AN EXPERIMENTAL INVESTIGATION OF CANTILEVER BEAM USING IMPULSE MODAL ANALYSIS TECHNIQUE

[1]Dr. Sushila Rani
[1]Assistant Professor
[1] Mechanical Production, Industrial and Automobile Engineering Department,
[1] Delhi Technological University, New Delhi, India

Abstract: In order to understand structural dynamics of failed structures in real time, the modal and harmonic analysis are used to obtain the dynamic characteristics of structures. Impulse modal analysis technique is an experimental technique used to derive the modal model of a linear time-invariant vibratory system. The theoretical basis of the technique is establishing a relationship between the vibration response at one location and excitation at the same or another location as a function of excitation frequency. This relation is known as frequency response function (FRF). A frequency response function (FRF) captures the unique dynamic characteristics of the structure between two degrees of freedom (DOFs); the response DOF and the excitation DOF. The frequency response functions (FRFs) of structures are obtained as output results by applying the system input artificially through some type of exciter, i.e. either impact hammer or magnetic shaker (*Richardson et al., 1990, 1993)*. An impact test generally results in measuring one of the rows of the frequency response function matrix whereas the shaker test generally results in measuring one of the columns of the frequency response function matrix.

In this research work a methodology Impulse modal analysis technique (IMAT) is discussed to determine the modal parameter i.e. natural frequencies of structures. A case study of cantilever beam analyzed with vibration analyzer OROS to determine its natural frequencies through Impulse modal analysis technique.

Key words: Impulse modal analysis technique, Cantilever beam, Frequency response function (FRF).

I. INTRODUCTION

Impulse modal analysis technique has become the most popular modal analysis method with FFT analyzer to compute FRF measurements directly. IMAT technique is a fast, convenient, and low cost method for evaluating modal parameters of structures. For conducting impulse response tests, an accelerometer, a force hammer and a data acquisition system/ analyzer are needed. The most important step in modal analysis is to decide the location or locations for stationary transducer. Generally, during impact testing the response transducer is the stationary one and the impact force is applied at suitable locations on the structure to define mode shapes. The analyzer consists of analog to digital converters and a system for performing a discrete finite fourier transformation. To compute FRFs, subsequent averaging and data manipulation has been carried out. The basic steps to obtain FRFs from vibration analyzer are shown in Fig. 1. The frequency response functions (FRFs) of structures are obtained as output results by applying the system input artificially through impact hammer (*Schwarz et al., 1999*).

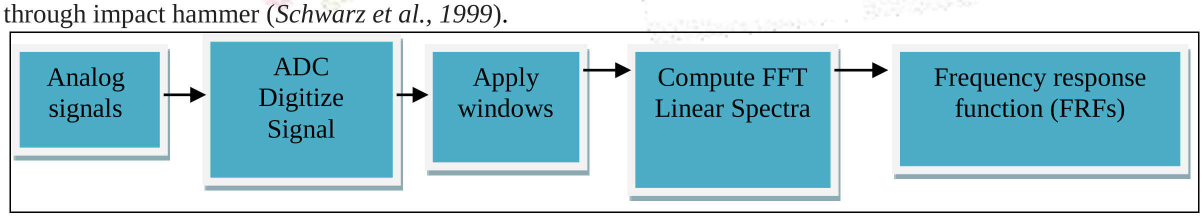

Fig. 1: Basic steps for measuring Frequency Response Functions (FRF's)

Yinming et al. (2004) analysed a cantilever beam by experimental modal analysis. They have compared controlled and uncontrolled impulse responses at the free end of the beam in time domain and frequency domain. They have found that this proposed procedure can be used for solving complex structures problems. *Boudjemai et al.(2012)* analysed a hexagonal honeycomb plate through impact hammer test. The experimental set up consisted of two accelerometers; these are placed on the top and on the core of beam to measure the bending and lateral modes respectively. The modal parameters are evaluated through resultant FRFs. Khan and *Parhi et al. (2013)* analysed a double cracked cantilever beam through experimental modal analysis. They have found that as crack depth increases, higher modes are excited. *Singh and Nanda (2013)* analysed a layered and tack- welded mild steel cantilever beam through experimental modal analysis. Frequency response functions are collected as output results and modal parameters of the structure are evaluated from resultant FRFs through vibration analyzer software.

This research work discusses one of the most frequently used techniques for experimental structural frequency response testing, which is based upon the excitation of structure with an impulsive force applied through an impact hammer and finally, vibration response is recorded through accelerometer. These signals are then fed into analyzer to compute frequency response functions (FRFs). As a case study, a cantilever beam has been analyzed to evaluate its natural/ resonant frequencies.

II. FORMULATION OF GOVERNING DIFFERENTIAL EQUATION FOR FREQUENCY RESPONSE OF TURBINE BLADE

Mathematically, the modes of vibration are defined by certain parameters of a linear dynamic model of a blade. The dynamic properties of a blade can be written as a set of differential equations in the time domain, as presented in Eq. (5.1). This equation can also be represented as a set of equations containing transfer functions in the Laplace (frequency) domain as given in Eq. (5.2)

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{f(t)\} \tag{1}$$

Where [M],[C] and [K] are the mass, damping and stiffness matrices respectively along with the corresponding acceleration $\{\ddot{x}\}$ and the external force $\{f(t)\}$ applied to the system.

By taking Lapalace transform of Eq.(5.1) , one may write the equation as presented-

$$\left[[M]s^2+[C]s+[K]\right]\{X(s)\}=\{F(s)\}$$

$$[B(s)]\{X(s)\}=\{F(s)\} \tag{2}$$

The frequency response function is defined as the system transfer function along the frequency axis, and is defined as the inverse of the system matrix

$$[B(s)]^{-1}=[H(s)]=\frac{Adj[B(s)]}{\det[B(s)]}=\frac{[A(s)]}{\det[B(s)]} \tag{3}$$

In partial fraction form, frequency response function is written as

$$[H(s)]_{s=j\omega}=[H(j\omega)]=\sum_{k=1}^{m}\frac{[A_k]}{(j\omega-p_k)}+\frac{[A_k^*]}{(j\omega-p_k^*)} \tag{4}$$

Thus, transfer function is

$$h_{ij}(j\omega)=\sum_{k=1}^{m}\frac{a_{ijk}}{(j\omega-p_k)}+\frac{a_{ijk}^*}{(j\omega-p_k^*)} \tag{5}$$

Where, $P_k=-\sigma_k+j\omega_d=K^{th}$ pole

$a_{ijk}=$ residue for K^{th} pole

σ_k=Damping co-efficient,

ω_d = Damped natural frequency

ω = Undamped natural frequency

III. CASE STUDY: IMPULSE MODAL ANALYSIS OF CANTILEVER BEAM

The experimental modal analysis of a cantilever beam has been carried out using vibration analyzer *OROS®*. The boundary condition of the beam (cantilever beam) is obtained by having one end of beam fully built-in using a C-clamp. The experimental set up for experimental modal analysis is shown in Fig. 2. It consists of a mild steel cantilever beam, a *PCB-78534* accelerometer, a C-clamp, a *PCB-086C03* impact hammer and a vibration analyzer *OROS®*. Firstly, hammer tip (plastic), inputs (force and acceleration), the frequency range (*0-5 KHz*), triggering (start delay: *10ms*), windows (force: hamming and acceleration: response), sampling rate (*51.2kilo samples / sec*) and the number of FFT lines *(401)* are selected. The accelerometer is placed at the top of free end of cantilever beam as shown in Fig. 3.

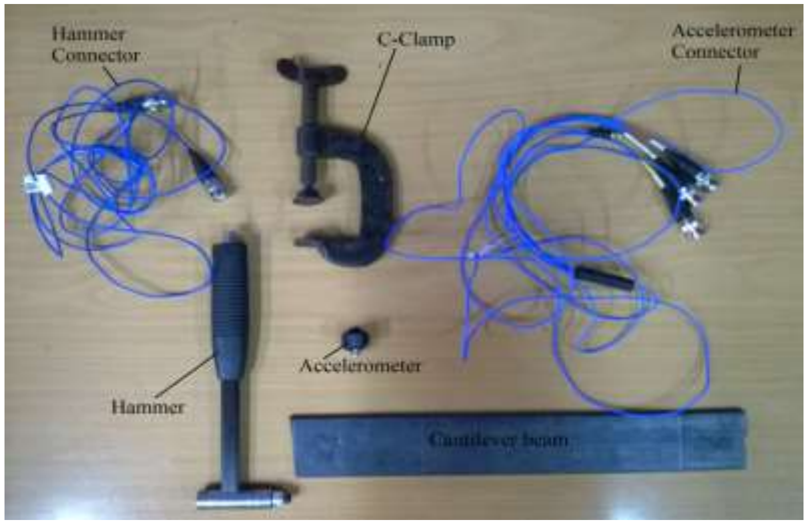

Fig. 1: Components used for modal analysis of cantilever beam

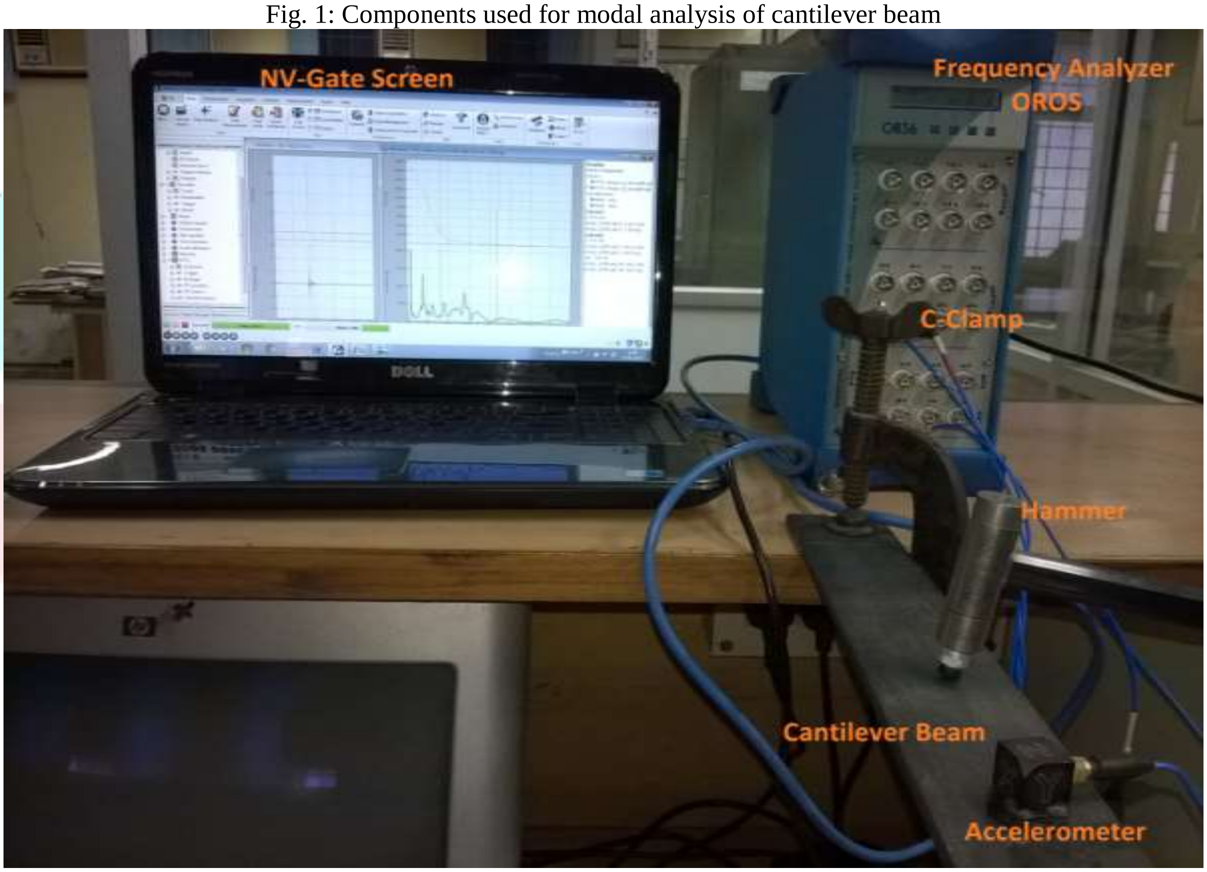

Fig. 2: Experimental set up for modal analysis of cantilever beam

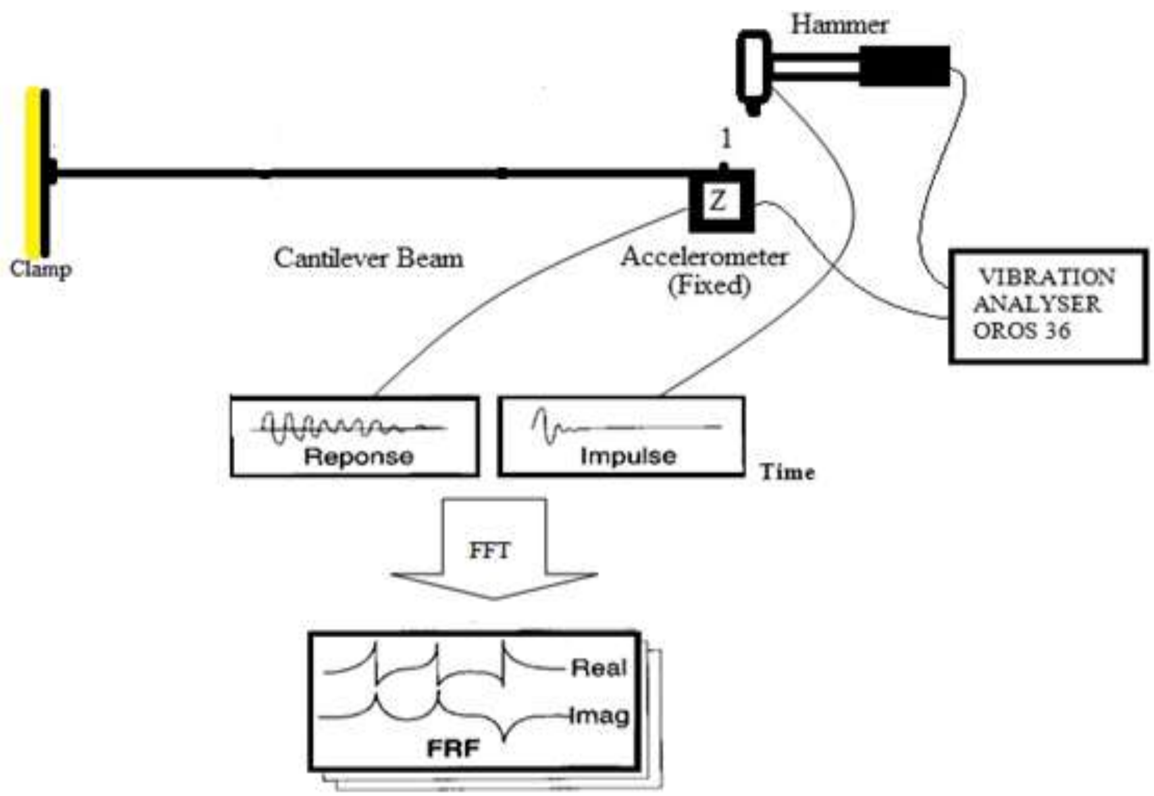

Fig. 3: Impulse modal analysis technique (IMAT)

IV. RESULT AND DISCUSSION

(i) Recorded signal

The input signal is recorded for 10 seconds. A recorded signal obtained by hitting impact hammer at measured location 1 is shown in Fig. 4. The trigger is set at start delay of *-10ms*. Triggered block of signals at measurement location1 is shown in Fig. 5.

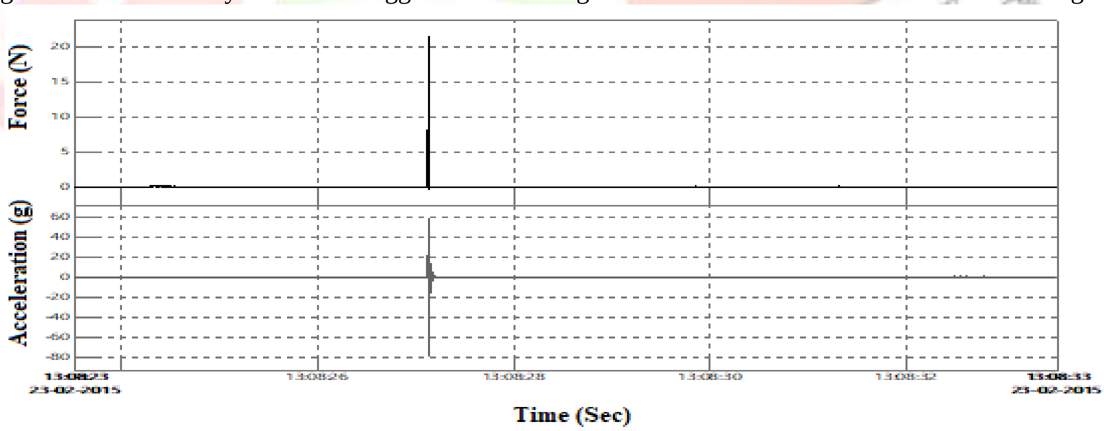

Fig. 5: Recorded signal measured at location 1

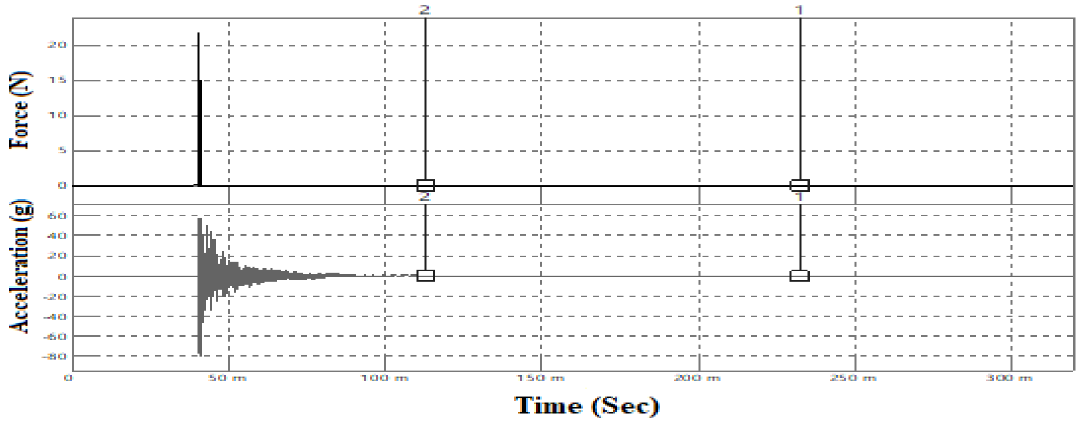

Fig. 5: Triggered signal measured at location 1

(ii) Fast fourier transformation (FFT)

The signals are fed into analyzer to compute frequency response function (FRF). The analyzer consists of analog to digital converters and a system for performing a discrete finite fourier transformation. The finite fourier transformation (FFT) of recorded signal is shown in Fig. 6. The force Vs frequency plots present the amplitude of force, which will be decaying with time. Peak values of acceleration Vs frequency plots are correspond to natural/resonant frequencies of cantilever beam. The four dominant resonance peaks appear clearly in FRF at frequencies around *49 Hz*, *302 Hz*, *467 Hz* and *850 Hz*. Therefore, the first four natural/ resonant frequencies of cantilever beam are *49 Hz*, *302 Hz*, *467 Hz* and *850 Hz*.

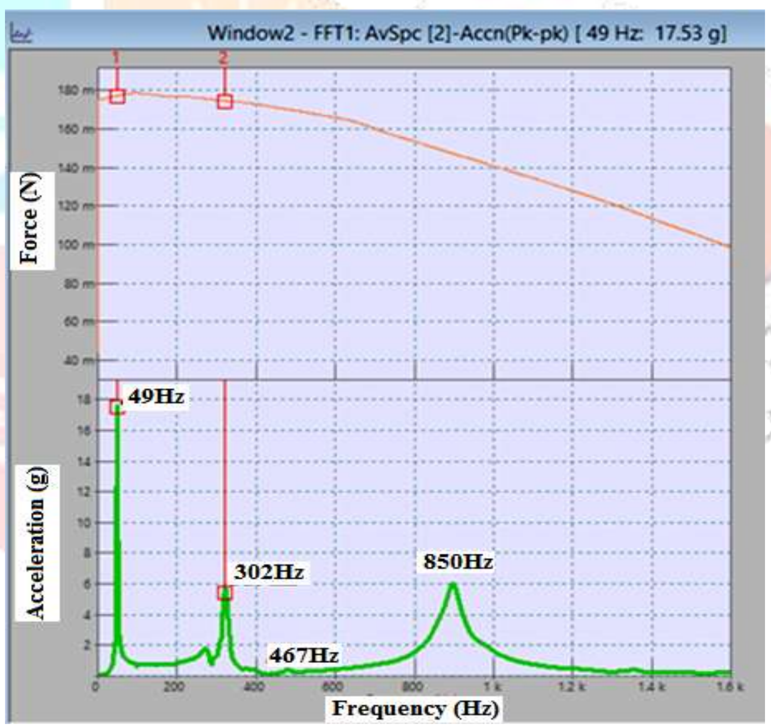

Fig. 4: FFT measured at location 1

(iii) Frequency response function (FRF)

The frequency response function (FRF) of cantilever beam is obtained as output results by impacting the beam by force hammer.The frequency response function (FRF) at measured location 1 is shown in Fig. 7. The frequency response function (FRF) plot is the representation of the ratio of output to the input signal Vs frequency. Fig. 7 present the plots of (Acceleration/ Force) Vs frequency and phase change Vs frequency. "Peak" values seen in frequency response function plots are corresponding to natural frequencies of the cantilever beam. It is clearly evident from Fig. 7 that the response is changing its sign at resonant frequencies. The frequency response functions (FRFs) plots are further used to calculate modal damping and mode shape of beam.

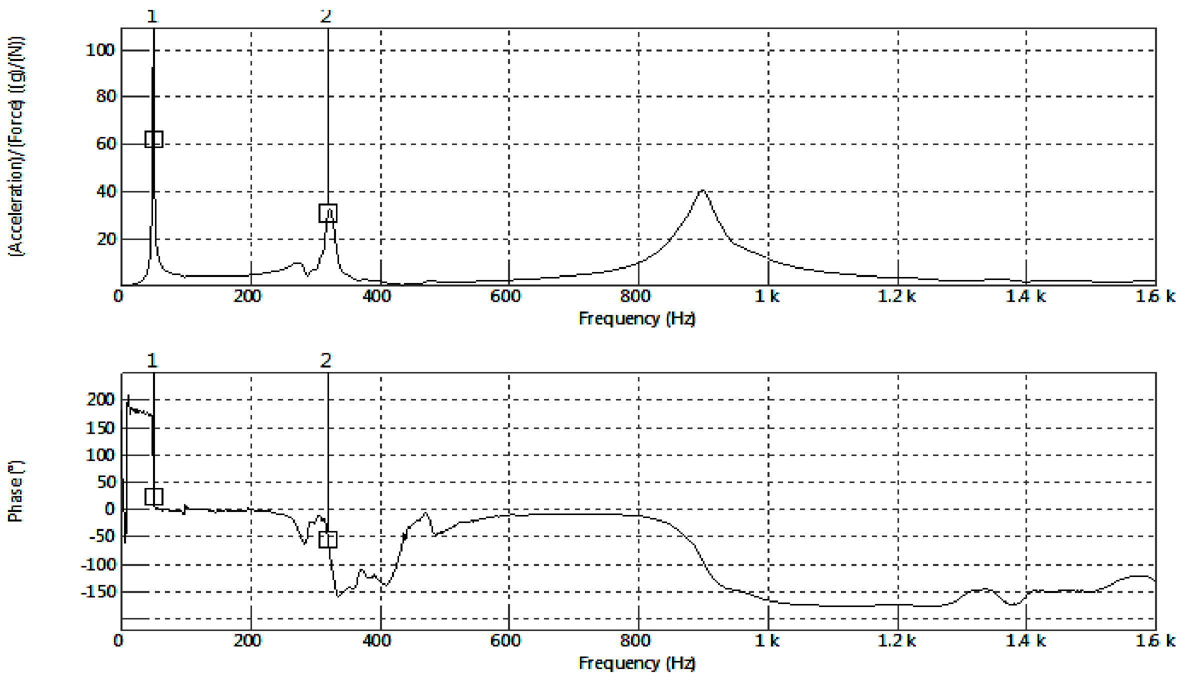

Fig. 7: Frequency response function measured at location 1

V. CONCLUSIONS

Impulse modal analysis technique has been discussed and applied on cantilever beam to evaluate its natural frequencies. The following conclusions are drawn:

- Impulse modal analysis technique is a fast, convenient, and low cost method for evaluating modal parameters of structures.
- Impulse modal analysis technique has been applied to analyze a cantilever beam. In the resultant Frequency response function (FRF), the four dominant resonance peaks appear clearly at frequencies around *49 Hz, 302 Hz, 467 Hz* and *850 Hz*. Therefore, the first four natural/ resonant frequencies of cantilever beam are *49 Hz, 302 Hz, 467 Hz* and *850 Hz*.
- Knowledge of modal parameters is helpful in damage detection in various structures and hence used as an assessment tool for structure.

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