Optimal Two-Warehouse Inventory level for Deteriorating Items with Variation Time

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Abstract: The proposed model represents the optimal inventory level for different interval time with two-warehouse demand, optimal total cost, special consideration the offering rate is less than demand rate within first time interval and takes opposite that within second time interval with these assumptions the deterioration occurs for deteriorating items, the finite horizon planning, without shortage cost, inventory level is non-zero before the replenishment .Sensitivity analysis for the proposed model was represented the many values lies in range of the deterioration rate, the represented figures explained the performance of optimal inventory level and optimal total cost for required time, the difference between the optimal total cost and actual total cost was proposed.

I. INTRODUCTION

The inventory control system with deterioration rate was represented by researchers as Bassok, Anupindi and Akella (1999) porposed Single period multiproduct inventory models with situation. Bose,Goswami,and Chaudhri (1995) developed an EOQ model for dteriorating items with linear time dependent demand rate and shortages under inflation and time discounting. Das ,Maity and Maiti (2007) formulated the two warehouse supply chain model under possiblity necessary ceridetibility measures. Goh ,Greenberg and Matsuo (1993) studied two-stage perishable inventory models . Haneveld ,Teunter (1992) invetigated the effects of discounting and demand rate variability on the EOQ. Lee and Hsu (2009) porposed model as two warehouse production model for deteriorating inventory items with time dependent demands .Philip (1974) assumed Weibull distribution deterioration to developed generalized EOQ model for deteriorating items. Rong, Mahapatra and Maiti (2008). Sana, Chaudhuri (2008), Shinn, Hwang (1996) developed Jiont price and alot size determintion under conditions of permissible delay in payment and quantity discount for freight cost.In real life deteriorating items required model to determine the optimal size of the inventory level considers the offering and demand rates according to the seasonal consume the items have deterioration when offering rate is more than demand rate and opposite that to minimize the total cost of deteriorating items over the variant times when minimum inventory level is non-zero to satisfied the demand especially the almost stock used advanced software to manage the inventory in stock and alarm the manager of stock when inventory level equal 10% of total quantity for the new replenishment. The proposed model consider the inventory level when the quantity of deteriorating items not equal zero with several values of deterioration rate over the variant times of finite planning horizon for any required time the planned model assumed that two-warehouse inventory and demand rate .

II. MATERIAL AND METHODS

2.2. Assumptions and notions

2.2.1. Assumptions

In this paper the mathematical model is developed with the following assumptions

1) Planning horizon is finite.
2) Replenishment rate is infinite.
3) Single item inventory control.
4) Demand and deterioration rate are constant.
5) The offering rate for items is less than demand rate within $[0,t_1]$.
6) The offering rate for items is more than demand rate within$[t_1, T]$.
7) Deteriorating occurs as soon as the items are received into inventory within [0, T].
8) Shortage is not allowed.
9) The lead time is zero.
10) The inventory level at the end of planning horizon is variation will be non-zero.
11) The total relevant cost consists of fixed ordering, purchasing and holding cost.

2.3. Notation

D_1= The demand rate quantity in period $[0,t_1]$.
D_2= The demand rate quantity in period $[t_1, T]$.
b_1= The first level of inventory in $[0,t_1]$.
b_2= The second level of inventory in $[t_1, T]$.
C= The present value of purchasing cost.
TC_A= The total fixed ordering cost during [0, T].
I_h =The holding cost during [0, T].
TC_{h1}= The total holding cost during $[0,t_1]$.
TC_{h2}= The total holding cost during $[t_1, T]$.
TC_{P1}= The total purchasing cost during $[0,t_1]$.
TC_{P2}= The total purchasing cost during $[t_1, T]$.
TC_1= The total relevant cost during $[0,t_1]$.
TC_2= The total relevant cost during $[t_1, T]$.

2.4. Parameters

T = The length of the finite planning horizon.
$I_1(t)$ = The inventory level at time $[0,t_1]$.
$I_2(t)$ = The inventory level at time $[t_1, T]$.
t_1 = The time at which the inventory level reduce to b_1 .
T= The time at which the inventory level improved to b_2 .
Θ = The constant deteriorating rate units/unit time during [0, T].

III. MATHEMATICAL MODEL

Let $I_1(t)$ is the inventory level at any time t, $0 \le t \le t_1$, lessening inventory level to b_1 due to demand and deterioration rate in keeping with the assumption. The first order differential equation that describes the instantaneous state of $I_1(t)$ over the open interval $[0,t_1]$ is given by.

$$\frac{dI_1(t)}{dt} + \Theta I_1(t) = -D_1, 0 \le t \le t_1,\ 0 \le \Theta \le 1, I_0(t) = e^{-\theta t} \tag{1}$$

Let $I_2(t)$ is the inventory level at any time t, $t_1 \le t \le T$, augment inventory level to b_2 due to demand and deterioration rate in line with the assumption. The first order differential equation that describes the instantaneous state of $I_2(t)$ over the open interval $[t_1, T]$ is specified by.

$$\frac{dI_2(t)}{dt} + \Theta I_2(t) = -D_2, t_1 \le t \le T,\ 0 \le \Theta \le 1, I_0(t_1) = e^{-\theta t_1} \tag{2}$$

$$I_1(t) = I_0(t)[\int_t^{t_1} D_1 e^{\theta u} - b_1 t_1] = \frac{D_1}{\theta}\left(e^{\theta(t_1-t)} - 1\right) - b_1 t_1 e^{\theta t_1}, I_0(t) = e^{\theta t_1} \quad (3)$$

$$I_1(0) = \frac{D_1}{\theta}\left(e^{\theta(t_1-t)} - 1\right) - b_1 t_1 e^{\theta t_1}$$

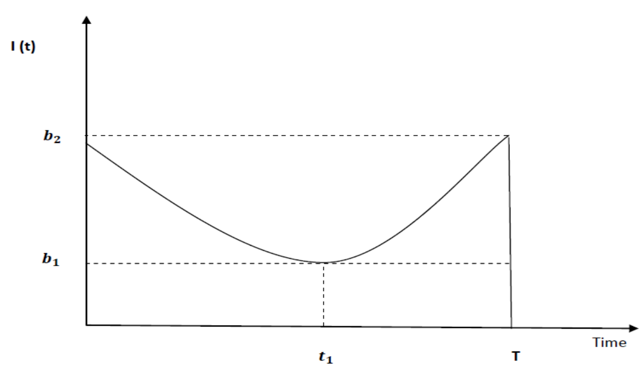

Fig.1 Graphical demonstration of two warehouse inventory control diagram

$$b_1 \approx \frac{D_1}{\theta}\left(e^{\theta(t_1-t)}-1\right), t_1 \approx 0$$

$$\frac{dI_2(t)}{dt}+\Theta I_2(t)=-D_1, t_1 \le t \le T,\ 0 \le \Theta \le 1, I_0(t_1)=e^{-\theta t_1}$$

$$I_2(t)=I_0(t_1)[(\int_{t_1}^{T} D_2 e^{\theta u})-b_1(T-t_1)]=\frac{D_2}{\theta}\left(e^{\theta(T-t_1)}-1\right)\text{-}b_1(T-t_1)e^{\theta t_1} \quad (4)$$

$$I_2(0)=\frac{D_2}{\theta}\left(e^{\theta(T-t_1)}-1\right)-b_1(T-t_1)e^{\theta t_1}$$

$$b_2 \approx \frac{D_2}{\theta}\left(e^{\theta(T-t_1)}-1\right), T \approx t_1$$

3.1. Fixed ordering cost

The fixed ordering cost in the length of finite horizon [0, T]

$$TC_A = A \quad (5)$$

3.2. Purchasing cost

According to fig.1 of inventory level the purchasing cost of

$$TC_{P1}=\frac{CD_1(e^{\theta t_1}-1)}{\theta}-Cb_1t_1e^{\theta t_1} \quad (6)$$

$$TC_{P2}=\frac{CD_2(e^{\theta(T-t_1}-1)}{\theta}-Cb_1(T-t_1)\,e^{\theta t_1} \quad (7)$$

3.3. Holding cost excluding interest cost

We locate the average inventory quantity to obtain holding cost

$$TC_{h1}=I_h\int_0^{t_1} I_1(t)dt=I_h\int_0^{t_1}\left[\frac{D_1}{\Theta}\left(e^{\Theta(t_1-t)}-1\right)-b_1t_1e^{\theta t_1}\right]dt=\ [\frac{I_hD_1}{\theta^2}\left(e^{\theta t_1}-\theta t_1-1\right)-I_hb_1(t_1e^{\theta t_1}-\frac{e^{\theta t_1}}{\theta^2}+\frac{1}{\theta^2})] \quad (8)$$

$$TC_{h2}=I_h\int_{t_1}^{T} I_2(t)dt$$

$$=I_h\int_{t_1}^{T}\left[\frac{D_2}{\Theta}\left(e^{\Theta(T-t_1)}-1\right)-b_1(T-t_1)e^{\theta t_1}\right]dt=\ \frac{I_hD_2}{\theta^2}\left(e^{\Theta(T-t_1)}-\Theta(T-t_1)-1\right)-I_hb_1\left(\frac{T^2}{2}-t_1T+\frac{{t_1}^2}{2}\right)e^{\theta t_1} \quad (9)$$

3.4. Optimal inventory level and optimal time

3.4.1. Optimal inventory level and optimal time [0,t_1]

To optimal inventory level and optimal its time by minimizing the total cost

$TC_1 = TC_A + TC_{h1} + TC_{P1}$ (10)

By subsisting Eq. (5, 6, 8) in Eq. (10)
Then

$$TC_1=A\ +\left[Cb_1-Cb_1t_1e^{\theta t_1}+[\frac{I_hD_1}{\theta^2}\left(e^{\theta t_1}-\theta t_1-1\right)-I_hb_1\left(t_1e^{\theta t_1}-\frac{e^{\theta t_1}}{\theta^2}+\frac{1}{\theta^2}\right)\right]$$

Based on Taylor's series about $t_1 = 0$ then,

$$t_1=\frac{b_1}{D_1}$$

$$TC_1=A\ +\left[Cb_1-\frac{C{b_1}^2}{D_1}(1+\theta\frac{b_1}{D_1})-I_hb_1\left(\frac{b_1}{D_1}\right)^2\right] \quad (11)$$

Deviating Eq. (11) with respect to b_1

$\frac{dTC_1}{db_1} = \left(C - \frac{2Cb_1}{D_1} - \frac{3\theta C{b_1}^2}{{D_1}^2} - \frac{3I_h{b_1}^2}{{D_1}^2}\right)$=0

$${b_1}^* = \frac{D_1}{3}\left[\frac{(C^2+3C(\theta C+I_h))^{\frac{1}{2}}-C}{\theta C+I_h}\right] = q_1 \quad (12)$$

$${t_1}^* = \frac{{b_1}^*}{D_1} \quad (13)$$

3.4. 1.Economic order quantity during $[t_1, T]$

To optimal inventory level and optimal its time by minimizing the total cost

$TC_2 = TC_A + TC_{h2} + TC_{P2}$ (14)

By subsisting Eq. (5, 7, 9) in Eq. (14)
Then

$$TC_2 = A + \left[\frac{CD(e^{\theta(T-t_1)}-1)}{\theta} - Cb_1(T-t_1)e^{\theta t_1} + [\frac{I_hD_2}{\theta^2}\left(e^{\theta(T-t_1)} - \theta(T-t_1)-1\right) - I_hb_1\left(\frac{T^2}{2} - t_1T + \frac{{t_1}^2}{2}\right)e^{\theta t_1}\right] \quad (15)$$

Similarly the Taylor's series for $e^{\theta(T-t_1)} about T = t_1$ then,

$$T = \frac{b_2D_1}{D_2(D_1-b_1)} + \frac{b_1}{D_1}$$

$$TC_2 = A + \left[Cb_2 - Cb_1\left(\frac{b_2D_1}{D_2(D_1-b_1)} + \frac{b_1}{D_1}\right)e^{\theta t_1} + [\frac{I_h}{\theta}\left(b_2 - \frac{D_{1b2}}{D_1-b_1}\right) - I_hb_1(\frac{{b_2}^2{D_1}^2}{2{D_2}^2(D_1-b_1)^2})(1+\frac{\theta b_1}{D_1})\right] \quad (16)$$

Deviating Eq. (16) with respect to b_2

$$\frac{dTC_2}{db_2} = (C - \frac{Cb_1D_1(1+\frac{\theta b_1}{D_1})}{D_2(D_{1-b_1})} + \frac{I_h}{\theta} - \frac{I_hD_1}{\theta(D_1-b_1)} - \frac{I_hb_1b_2{D_1}^2(1+\frac{\theta b_1}{D_1})}{{D_2}^2(D_1-b_1)^2}$$

Here

$${b_2}^* = \frac{{D_2}^2(D_1-{b_1}^*)^2}{I_h{b_1}^*{D_1}^2(1+\frac{\theta {b_1}^*}{D_1})}\left[C - \frac{Cb_1D_1(1+\frac{\theta b_1}{D_1})}{D_2(D_{1-b_1})} + \frac{I_h}{\theta} - \frac{I_hD_1}{\theta(D_{1-b_1})}\right] = q_2 \quad (17)$$

$$T^* = \frac{{b_2}^*D_1}{D_2(D_1-b_1)} + \frac{b_1}{D_2} = l_2 (18)$$

Lemma:

1) (l_1, q_1) is optimal solution intended for TC_1 .

2) (l_2, q_2) is optimal solution designed for TC_2.

Proof:
Since

$$\frac{dTC_1}{db_1} = \left(C - \frac{2Cb_1}{D_1} - \frac{3\theta C{b_1}^2}{{D_1}^2} - \frac{3I_h{b_1}^2}{{D_1}^2}\right) \quad (19)$$

Deviating the Eq. (19) with respect to b_1

$\frac{d^2TC_1}{{db_1}^2}|_{(l_1,q_1)} = -\left(\frac{2C}{D_1} + \frac{6\theta C{b_1}^*}{{D_1}^2} + \frac{6I_h{b_1}^*}{{D_1}^2}\right) < 0, \frac{2C}{D_1} + \frac{6\theta C{b_1}^*}{{D_1}^2} + \frac{6I_h{b_1}^*}{{D_1}^2} > 0$
Lemma (1) is holding.

$$\frac{dTC_2}{db_2} = (C - \frac{Cb_1D_1(1+\frac{\theta b_1}{D_1})}{D_2(D_1-b_1)} + \frac{I_h}{\theta} - \frac{I_hD_1}{\theta(D_1-b_1)} - \frac{I_hb_1b_2D_1^2(1+\frac{\theta b_1}{D_1})}{D_2^2(D_1-b_1)^2}$$

Similarly we found out that

$\frac{d^2TC_2}{db_2^2}|_{(l_2,q_2)} = -\left(\frac{I_hb_1^*D_1^2\left(1+\frac{\theta b_1^*}{D_1}\right)}{D_2^2(D_1-b_1^*)^2}\right) < 0, \frac{I_hb_1^*D_1^2\left(1+\frac{\theta b_1^*}{D_1}\right)}{D_2^2(D_1-b_1^*)^2} > 0$

Lemma (2) is holding.

IV. Sensitivity analysis

The conjecture of the parameters of total cost in for the two inventory models as follows:

Example1

$D_1 = 350$ unite, $I_h = 3\$$ per unite , $C =, 300\$$per unite, $A = 200\$$per unit

Table1. The sensitivity analysis for first interval

θ	b_1^*	t_1^*	TC_1^*	TC_1	Difference
0.005	173.0743	0.494498	26256.37	87056.39	60800.02
0.05	167.7629	0.479322	25711.32	84384.73	58673.4
0.06	166.6667	0.47619	25596.83	83833.33	58236.51
0.07	165.5979	0.473137	25484.54	83295.76	57811.23
0.08	164.5554	0.470158	25374.38	82771.39	57397.01
0.09	163.538	0.467251	25266.26	82259.61	56993.35
0.1	162.5445	0.464413	25160.13	81759.89	56599.77
0.11	161.574	0.46164	25055.9	81271.71	56215.82
0.12	160.6254	0.45893	24953.51	80794.59	55841.08
0.13	159.6979	0.45628	24852.89	80328.06	55475.17
0.14	158.7907	0.453688	24754	79871.72	55117.72
0.15	157.9029	0.451151	24656.77	79425.14	54768.37
0.16	157.0337	0.448668	24561.15	78987.96	54426.82

Example2.

$D_2 = 500$ unite, $I_h = 3\$$ per unite , $C =, 300\$$per unite, $A = 200\$$per unite

Table2. The sensitivity analysis for second interval

θ	b_2^*	T*	TC_2	TC_2^*	Difference
0.005	11544.62	46.17038	5806944	-6207089	*
0.05	13420.59	52.02982	6750558	-31617.4	*
0.06	13822.75	53.25397	6952844	112652.8	6840191
0.07	14219.84	54.45239	7152578	224946.7	6927631
0.08	14611.97	55.62615	7349821	317212.7	7032608
0.09	14999.27	56.77625	7544633	396084.4	7148548
0.1	15381.85	57.90363	7737071	465539.2	7271532
0.11	15759.82	59.00916	7927191	528105.7	7399085
0.12	16133.29	60.09368	8115047	585467.7	7529579
0.13	16502.37	61.15797	8300690	638789.9	7661901
0.14	16867.14	62.20277	8484172	688902.7	7795269
0.15	17227.72	63.22877	8665541	736414.6	7929126
0.16	17584.18	64.23664	8844844	781781.3	8063062

IV. RESULTS AND DISCUSSION

The output of planned model within two class intervals founded out that, according fig.2 when the offering rate less than demand rate the first inventory level was decreasing when deterioration rate increased, fig.3 illustrate that the deterioration rate was decreasing when first optimal time increased this is support the possibility in real life with deteriorating items, fig.4 exemplify the gap between actual total and optimal costs was high that made the proposed model is applicable deterioration rate increased while the total was decreasing. According fig.5 the optimal second inventory level was increasing when the deterioration rate increased also to achieved that offering rate is more than demand rate under minimizing total cost related to second time interval, fig.6 showed when second optimal time increased the deterioration rate is increased, fig.7 exemplify the gap between actual total and optimal costs was high that made the proposed model is applicable to achieve optimal total cost under any deterioration rate lies [0.06,1] because at second time the planned model is not fitted when low risk as deterioration risk for item that makes the suggested model is more significance with high risk.

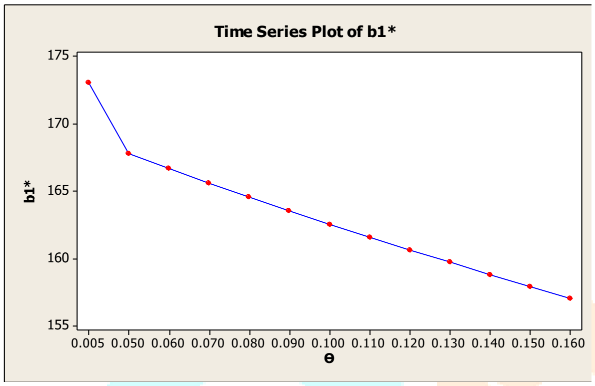

Fig2. Graph depiction optimal first inventory level VS deterioration rate

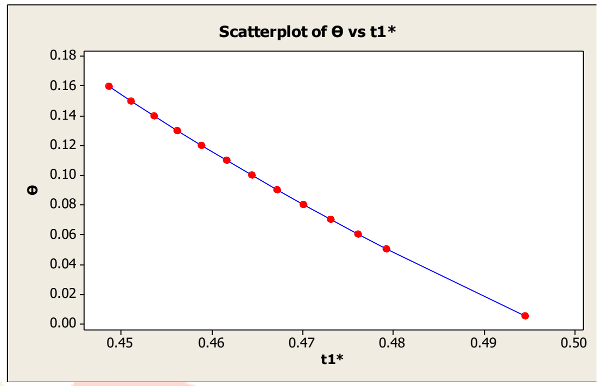

Fig3. Graph illustration optimal first time VS deterioration rate

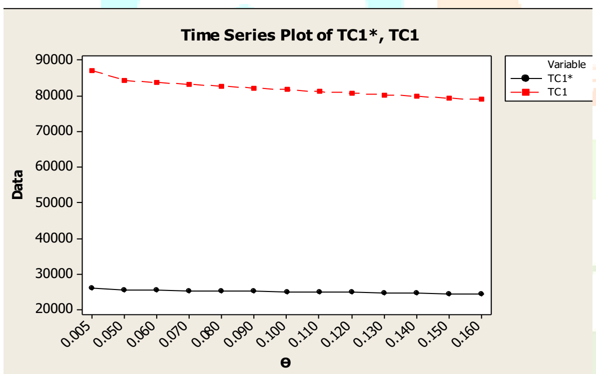

Fig4. Graph sign the optimal total and actual costs VS deterioration rate within first time

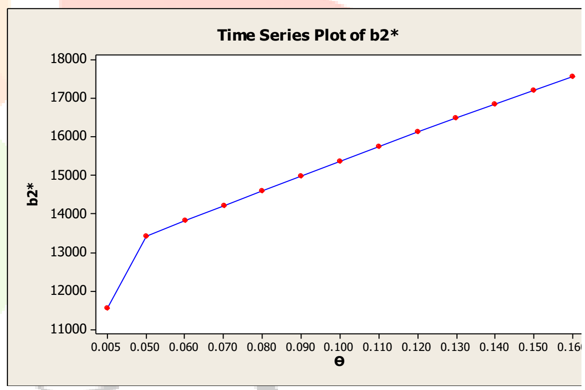

Fig5. Graph representation optimal second inventory level VS deterioration rate within second time

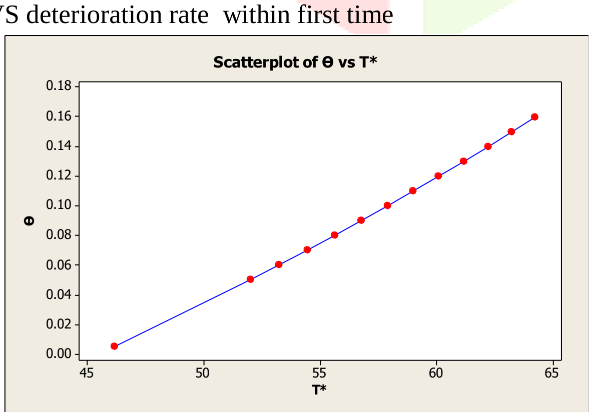

Fig6. Graph representation optimal second time optimal second time

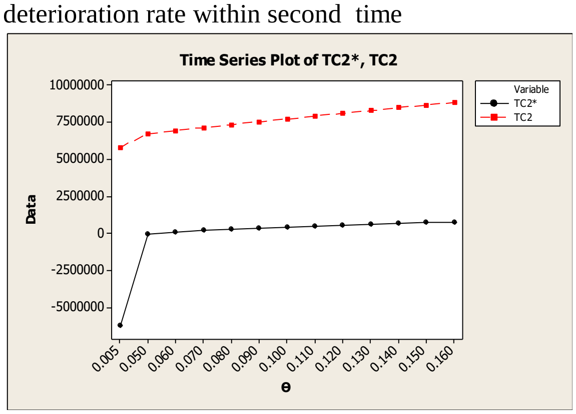

Fig7. Graph representation optimal total and actual total costs VS deterioration rate

V. ACKNOWLEDGMENTS

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