Study of Motion of the System in the Central Gravitational Field of Force for the Elliptical Orbit

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Abstract: ***In this article, it is tried to study the motion of the system in the central gravitational field of force when the centre of mass of the system in moving along elliptical orbit. The equation of the relative motion of the system in the central gravitational field of the force is derived in polar form for the elliptical orbit of the centre of mass. It is obtained in an approximate form of non-linear oscillations.***

Keywords: Center of Mass, Gravitational Field, Elliptic Orbit, Dumb-Bell Satellites, Polar Equations, and Differential Equations, etc.

1. INTRODUCTION

A system in the space is always subjected to perturbations and hence even an infinitesimal perturbation can change its circular motion to elliptical one. This very idea inspired us to take up the problem for the case of elliptical orbit of the centre of mass. To avoid certain inherent complexities of the elliptical motion of the centre of mass. We have restricted our consideration to the two dimensional motion of the problem. That is why we have assumed that the two particles are moving in the orbital plane of the centre of mass of the system. The relative motion of the system with respect to its centre of mass has been obtained in linearised and normalized from using the Nechvill's transformation.

II. MATHEMATICAL DISCUSSION:

We have the set of differential equations characterizing the motion of particle in the elegant from:

$$\begin{aligned} x'' - 2y'' - 3x\rho &= \overline{\lambda_a}\, \rho^x x \\ y'' + 2x' &= \overline{\lambda_a}\, \rho^4 y \\ z'' + z &= \overline{\lambda_a} z \qquad \ldots\ldots\ldots \qquad (1) \\ \text{Where, } \overline{\lambda_a} &= \frac{p^3}{m}\overline{\lambda_a} \end{aligned}$$

From the equation [1] the two dimensional motion of one of the particle in Nechvill's coordinate system when the centre of mass moves along the Keplarian elliptical orbit, are governed by the equations.

$$\begin{aligned} x'' - 2y'' - 3x\rho &= \overline{\lambda_a}\, \rho^4\, x \\ y'' + 2x' &= \overline{\lambda_a}\, \rho^4\, y \qquad \ldots\ldots\ldots\ldots\ldots \qquad (2) \end{aligned}$$

Here, x-axis is taken along the radius vector joining the centre of the mass and attracting centre and y-axis along the direction perpendicular to the radius vector in the orbital plane of the centre of mass and the condition for the constraint is given by the inequality.

$$x^2 + y^2 \le \frac{1}{\rho^2} \qquad \ldots\ldots\ldots\ldots \qquad (3)$$

Where, $\rho, e, \upsilon,\ and\ \overline{\lambda_a}$ have their usual meaning as,

$$\begin{aligned} \rho &= \text{parameter(radius vector)} \\ e &= \text{eccentricity} \\ \upsilon &= \text{true anamoly} \\ \overline{\lambda_a} &= \text{constraint} \end{aligned}$$

and (') denote the differentiation with respect to v. If motion of one of the satellites be determined with the help of above equations, then it is easy to determine the motion of the other satellite from the identity.

$$m_1\, \overrightarrow{\rho_1} + m_2\, \overrightarrow{\rho_2} = 0$$

Where $\overrightarrow{\rho_1}$ and $\overrightarrow{\rho_2}$ are radius vectors of the particles of the mass m_1 and m_2 respectively with respect to centre of the mass of the system. Obviously the actual motion of the system will be the combination of three types of motions.

(a) Free motion i.e. , $\lambda_a = 0$
(b) Constrained motion i.e. , $\lambda_a \neq 0$
(c) Mixed motion i.e. combination of free and constrained motion.

We are, here interested in the constrained motion because the free motion are bound to be converted into constrained motion with the lapse of time. In case of the constrained motion the quality sign bolds in the equation [3]. That is the particle is moving along the circle of variable radius given by.

$$x^2 + y^2 \leq \frac{1}{\rho^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (4)$$

Let us transform the equation of motion to the polar form by the substitution:

$$x = (1 + e\cos\upsilon)\cos\psi\text{, we get}$$

$$x = (1 + e\cos\upsilon)\cos\psi \qquad \ldots\ldots$$

$$y = (1 + e\cos\upsilon)\sin\psi \qquad \ldots\ldots\ldots \qquad (5)$$

Where ψ is the angle which the line joining the centre of mass with attracting centre makes with the X-axis and Y is the true anomaly of the orbit of the centre of mass,
Differentiating the two relations given by [5] with respect to v, we get

$$x' = \frac{\psi \sin\psi}{\rho} - e\sin\psi\ \cos\psi$$

$$x'' = \frac{\psi''\sin\psi}{\rho} - \frac{(\psi')^2\cos\psi}{\rho} + 2e\psi'\ \sin\upsilon\sin\psi - e\cos\upsilon\ \cos\psi$$

$$y' = \frac{\psi\cos\psi}{\rho} - e\sin\upsilon\ \sin\psi$$

$$y'' = \frac{\psi''\cos\psi}{\rho} - \frac{(\psi')^2\ \sin\psi}{\rho} + 2e\psi'\sin\upsilon.\cos\psi - e\cos\upsilon\ \sin\psi$$

$$\text{where, } \rho = \frac{1}{1 + e\cos\upsilon}$$

$$\text{and} \quad \rho' = \rho^2 \, e \cos \upsilon$$

Substituting the values of the derivations obtained above in the equation [2], we get;

$$-\frac{1}{\rho}\psi'' \sin\psi - \frac{1}{\rho}(\psi')^2 \cos\psi + 2e\psi' \sin\upsilon . \sin\psi - e\cos\upsilon \cos\psi - \frac{2}{\rho}\psi' \cos\psi$$

$$-2e \sin\upsilon \sin\psi - 3\cos\psi = \overline{\lambda_a}\, \rho^3 \cos\psi \qquad \dots\dots\dots\dots \qquad (6)$$

Again , we can write,

$$-\frac{1}{\rho}\psi'' \sin\psi - \frac{1}{\rho}(\psi')^2 \sin\psi + 2e\psi' \sin\upsilon \cos\psi - e\cos\upsilon \sin\psi - \frac{2}{\rho}\psi' \sin\psi - 2e\sin\upsilon \sin\psi - 3\sin\psi = \overline{\lambda_a}\, \rho^3 \sin\psi \qquad \dots\dots\dots \qquad (7)$$

Multiplying [6] by $\sin y$, **and [7] by** $\cos y$ **and subtracting, we get;**

$$(1 + e\cos\upsilon)\psi'' - 2e\psi' \, 2e \sin\upsilon + 3\sin\psi \cos\psi = 2e\sin\upsilon \qquad \dots\dots\dots \qquad (8)$$

Again multiplying [6] by $\cos y$ **and [7] by** $\sin y$ **and adding these together, we get;**

$$(1 + e\cos\upsilon)\left(\psi'^2 + 2\psi'\right) + \cos^2\psi + e\cos\upsilon = \overline{\lambda_a}\rho^3 \qquad \dots\dots\dots \qquad (9)$$

The equation [9] determines undetermined Lagrange's multiplier $\overline{\lambda_a}$, **the mechanical implications of undetermined Lagrange's multiplier's is the motion will be constrained so long** $\lambda(t) \geq 0$ *i.e.*, $\overline{\lambda_a}(t \leq 0)$
As soon as the condition is violated the particle will start moving within the circle [4]. Thus, clearly the motion will be constrained and will be determined by the equation [7], so long the inequality,

$$(1 + e\cos\upsilon)\left(\psi' + 1\right)^2 + 3\cos^2\psi - 1 \geq 0 \qquad \dots\dots \qquad (10)$$

is satisfied. It shows that there exists a vast region of phase space (ψ and ψ') in which this condition is satisfied. We are, therefore, interested in analyzing the behavior of the system described by [08] assuming that the inequality [10] is satisfied. We are, therefore, interested in analyzing the behavior of the system described by (8), assuming that the inequality (10) is satisfied. The equation [08] is nothing but the equation of dumb-bell satellites in the central gravitational field of force and is in a suitable form of a non-linear oscillation of the system about its equilibrium position.

III. CONCLUSION

The present research work deals with polar form of equations of motion of the system the central gravitational field of force for the elliptical orbit of the centre of mass. In this article it is studied the motion of the system in the central gravitational field of force when the centre of mass of the system in moving along elliptical orbit.
We found that the equation of dumb- bell satellite in the central gravitational field of force and is in a suitable form of a non-linear oscillator. This describes the non- linear oscillation of the system about its equilibrium position. This research work is in progress. It is hoped that the research may be very useful in the future applications in satellites.

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