6.244998	6	0.244998	0.081665999	0.489996
40	36	4	6.324555	6	0.324555	0.08113883	0.486833
41	36	5	6.403124	6	0.403124	0.080624847	0.483749
42	36	6	6.480741	6	0.480741	0.08012345	0.480741
43	36	7	6.557439	6	0.557439	0.079634075	0.477804
44	36	8	6.63325	6	0.63325	0.079156198	0.474937
45	36	9	6.708204	6	0.708204	0.078689326	0.472136
46	36	10	6.78233	6	0.78233	0.078232998	0.469398
47	36	11	6.855655	6	0.855655	0.077786782	0.466721
48	36	12	6.928203	6	0.928203	0.077350269	0.464102
49	NA	NA	NA	NA	NA	NA	NA
50	49	1	7.071068	7	0.071068	0.071067812	0.497475
51	49	2	7.141428	7	0.141428	0.070714214	0.494999
52	49	3	7.211103	7	0.211103	0.070367517	0.492573
53	49	4	7.28011	7	0.28011	0.070027472	0.490192
54	49	5	7.348469	7	0.348469	0.069693846	0.487857

56	49	7	7.483315	7	0.483315	0.06904497	0.483315
57	49	8	7.549834	7	0.549834	0.0687293	0.481105
58	49	9	7.615773	7	0.615773	0.06841923	0.478935
59	49	10	7.681146	7	0.681146	0.06811457	0.476802
60	49	11	7.745967	7	0.745967	0.06781515	0.474706
61	49	12	7.81025	7	0.81025	0.06752081	0.472646
62	49	13	7.874008	7	0.874008	0.06723137	0.47062
63	49	14	7.937254	7	0.937254	0.06694671	0.468627
64	NA	NA	NA	NA	NA	NA	NA
65	64	1	8.062258	8	0.062258	0.06225775	0.498062
66	64	2	8.124038	8	0.124038	0.0620192	0.496154
67	64	3	8.185353	8	0.185353	0.06178426	0.494274
68	64	4	8.246211	8	0.246211	0.06155281	0.492423
69	64	5	8.306624	8	0.306624	0.06132477	0.490598
70	64	6	8.3666	8	0.3666	0.06110004	0.4888
71	64	7	8.42615	8	0.42615	0.06087854	0.487028
72	64	8	8.485281	8	0.485281	0.06066017	0.485281
73	64	9	8.544004	8	0.544004	0.06044486	0.483559
74	64	10	8.602325	8	0.602325	0.06023253	0.48186
75	64	11	8.660254	8	0.660254	0.06002309	0.480185
76	64	12	8.717798	8	0.717798	0.05981649	0.478532
77	64	13	8.774964	8	0.774964	0.05961265	0.476901
78	64	14	8.831761	8	0.831761	0.05941149	0.475292
79	64	15	8.888194	8	0.888194	0.05921296	0.473704
80	64	16	8.944272	8	0.944272	0.05901699	0.472136
81	NA	NA	NA	NA	NA	NA	NA
82	81	1	9.055385	9	0.055385	0.05538514	0.498466
83	81	2	9.110434	9	0.110434	0.05521679	0.496951
84	81	3	9.165151	9	0.165151	0.05505046	0.495454
85	81	4	9.219544	9	0.219544	0.05488611	0.493975
86	81	5	9.273618	9	0.273618	0.0547237	0.492513
87	81	6	9.327379	9	0.327379	0.05456318	0.491069
88	81	7	9.380832	9	0.380832	0.0544045	0.489641
89	81	8	9.433981	9	0.433981	0.05424764	0.488229
90	81	9	9.486833	9	0.486833	0.05409255	0.486833
91	81	10	9.539392	9	0.539392	0.0539392	0.485453
92	81	11	9.591663	9	0.591663	0.05378755	0.484088
93	81	12	9.643651	9	0.643651	0.05363756	0.482738
94	81	13	9.69536	9	0.69536	0.05348921	0.481403
95	81	14	9.746794	9	0.746794	0.05334245	0.480082
96	81	15	9.797959	9	0.797959	0.05319726	0.478775
97	81	16	9.848858	9	0.848858	0.05305361	0.477483
98	81	17	9.899495	9	0.899495	0.05291147	0.476203
99	81	18	9.949874	9	0.949874	0.0527708	0.474937
100	NA	NA	10	NA	NA	NA	NA
101	100	1	10.04988	10	0.049876	0.04987562	0.498756

102	100	2	10.0995	10	0.099505	0.04975247	0.497525	
103	100	3	10.14889	10	0.148892	0.04963052	0.496305	
104	100	4	10.19804	10	0.198039	0.04950976	0.495098	
105	100	5	10.24695	10	0.246951	0.04939015	0.493902	
106	100	6	10.29563	10	0.29563	0.04927169	0.492717	
107	100	7	10.34408	10	0.34408	0.04915435	0.491543	
108	100	8	10.3923	10	0.392305	0.04903811	0.490381	
109	100	9	10.44031	10	0.440307	0.04892295	0.489229	
110	100	10	10.48809	10	0.488088	0.04880885	0.488088	
111	100	11	10.53565	10	0.535654	0.0486958	0.486958	
112	100	12	10.58301	10	0.583005	0.04858377	0.485838	
113	100	13	10.63015	10	0.630146	0.04847275	0.484728	
114	100	14	10.67708	10	0.677078	0.04836273	0.483627	
115	100	15	10.72381	10	0.723805	0.04825369	0.482537	
116	100	16	10.77033	10	0.77033	0.0481456	0.481456	
117	100	17	10.81665	10	0.816654	0.04803846	0.480385	
118	100	18	10.86278	10	0.86278	0.04793225	0.479322	
119	100	19	10.90871	10	0.908712	0.04782695	0.47827	
120	100	20	10.95445	10	0.954451	0.04772256	0.477226	
121	NA	NA	NA	NA	NA	NA	NA	
122	121	1	11.04536	11	0.045361	0.04536102	0.498971	
123	121	2	11.09054	11	0.090537	0.04526825	0.497951	
124	121	3	11.13553	11	0.135529	0.04517624	0.496939	
125	121	4	11.18034	11	0.18034	0.04508497	0.495935	
126	121	5	11.22497	11	0.224972	0.04499443	0.494939	
127	121	6	11.26943	11	0.269428	0.04490461	0.493951	
128	121	7	11.31371	11	0.313708	0.0448155	0.49297	
129	121	8	11.35782	11	0.357817	0.04472709	0.491998	
130	121	9	11.40175	11	0.401754	0.04463936	0.491033	
131	121	10	11.44552	11	0.445523	0.04455231	0.490075	
132	121	11	11.48913	11	0.489125	0.04446594	0.489125	
133	121	12	11.53256	11	0.532563	0.04438022	0.488182	
134	121	13	11.57584	11	0.575837	0.04429515	0.487247	
135	121	14	11.61895	11	0.61895	0.04421072	0.486318	
136	121	15	11.6619	11	0.661904	0.04412692	0.485396	
137	121	16	11.7047	11	0.7047	0.04404374	0.484481	
138	121	17	11.74734	11	0.74734	0.04396118	0.483573	
139	121	18	11.78983	11	0.789826	0.04387923	0.482672	
140	121	19	11.83216	11	0.83216	0.04379787	0.481777	
141	121	20	11.87434	11	0.874342	0.0437171	0.480888	
142	121	21	11.91638	11	0.916375	0.04363692	0.480006	
143	121	22	11.95826	11	0.958261	0.04355731	0.47913	
144	NA	NA	NA	NA	NA	NA	NA	

The series is ended here and we take some example of higher order of digits to find some interesting trend.

1000	961	39	31.62278	31	0.622777	0.01596863	0.495028	
1001	961	40	31.63858	31	0.638584	0.0159646	0.494903	
1002	961	41	31.65438	31	0.654384	0.01596058	0.494778	
1003	961	42	31.67018	31	0.670175	0.01595655	0.494653	
1004	961	43	31.68596	31	0.685959	0.01595254	0.494529	

10005	10000	5	100.025	100	0.024997	0.004999	0.499938
10006	10000	6	100.03	100	0.029996	0.004999	0.499925
10007	10000	7	100.035	100	0.034994	0.004999	0.499913
10008	10000	8	100.04	100	0.039992	0.004999	0.4999
10009	10000	9	100.045	100	0.04499	0.004999	0.499888
10010	10000	10	100.05	100	0.049988	0.004999	0.499875
10011	10000	11	100.055	100	0.054985	0.004999	0.499863
10012	10000	12	100.06	100	0.059982	0.004999	0.49985
10013	10000	13	100.065	100	0.064979	0.004998	0.499838
10014	10000	14	100.07	100	0.069976	0.004998	0.499825
10015	10000	15	100.075	100	0.074972	0.004998	0.499813
10016	10000	16	100.08	100	0.079968	0.004998	0.4998
10017	10000	17	100.085	100	0.084964	0.004998	0.499788
10018	10000	18	100.09	100	0.08996	0.004998	0.499775
10019	10000	19	100.095	100	0.094955	0.004998	0.499763
10020	10000	20	100.1	100	0.09995	0.004998	0.49975
10021	10000	21	100.1049	100	0.104945	0.004997	0.499738

OBSERVATION

By studying table 2, we make the following observations:

- The theorem holds good with its first statement as we can observe that by increasing the difference between the given number and its nearest perfect square root, there is a gradual increase in the difference between their square roots and vice versa.
- As in case of the second statement of the theorem which states that the divisibility constant must also remain approximately constant. This is only possible when we multiply the ratio with the square root of the NLP integer (as seen in the last column of table 2). There is an interesting reason why such an observation is made and this is explained in the inference section below.
- The divisibility constant in this case is obtained from the last column can be used to derive the final equation for this case.

MATHEMATICAL EQUATION

Hence from the observation made and the statement of the theorem we can say that equation 3a for this particular scenario can be modified as follows:

From table 2 we got that the approximate value of **K' = 0.5**.

Now, $K' = K \times S$ [From the table]

or, $K = \frac{K'}{S}$ [EQUATION NO. - 5]

Putting the value of K' in the above equation we get,

Or, $K = \frac{1}{2 \times S}$ [EQUATION NO.-6]

Hence putting this value of K in equation no. - 3a, we get:

$$A = \frac{M-N}{2\times S} + S \qquad \text{[EQUATION NO.-7]}$$

INFERENCE

From the observation made the following conclusions can be drawn,

- As mentioned in the quantization part in the introduction to this paper that the number line is quantized and it is explained in that section on how it is done. However from the table data we can say that the idea of quantization is not only limited to the order of the number but can be observed even in between different perfect square roots. This is observed in the table 2, as we proceed from one perfect square to the next, the number of whole numbers between them increases. For example, if we see the table, between 16 and 25 there are 8 whole numbers but from 25 to 36 it is 10.
- This actually shows that the perfect square indeed plays a crucial role in quantizing the number line in a deeper level. On further investigation it is found that this number can also be quantized.
- $\textit{Number of Integers between the next perfect square} = 2 \times S$ [Eq. No.- 8]

Where S is the square root of the NLP integer.

Hence we can predict that the number of integers between 36 to 49 will be given be,

No. of integers = 2 x $\sqrt{36}$ = 2x6 = 12.

This can be verified from the data of table 2. This information was used in finding the divisibility constant in this particular case, as it was observed that if the ratio of (A-S) & (M-N) is divided by eq. no. - 8, we get our perfect result.

To prove the findings a few examples are included here,

Q2. - Find the square root of the following number

a) **58, b) 654**

Sol. - a) Here M= 58, N= 49, S= 7.
Putting the formula of eq.-7 we get
A= 7.64.
<u>Actual value= 7.6157</u>
<u>Percentage of accuracy =99.64%</u>

b) Here, M= 654, N= 625, S=25
Putting the formula of eq.-7 we get
A= 25.58.
<u>Actual Value= 25.573.</u>
<u>Percentage of Accuracy = 99.97%</u>

<u>IT SHOULD BE NOTED THAT THIS FORMULA CAN ALSO BE APPLIED TO THAT OF SINGLE DIGIT NUMBER.</u>

Now, as mentioned before there are certain exception to this theorem, and to modify them a few modification are done to the original formula of equation- 3a. These modifications are mentioned under the following 2 corollaries.

<u>Corollary 1: When the nearest perfect square number is 1.</u>

The number 1 is a very interesting integer, because almost all mathematical functions applied on it will lead to itself, including exponential, square root and cube root. Not only is that it's the only number which when divided by itself will give itself as the result, while all other numbers when do so gives 1 as the final result. We cannot truly say that it is a prime number nor is it a true composite number. So somewhere it is rightly said that "*one is the loneliest number.*"

It should be noted that this corollary will lead quite accurate result however the accuracy will be less than that of the original theorem but to apply the original theorem we cannot consider the number 1 as the nearest lowest perfect integer because of its special property instead we have to move to at least one decimal accuracy of the nearest lowest perfect number. So for example, for the number 2 the nearest lowest prefect integer should be consider as 1.96 with square root of 1.4. But since it will be tiresome and difficult to predict the perfect square which is a fractional number and not a whole number, this corollary is developed.

Here we sacrifice a little accuracy for simplicity. A graphical representation of the accuracy of this corollary is discussed in the derivation portion.

Hence the formula was modified as under:

$$\mathbf{A = 1 + 0.02n} \qquad \text{[Eq.-9]}$$

Where, $n = 2x + 1$
And $x = \frac{M-1}{0.1}$
Where M= the given number,
A= the square root value to be calculated.
x amd n are $variables$ $used$ to $simplify$ the $equation$
Although the formula may look a little tricky it will be much clearer if we observe some examples.
Q3- Find the square root of the following integers:

- **a) 2**
- **b) 3**

Sol. - a) Here M= 2, hence $x = \frac{2-1}{0.1}$ = 10,
$n = 2 \times 10 + 1 = 21$
$A = 1 + 0.02n = 1.42$
Actual value = 1.4142~ 1.41
Hence percentage of accuracy = 99.29%
b) Here M= 3, hence $x = \frac{3-1}{0.1} = 20$
$n = 2 \times 20 + 1 = 41$
$A = 1 + 0.02n = 1.82$
Actual value= 1.732
Accuracy = 94.92%.
However if we solve the same problem using the original formula we get a more accurate result but it is really difficult in finding the nearest lowest perfect integer which is not a whole number.
In this case it will be equal to 2.89 with perfect square root of 1.7. Now applying the first theorem we get,
$A = (M - N) \times 0.2 + S$
$A = (3 - 2.89) \times 0.2 + 1.7 = 1.722$
Percentage of accuracy = 99.42%.

Derivation

The derivation of this corollary is done by employing constructive methods of mathematics which enable us to prove a theorem based on logical reasoning of data collected. Included below is the data collected for the square root of some numbers whose NLP is 1.

Number (M)	**values of x (x= (M-1)/0.1)**	**value of n (n=2x+1)**	**calculated sq. Root (1+0.02n)**	**actual sq. Root**
1	0	0	1	1
1.1	1	3	1.06	1.05
1.2	2	5	1.1	1.1
1.3	3	7	1.14	1.14
1.4	4	9	1.18	1.18
1.5	5	11	1.22	1.22
1.6	6	13	1.26	1.26
1.7	7	15	1.3	1.3
1.8	8	17	1.34	1.34
1.9	9	19	1.38	1.38
2	10	21	1.42	1.41
2.1	11	23	1.46	1.45
2.2	12	25	1.5	1.48
2.3	13	27	1.54	1.52
2.4	14	29	1.58	1.55
2.5	15	31	1.62	1.58
2.6	16	33	1.66	1.61
2.7	17	35	1.7	1.64
2.8	18	37	1.74	1.67
2.9	19	39	1.78	1.7
3	20	41	1.82	1.73

3.1	21	43	1.86	1.76
3.2	22	45	1.9	1.79
3.3	23	47	1.94	1.82
3.4	24	49	1.98	1.84
3.5	25	51	2.02	1.87
3.6	26	53	2.06	1.9
3.7	27	55	2.1	1.92
3.8	28	57	2.14	1.95
3.9	29	59	2.18	1.97

TABLE 3: Data for numbers whose NLP is 1

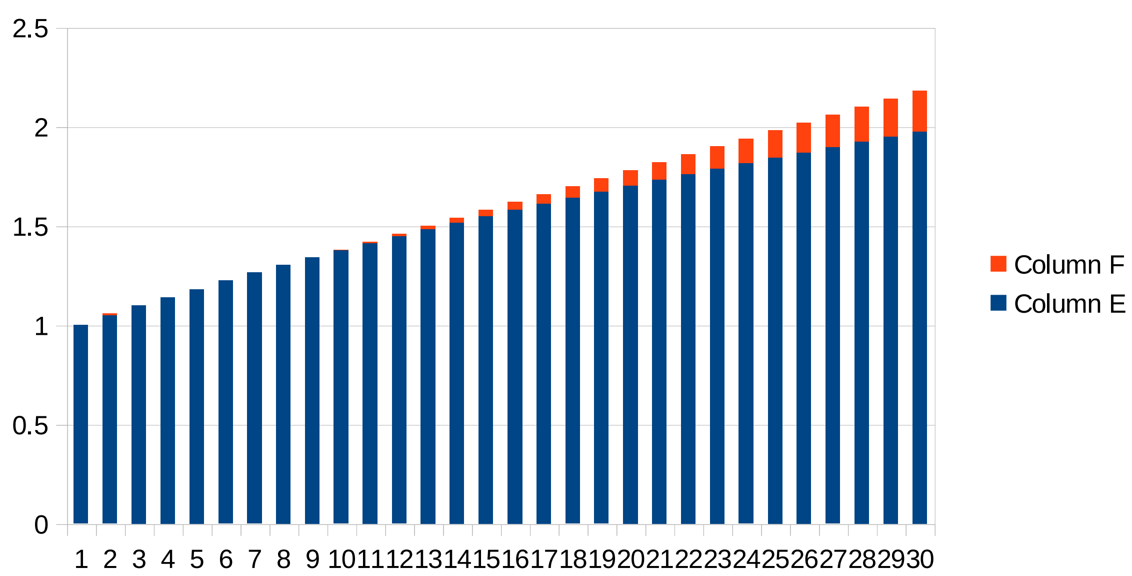
Let us consider that **M** is a given number whose square root is A**,** so mathematically:

$$\sqrt{M} = A \qquad \text{[Eq.-9.1]}$$

Now from the data collected in the table no.-3 we can say that that there exist a quantization pattern and for every increase of 0.1, we observe a fixed pattern of change for the actual square root values mentioned in the 5th column.
A rigorous work was put in to find the pattern in the series of the square roots mentioned in the 5th column and thereby by trial and error method the given pattern was found and was tested to be pretty accurate.

Observation

From a graphical plotting of the data it is observed that the results are quite satisfactory with a good percentage of accuracy.



Where the blue line shows the calculated value and the orange line shows the actual value. The deviation at the end shows that it is reaching close to the next NLP integer which is 2, and from henceforth the original theorem as mentioned in equation 3a should be used.
From the analysis of the data of table 3 we can say that the formula of this corollary can give a data accuracy of up to 100% and even in the worst scenario near reaching the higher values will give an accuracy of up to 90% with extreme ease as compared to the traditional trial and error method.

Corollary 2: For any number which is less than 1

The original formula was developed with the limitation that the nearest lowest perfect square has to be a whole number but when dealing with figures less than 1 it is to be noted that the formula is not applicable. Hence this corollary will deal with this particular type of situation. This formula was developed by studying an important similarity of quantization of the number line which is discussed in detail in the derivation section of this corollary.

If Z is the given number whose square root is to be calculated then the value will be given by

$$\sqrt{Z} = 0.1 \times \sqrt{y} \qquad [\text{Eq.} - 10]$$

Where y= $100 \times Z$

So we have derive the square root of **y** by employing any of the previous formula of this theorem as per the criteria and conditions and then putting the value in equation 10 we get the final value of the square root of **Z.** Let us study an example to understand it better.

Q4- Find the square root of 0.75

Sol- Here $y = 100 \times 0.75 = 7.5$

Now we can find the square root of 75 using the original formula of case 1 and then put the value in equation-7 to obtain the value of square root of 0.75,

Hence applying the original theorem and equation - 7, we can say that,

M= 75, N=64, S=8. Hence

A= 0.86875.

(Accuracy=99.68%)

Derivation

In developing many scientific experiments the final conclusion are made solely by means of the experimental data collected and not by theoretical arguments. In mathematics one such popular approach in group theory is proof by induction [35] method. Another such method exists, known as proved by construction to show by a set of data that a particular property exists. Joseph Liouville [36] for instance proved the existence of transcendental numbers using this method. This is the method applies to develop this corollary. [37]

Hence our objective is to prove that;

$$\sqrt{Z} = 0.1 \times \sqrt{y}$$

Where y= $100 \times Z$

If Z is the given number whose square root is to be calculated then we can modify equation number 10 by squaring both sides as follows:

0.01Y = Z [Eq. No. - 10.1]

Hence we can say that the square root of Z is equal to the square root of (0.01 x Y) which is exactly what equation 10 is. Hence proved.

Observation

- So from the derivation we can say that the formula for this corollary holds good as per the definition of the theorem and delivers perfect square root values as intended to.
- The efficiency of this corollary is more than 97% as seen from various solved cases.

III. INTERPRETATION ,CONCLUSION AND APPLICATIONS

The aim of this theorem was to develop a simpler and scientific approach to derive the square root of any given number. It is fact of surprise that living in a developed scientific era, our approach to solve square root which is one of the most fundamental problem in algebraic mathematics is by trial and error method. Not only is this tiresome and difficult for students who have to manually solve them during examination but trial and error method is a very barbarian approach and should be avoided in a modern world as much as possible. We should strive to make each of our approach as much scientific as possible. Moreover it can also be designed specifically for computer programs which currently employs Newton-Raphson's or complex algorithm like the Goldschmidt's algorithm to calculate the square root of a input number, we more research and findings the divisibility theorem (Aditya's Method) can be employ directly or indirectly to reduces such complexity in computer programming.

However while working to derive a simpler method to derive the square root function, I have found the concept of quantization in the number theory and how by manipulation of this concept we can solve such an age old problem. As of today its application is restricted in providing a simpler approach to derive square root but in future this innovative approach may be used in some other way to solve even more complicated problems.

As of the success of the aim of this experiment, from the data collected along with the observation made it is pretty clear that the theorem had successfully accomplished its goal. The small variation in the accuracy is due to fact that many higher order decimals are omitted since they are too small and won't affect the result by much and will also make the process much simpler.

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